When crowdfunding meets bank financing: Substitute or supplement?[1]

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Abstract

Crowdfunding is a new financing method that is widely utilized by startup firms to substitute or complement bank loans. We consider the optimal financing problem for a profit-maximizing startup firm, given that both the crowdfunding demand and retail market demand are uncertain and correlated. We find four types of financing strategies: single-source (crowdfunding only) and dual-source (crowdfunding and bank loan) financing, with prudent or aggressive goal-setting in crowdfunding. The four strategies each can be optimal for a specific parameter range with respect to the uncertainty ratio of crowdfunding and retail markets and the cost of crowdfunding. Upon deriving optimal strategies, we analyze comparative statics for the strategies and optimal profits. We show that, among other interesting results, market expansion (through either crowdfunding or the retail market) always calls for a higher crowdfunding goal but may raise or reduce the optimal bank loan depending on which market expands, and market expansion generally increases optimal profit except when the crowdfunding market grows too large which intensifies the overfinancing risk. Moreover, a high correlation between crowdfunding and the retail market does not necessarily favor the use of crowdfunding capital, nor does it always increase profits, and the impacts must be co-examined with the underfinancing and overfinancing risks.

Keywords: crowdfunding, capital constraint, capital supply uncertainty, demand uncertainty

1. Introduction

Spurred by developments in information technology, a financing innovation known as crowdfunding is being increasingly deployed by a diverse range of entrepreneurs, which is significantly different from the typical way investors would participate in financial markets (Agrawal et al. 2015, Cornelius and Gokpinar 2020, Kim and Viswanathan 2019, Wei et al. 2020, Xiao et al. 2021). Though partially functioning as a financing source, backers of a successful crowdfunding project (i.e., the investors) do not claim any right on the cash-flows of the firm,[2] but will receive the product that they pledged for after the campaign succeeds and proceeds to the production stage. This is the so-called reward-based crowdfunding, The reward to backers, therefore, depends on their expected utility from the delivered product. This novel form of investment makes financing through crowdfunding a different story from other well-known conventional financing channels. Compared with the traditional financing paradigm that separates financing markets

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[2]We focus on reward-based crowdfunding which is popular on the platforms of Kickstarter, Indiegogo, etc. Though equity and debt crowdfunding also have wide applications, they are outside the scope of this paper.

from the product market environment, investment financing via crowdfunding is one that needs to be integrated with the product market side (Chemla and Tinn 2020, Chen et al. 2022).

Not only do small startups utilize crowdfunding for their novel projects, but the fact that crowdfunding allows firms to finance directly from their potential customers as a way to test and reveal the market demand (Chemla and Tinn 2020) also makes it appeal to established companies. For instance, Sony, one of the world's largest electronics manufacturers, created a crowdfunding platform called First Flight to raise funds for and obtain market feedback on its internal projects (i.e., projects developed by Sony employees). There are also plenty of startup successes on crowdfunding platforms. Founded in 2016, the startup e-bike company Fiido utilized crowdfunding multiple times and in the most recent campaign, raised over $1.5M for its new model Fiido X on the Indiegogo platform in 2021.

The increasingly wide application of crowdfunding brings an imperative to analyze its dual role in alleviating financing constraints and revealing market feedback, which is related to the financial-operational management of the firm (Kumar et al. 2020). Specifically, this dual role of crowdfunding calls for incorporating crowdfunding as part of the *financing market* where it synergizes (and competes) with traditional capital sources such as bank loans, and as part of the *product market* where it raising awareness and boosts sales of the product before the latter formally enters the retail market. Each role of crowdfunding has been examined separately in past studies–for the former, crowdfunding has been contrasted with bank or supply chain financing (Chen et al. 2022, Xu et al. 2023) and analyzed along with venture capital (Babich et al. 2021), while for the latter, crowdfunding's market learning value has been explicitly discussed (Chemla and Tinn 2020) and so does the profit-maximizing campaign/menu design problem (Ellman and Hurkens 2019, Hu et al. 2015, Liu et al. 2021)–but not analyzed together, especially with the potential correlation between the crowdfunding and retail markets. Therefore, the main research question this paper seeks to answer is, how crowdfunding's dual role as described above affects the entrepreneur's optimal financing strategy. Specifically, how should a profit-maximizing entrepreneur design its financing strategy in terms of the crowdfunding goal and the amount of bank loan to obtain (given accessibility), when sales come from both the crowdfunding market and the post-campaign retail market, with the two markets potentially correlated? In a reduced form, will crowdfunding supplement, or completely substitute the traditional bank capital?

At the starting point, we consider a firm that has access to both traditional bank loans and crowdfunding. Access to bank loans is not a problem for big firms who may also participate in the crowdfunding market, such as Sony and its crowdfunding platform First Flight. It is difficult for small startups to get large amounts of business loans due to a lack of credit or collateral, but small amounts of personal debt are usually accessible. In fact, personal debt (including personal credit card and bank loans) constitutes the second most important capital source besides the owner's equity funds at the startup stage for a business (see Table 2 in Cotei and Farhat 2017). Moreover, most crowdfunding projects have a moderate monetary goal that is comparable to small personal loans as opposed to large business loans.[3] Among the two capital sources, crowdfunding is usually considered a cheaper alternative to bank loans with an average platform fee of 5% (on both Indiegogo and Kickstarter). In comparison, unsecured bank loans (i.e., charged for walk-in customers having no other banking relationship with the lending institution and lacking collateral) can have an average interest rate as high as 16.06% (Kahn et al. 2005), while credit

[3]According to data provided on webrobots.io, for 22,731 Indiegogo projects which are active in October 2023, have USD as their currency and are not in InDemand status (i.e., continuing to raise funds after the deadline), the average goal and money raised are $6,474 and $5,280. The data is available from the authors upon request.

card debts have even higher APRs.[4]

Although crowdfunding has a cost advantage over bank loans, crowdfunding capital supply is uncertain as the amount of funds attainable depends not only on the monetary goal set by the entrepreneur but also on the actual crowdfunding demand of the product. More importantly, as we also consider potential sales in the post-campaign retail market (such as Amazon, company website, or Indiegogo InDemand) where many entrepreneurs continue to sell their products beyond the crowdfunding stage, the crowdfunding market partially reveals the retail market's demand for the novel product (Chemla and Tinn 2020). So the problem analyzed here involves a single product in a single production run, where the firm, faced with capital supply uncertainty (from crowdfunding) and demand uncertainty (from the retail market) which are potentially correlated, needs to decide its optimal financing strategy.

A two-stage model is developed. In the first stage, both the crowdfunding capital supply and the retail market demand are unknown except for their respective distributions, while the firm decides the crowdfunding goal and the amount of bank loan to obtain. In the second stage, the crowd funds and bank loans are obtained, the retail market demand is revealed and the firm maximizes profits over the production quantity, subject to the capital constraints determined by its first-stage financing decision. The trade-off between the two capital sources is: that bank loans are predictable but relatively expensive, while crowdfunding is cheaper but uncertain.

An important part of the financing strategy is the crowdfunding goal-setting problem, with uncertainties from both the capital supply and the retail demand. In our model, we derive two possible goal-setting strategies: *aggressive goal setting*, where the firm sets the goal at the maximal possible value of crowdfunding capital (in a Uniform distribution), and *prudent goal setting*, where the goal is set less than that maximal possible value.[5] Note that whether the goal setting is aggressive or prudent does not depend on the absolute magnitude of the goal, but on the relative magnitude of the goal to the maximal possible crowdfunding capital supply for the focal product—so it is campaign-specific. In practice, Indiegogo recommends their users to be prudent in goals by setting "the smallest amount you need to complete your project and fulfill your perks", but sometimes it could also be optimal for the entrepreneur to set an aggressive goal to exhaust the potential of crowdfunding capital supply.[6] With an uncertain retail market demand, the risks here are *overfinancing*, namely financing more capital than what production needs and thereby paying unnecessary financing costs, and *underfinancing*, namely financing less capital than what production needs and thereby unable to satisfy the total market demand. Notice that such risks originate from the uncertainty of retail market demand because crowdfunding is always self-sufficient in production for its platform demand as long as the product is sold at a premium over the production cost. The correlation between the crowdfunding capital supply and retail market demand considered in this paper serves as a double-edged sword in alleviating these financing risks, depending on the premium $p-c$,

[4] On top of the bank interest rate and crowdfunding platform fees, banks may charge other transaction fees (e.g., origination fee, late fee) while crowdfunding platforms may also charge a processing fee (3% on both Indiegogo and Kickstarter). Adding up all costs, the bank loan is in general more expensive than crowdfunding for startup businesses.

[5] As will be clarified in the modeling section, we only consider the Keep-It-All mechanism in our paper, so setting the goal at the maximal value does not necessarily lead to campaign failure.

[6] As an example of aggressive goal setting, Smosh started an Indiegogo campaign in 2013 for a video game "Food Battle: The Game" setting the goal at $250,000 (a number on the higher end in terms of goals compared to average campaigns), soon to close as one of the biggest successes on Indiegogo in that year. The goal almost exhausted its crowdfunding potential as it was fulfilled by 3,937 backers with $258,517 raised in the end. Indiegogo encourages prudent goals, by meanwhile assisting the entrepreneur to raise funds after the campaign ends through InDemand (resembling a retail stage). It is easy to discover on the Indiegogo InDemand category that many projects doubled, or even tripled their goal with the InDemand status, indicating that the initial goal is a prudent one compared to the actual platform demand.

as high correlation plus high premium can lead to overfinancing while high correlation plus low premium can lead to underfinancing, which are moderated by the bank loan. Therefore, despite of crowdfunding's low cost, it is not always prioritized—sometimes a prudent crowdfunding goal plus bank loans is optimal.

This paper has the following two contributions to the crowdfunding literature. First, we consider not only the funding phase but also the post-campaign retail market when analyzing the firm's financing strategy, with the two markets' correlation explicitly modeled. Second, we extend the focus of crowdfunding literature by studying its interaction with the more traditional financial source (i.e., bank capital). We derive four kinds of strategies, single/dual-source financing with prudent/aggressive crowdfunding, each optimal in a parameter space depending on the relative bank&crowdfunding costs and the relative magnitude of retail&crowdfunding markets' uncertainties. For the latter, we derive a magnitude-ratio threshold under (above) which crowdfunding substitutes (supplements) bank loans, and we analyze how the correlation between these two uncertain markets affects the optimal financing strategy, in addition to some important comparative statics.

Our results show that market expansion through either crowdfunding or the retail market always calls for a higher crowdfunding goal but may raise or reduce the optimal bank loan depending on which market expands (with some exceptions). Market expansion generally increases optimal profit, except when the crowdfunding market grows too large, which intensifies the overfinancing risk, so it is sometimes the case that the optimal profit increases in crowdfunding market size first and then decreases. A higher correlation between the crowdfunding and retail market does not always favor the use of crowdfunding capital, nor does it always increase the optimal profit. Depending on the profit margin and retail market demand, a high correlation may intensify underfinancing or overfinancing risks, call for a lower crowdfunding goal, and lead to a decrease in the optimal profit. So the correlation is a double-edged sword and must be co-examined with the financing risks. In addition, using a counterfactual deterministic-demand model, we show that it is the demand uncertainties that give rise to the optimality of SA and DP strategies, and that make f an important determinant for the optimality of each financing strategy as well as for crowdfunding to be prioritized when the retail market size grows in its relative size.

The remaining paper is organized as follows. In Section 2, we review the related literature. In Section 3 we construct the theoretical model and in Section 4, we solve the model, analyze the optimal financing strategy, and conduct some comparative statics. In Section 5, we consider how uncertainty affects the optimal financing strategy from two different perspectives. Lastly we conclude in Section 6. All proofs are available in the Appendix.

2. Literature Review

Crowdfunding is a recent phenomenon that enables small and start-up businesses to seize the benefits of the sharing economy to address financial concerns, and it has generated significant interest in both empirical and theoretical literature. The prior empirical literature on crowdfunding mostly focuses on identifying factors that affect the crowdfunding campaigns' performances, such as the campaign-embedded signals of project quality and e-word of mouth (Bi et al. 2017), early investor's experience (Kim et al. 2020), social capital (Colombo et al. 2015, Zhang et al. 2022), entrepreneurial legitimacy (Frydrych et al. 2014), customer value propositions (Ma et al. 2022), effects of prefunding (Wei et al. 2020), etc. Another strand of empirical works focuses on the investors' behaviors in crowdfunding, such as geographical difference and social network effects (Agrawal et al. 2015), contributors' strategic information conceal (Burtch et al. 2016), the involvement of customers in product development (Cornelius and Gokpinar 2020), leading investor's adverse incentives (Hildebrand et al. 2017), herding and information disclosure (Tian et al. 2021, Xiao et al. 2021), etc.

As we focus on the use of crowdfunding as a financing tool in the presence of market uncertainties, our paper is more closely related to the theoretical literature of crowdfunding in which operation management and demand uncertainties are addressed. An abundance of theoretical papers in this strand has explored the selling, financing, and market testing functions of crowdfunding.

Viewing crowdfunding as an independent market of financing and selling, the crowdfunding menu design (for target and price) and product line analysis (in terms of product qualities), in comparison with traditional selling mode, are investigated by Hu et al. (2015) and Liu et al. (2021). Belleflamme et al. (2014) compare two different modes of crowdfunding, pre-ordering (as in reward-based crowdfunding) and profit-sharing (as in equity crowdfunding), with potential quality uncertainty and information asymmetry, and solve their optimality to a profit-maximizing entrepreneur. Considering demand uncertainty and moral hazard, Strausz (2017) utilizes mechanism design to show that an information-restricted, payout-deferred, all-or-nothing reward-based crowdfunding scheme can deal with moral hazard effectively, by motivating the entrepreneur to make investments and engage in the market instead of diverting the funds otherwise.

Viewing crowdfunding as an independent financing tool with a market testing function, Ellman and Hurkens (2019) set out to characterize the profit- and welfare-maximizing production mechanism and show that reward-based crowdfunding can implement both optimal mechanisms if allowed to set multiple prices, thus locating crowdfunding's key benefit in its market test role of adapting production to demand when compared with traditional selling. Chemla and Tinn (2020) model crowdfunding's market testing function more explicitly, by assuming that crowdfunding demand is a random sample drawn from the uncertain aggregate market demand, thus providing learning values to the firm. They analogize this learning value of crowdfunding to a real option that informs the firm's investment decision, which also helps the firm to overcome moral hazard.

This paper investigates the dual role of crowdfunding as both a financing tool that can supplement or substitute traditional (bank) financing, and a selling platform that can be correlated with the post-campaign retail market. In other words, crowdfunding mediates the traditional financing and retail markets by playing the dual role of raising capital and selling products.

The interaction of crowdfunding and traditional VC&bank financing has been investigated by (Babich et al. 2021), which instead focused on the beneficial effect of capital market competition on the entrepreneur and the VC investor (with noncontractible efforts and moral hazard in the agency problems), rather than the effect of crowdfunding's market testing function. To highlight the importance of institution investors, (Babich et al. 2021) also assume that crowdfunding cannot fund the project without the help of external funds, thus serving only as a supplemental tool, and it sometimes harms the entrepreneur and the VC investor, who may walk away from the deal entirely. But in reality, many small startups kick-started their business with the help of crowdfunding alone (e.g., some indie game/art production studios) and some even ended as blockbuster successes (e.g., the Indiegogo eBike campaign MATE X raised an astonishing 17M USD in its original campaign)—in these cases, crowdfunding may entirely substitute traditional financing, at least in its first-round production.

Chen et al. (2022), Xu et al. (2023), Kumar et al. (2020) further analyzed the entrepreneur's optimal financing problem with both crowdfunding and traditional financing, shedding light from different angles. Chen et al. (2022) studied a supply chain problem where the manufacturer may get funded from the supplier, the bank, or crowdfunding, and focused on analyzing the optimal component ready timing and wholesale pricing decisions under different financing strategies. Xu et al. (2023) and Kumar et al. (2020) both considered financing through crowdfunding and selling in the spot (retail) market via price discrimination, the former focusing on joint price and quality optimization with different financing modes, and the latter on how crowdfunding alleviates financing constraints and enables the execution of projects that could not be otherwise undertaken when only traditional financing is available. In these papers, crowdfunding and spot market are considered as a single (potentially uncertain) aggregate market, in a manner that eludes crowdfunding's market testing function, i.e., the possibility that the two markets may be correlated or drawn from the same distribution as in Chemla and Tinn (2020). Moreover, traditional bank financing is often treated as a separate or supplemental capital that is conveniently (and passively) available to obtain any insufficient funds, with its interaction with demand uncertainties abstracted away.

To fully explore how uncertainty affects the entrepreneur's optimal financing strategy, we assume that the crowdfunding market and retail market are two correlated markets, the latter's distribution conditional on the former's realization, and the entrepreneur needs to decide its financing strategy as a pair of the crowdfunding goal and the bank loan amount, before entering the production stage when both markets' demands are revealed. We novelly derive four kinds of financing strategies and emphasize the effects of capital costs and demand uncertainty in determining each strategy's optimality. Our paper highlights the double-edged role of crowdfunding's market testing function which may lead to a mismatch of financed capital and capital needed in production and intensify underfinancing or overfinancing risks.

3. The Model

We consider a startup firm with a recently developed product that needs to finance its production from two possible capital sources: bank and crowdfunding. Most crowdfunding platforms require the firm to set the funding goal, duration, reward prices, and to provide descriptions of the product before launching the campaign. Focusing on the firm's financing strategy, we reduce the campaign design problem by assuming that the campaign duration and product price p are exogenous,[7] and price discrimination is not considered, [8] and thus the focal decision of the firm is to set its funding goal F_c. After the campaign starts, buyers arrive and if interested, pledge the price p in anticipation of receiving the product as a reward upon the campaign's success.[9] Production always happens if the firm gets fully funded, thus moral hazard is not considered.

The demand for the product consists of two parts, the crowdfunding demand and the post-campaign retail demand. In practice, crowdfunding is usually the debut of a new product upon the success of which the firm may bring the product to traditional online markets such as Amazon or the crowdfunding's internal market such as Indiegogo InDemand.[10] However, both the crowdfunding and post-campaign retail market demands are uncertain, thus presenting a major financing difficulty to the firm.

Formally, we consider a two-stage optimization problem where the firm makes financing and production decisions to maximize profit. In the first stage (the financing stage), the crowdfunding demand and post-campaign retail market demand are unobservable and the firm needs to make its financing decisions in

[7] To reduce analytical complexity and focus on the firm's financing strategies, we eschew market competition and treat the price as exogenously determined in the market, for instance, by the product's close substitutes. Also, since the buyer's valuation and the market demand for the new product are by nature unknown to the firm in the launching phase, the entrepreneur may elect to set a price that ensures a regular return rate for products in the same category.

[8] We analyze a context where the transition from the crowdfunding phase to the retail phase is rather fast, so the firm does not adjust the price of the product. In fact, Indiegogo allows their campaign launchers to enter the "retail phase" immediately after the campaign closes with Indiegogo InDemand, where the reward price is not adjusted.

[9] In the All-Or-Nothing mechanism, buyers will get a full refund if the campaign fails. As will be specified later, this paper studies the Keep-It-All mechanism and thus the campaign always succeeds.

[10] On Kickstarter and Indiegogo, it is a common practice for successful campaigns to post hyperlinks on the original campaign website which guide interested buyers to some other online purchasing site.

advance, i.e., to determine the crowdfunding goal F_c and the bank loan F_b before the production stage starts.[11] In the second stage (the production stage), both demands are revealed and the firm makes the production decision under capital constraints. In the following, we first elaborate on the two capital sources, then specify two key uncertainties intimate to the optimization problem, and construct the model with more rigorousness in the last subsection.

3.1. Two capital sources

We assume the firm has access to two capital sources, traditional bank loans and crowdfunding capital. The firm can get bank loans at an exogenously negotiated interest rate. The firm, whether an established firm or a small startup, is guaranteed access to the bank loan, as the latter can get a moderate amount of personal loans even without collateral (see Introduction for more justifications).

As for reward-based crowdfunding, it resembles pre-ordering (Belleflamme et al. 2014) where the campaign backers play a dual role of buyer and capital provider. As its name indicates, reward-based crowdfunding does not pay the backers any interests (unlike debt-based crowdfunding) or promise a share of profits once the business takes off (unlike equity-based crowdfunding), the only reward to the backers is the product itself. The cost of crowdfunding is the platform fee charged to successful campaigns, usually 5% of the funds raised plus processing fees, which will be deducted in a lump sum before being transferred to the fundraiser's bank account. As mentioned in the Introduction, the capital cost of crowdfunding is generally considered to be much lower than the bank loan, especially for highly risky startups.

Besides its cost advantage, the crowdfunding market can also correlate with the retail market since crowdfunding backers can be viewed as part of the market demand. Because the profit margin of crowdfunding can be used in production to satisfy retail market demand, crowdfunding naturally matches capital raised to capital needed in production when the available crowdfunding capital correlates with the market demand. The firm does the best if it finances the exact needed amount of capital for production, which is however impossible with demand uncertainties. It is therefore likely that the firm may raise too much capital in crowdfunding if the post-campaign retail market demand turns out to be low, or if the price margin $p - c$ is too high so that not all crowdfunding profit margins can be utilized in production to satisfy retail market demand, despite paying platform fees for all these crowd-raised funds (i.e., overfinancing). And vice versa, when the production is constrained by capital thus hurting profitability (i.e., underfinancing). Some may argue that the entrepreneur should raise as much crowdfunding capital as possible to seize all potential buyers in the crowdfunding market. However, we assume that the crowdfunding buyers who valued the product but missed to pledge in the crowdfunding stage (out of various reasons such as strategic delay) can and will purchase in the post-campaign retail market. Crowdfunding platforms have made this easy for both the buyer and the entrepreneur by allowing the latter to guide remaining buyers to new purchase websites through hyperlinks. Therefore, it is sometimes optimal for the entrepreneur to encourage some crowdfunding buyers to delay their purchase to the retail market (by setting a low crowdfunding goal) so that these purchases are not subject to platform fees.

In a nutshell, although crowdfunding capital is cheaper than the bank loan, the buyer-investor dual role and its potential correlation with the post-campaign retail market can be a double-edged sword that helps to match capital raised to capital needed, but may also cause overfinancing or underfinancing issues due to market uncertainties. The following table contains all the notations used in this paper.

[11] In reality, chances are that seeking financing opportunities (especially applying for a bank loan) can take a rather long time, so the firm has to arrange for financing before it gets to learn the market demand (which can also take time, for instance, by conducting market surveys) and enters the production stage.

Table 1: Notations

Parameter	Description	Category
F_c	Crowdfunding campaign goal.	Decision variable
F_b	Bank loan.	Decision variable
f	Crowdfunding service fee rate (%).	Parameter
r	Bank loan interest rate (%).	Parameter
c	Marginal cost of production.	Parameter
p	Product price per unit.	Parameter
$\tilde{S}_c$	Crowdfunding market demand, $\tilde{S}_c \sim U[0, S_c]$, with realization s_c.	Random variable
$\tilde{D}$	post-campaign retail market demand, $\tilde{D}\|s_c \sim U[ks_c, (1-k)D+ks_c]$, with realization d.	Random variable
k	A proxy of $\tilde{S}_c$ and $\tilde{D}$'s correlation.	Parameter
ρ	Magnitude ratio, $\rho = \frac{D}{S_c}$.	Parameter

3.2. Uncertainties in crowdfunding and post-campaign retail market

We have divided the market demand into the crowdfunding buyers and the post-campaign retail buyers. There are also two types of uncertainty associated with the two markets.

First, the crowdfunding demand (also crowdfunding capital supply due to the buyer-investor dual role of crowdfunding backers) is uncertain. Indeed, it depends on many campaign-specific and unpredictable factors such as product quality, marketing scheme, platform traffic, the entrepreneur's social network, etc. (Agrawal et al. 2015). To model the uncertainty, we use random variable $\tilde{S}_c$ to denote the crowdfunding demand. Ignoring donation and price discrimination, then given product price p, the crowdfunding capital supply is $p\tilde{S}_c$. Note that the capital supply equals crowdfunding demand times price, which reflects the dual role played by crowdfunding backers. Therefore, the crowdfunding uncertainty is also two-fold: crowdfunding demand uncertainty refers to the randomness in $\tilde{S}_c$ and crowdfunding capital supply uncertainty (or simply capital supply uncertainty) refers to the randomness in $p\tilde{S}_c$. Due to such uncertainties, the crowdfunding goal setting F_c is a nontrivial problem as the firm aims to finance the exact amount of capital needed for production.

The other uncertainty is the post-campaign retail market demand, which is also unknown to the firm in the financing stage. Let random variable $\tilde{D}$ denote the post-campaign retail market demand, so the total market demand equals $\tilde{S}_c + \tilde{D}$. The two markets can be highly related because the crowdfunding demand, which is realized earlier in the funding stage, is representative of the aggregate market and may confer information about the post-campaign retail market demand. And to model this, we allow the two markets' demand to be correlated.[12] Formally, we assume that the crowdfunding demand $\tilde{S}_c$ follows a uniform distribution on the interval $[0, S_c]$, with a typical realization value denoted by s_c. To model the correlation, we assume that the post-campaign retail demand $\tilde{D}$, with a typical realization value denoted by d, follows a conditional distribution uniform on the interval $[ks_c, (1-k)D+ks_c]$ given some realization

[12] For example, if the campaign has a high crowdfunding demand, with fast word-of-mouth sharing/marketing on the Internet, the crowdfunding buyers can help to promote the product through the social networks, and the product's information can thus reach more potential buyers and the post-campaign retail demand will grow.

s_c, i.e., $\tilde{D}|s_c \sim U[ks_c, (1-k)D + ks_c]$, where $0 \le k \le 1$.[13] Such modeling is consistent with Chemla and Tinn (2020) that the demand revealed in the crowdfunding stage delivers information about the product's post-campaign retail demand as a way to help the firm update its belief of the uncertain market demand.[14] Intuitively, k is a proxy of $\tilde{S}_c$ and $\tilde{D}$'s correlation (Lemma 3 in Section 5 shows that the two variables' correlation increases in k) and thus reflects how the crowdfunding and post-campaign retail demands are positively related. When $k = 0$, $\tilde{D}|s_c \sim U[0, D]$ where the parameter D represents the part of retail demand that is independent of the crowdfunding demand; in this case, the crowdfunding and retail demands are independent. When $k = 1$, $\tilde{D}|s_c = s_c$, thus the post-campaign retail demand equals the realized crowdfunding demand. In addition, the demand $\tilde{S}_c$ and $\tilde{D}$ represent the number of buyers whose valuation is greater than p, so they always purchase the product either through crowdfunding or in retail market, and crowdfunding buyers in particular can postpone their purchase to the retail stage if the goal is already achieved. In the following analysis, we refer to capital supply uncertainty as the randomness of $p\tilde{S}_c$ and demand uncertainty as the randomness of $\tilde{S}_c + \tilde{D}$.

3.3. Model construction

As mentioned at the beginning of this section, we study the following two-stage model.

Stage 1 (financing stage). Knowing only the distributions of $\tilde{D}$ and $\tilde{S}_c$, the firm sets its crowdfunding goal F_c and the amount of bank loan to obtain F_b.

Stage 2 (production stage). Both demands are realized and observable. The firm maximizes profit by choosing production quantity subject to capital constraints determined by the first-stage decision.

With the random crowdfunding capital supply $p\tilde{S}_c$ and funding goal F_c, the actual fund received by the firm depends on the chosen fundraising mechanism. In this paper, we consider a less aggressive mechanism called Keep-It-All (aka Flexible Goal on the Indiegogo platform). As the name suggests, the firm keeps all pledged funds no matter whether the funding goal is achieved or not.[15] In the Keep-It-All mechanism, the campaign goal plays a slightly different role than what it does in All-or-Nothing: since the firm can always keep all funds, the goal does not affect whether the firm gets funded or not, but affects how much the firm gets funded. Indeed, crowdfunding data suggests that most successful crowdfunding campaigns end up collecting exactly the goal amount (Mollick 2014, p.6). In other words, despite the existence of blockbuster successes, the over-achieving outcome is not commonly observed in crowdfunding due to a lot of reasons. For instance, Strausz (2017) shows that potential buyers have the incentive to elect a conditional pledging strategy as way to prevent moral hazard from the entrepreneur—they only pledge when the goal is not achieved, stop pledging as soon as the goal is achieved and in such cases postpone their purchases to the post-campaign retail market. This conditional pledging strategy ensures that the entrepreneur learns only that demand is high enough to make the project profitable, but not by how much, which effectively reduces the risk of moral hazard. Therefore, in Keep-In-All the pre-specified goal may serve to set an upper limit to how much the firm can receive from backers. Along this logic, we assume that if the crowdfunding goal is lower than the available capital, $F_c \le p\tilde{S}_c$, buyers stop pledging

[13]The distribution $\tilde{D}|s_c$ is designed both to reflect correlation and to assure tractability. In trial and error, we found that tractability of the model requires the length of $\tilde{D}$'s conditional uniform distribution, i.e., $(1-k)D$, to be independent of s_c.

[14]The major difference between this paper and Chemla and Tinn (2020) is that the relation between crowdfunding and post-campaign retail demand is modeled by two correlated random variables in this paper, but is treated as drawn from the same unknown demand distribution in Chemla and Tinn (2020).

[15]Another popular crowdfunding mechanism is All-Or-Nothing (AON): the firm only receives all funds if the funds exceed the pre-specified goal, nothing otherwise. AON is a riskier fundraising mechanism from the firm's perspective than Keep-It-All (KIA). While AON is the popular mechanism on Kickstarter, KIA is the popular mechanism on Indiegogo.

once the goal is achieved and the firm receives F_c from crowdfunding. The remaining buyers will make their purchase in the retail stage (i.e., a conditional pledging strategy as in Strausz 2017). If the goal is greater than the available capital, $F_c > p\tilde{S}_c$, all buyers pledge and the firm receives $p\tilde{S}_c$. Such model setups are both practically relevant and theoretically tractable.[16] Therefore, the raised capital through crowdfunding equals $\min\{F_c, p\tilde{S}_c\}$, and the total capital including bank loans equals $\min\{F_c, p\tilde{S}_c\} + F_b$.

The uncertainties entailed in the crowdfunding capital supply $p\tilde{S}_c$ and the post-campaign retail market demand $\tilde{D}$ bring in two types of risks concerning the optimal financing strategy, one related to capital sufficiency and the other related to goal setting. First, as the total demand is uncertain, the firm may either finance too much capital (thus paying extra capital costs) or too little capital (thus failing to capture the entire demand-induced profits). We use the terms *overfinancing* and *underfinancing* to address such situations: overfinancing means the total raised capital exceeds the total needed capital, or $\min\{F_c, p\tilde{S}_c\} + F_b > c(\tilde{D} + \tilde{S}_c)$, while underfinancing means the total raised capital falls short of the total needed capital, or $\min\{F_c, p\tilde{S}_c\} + F_b < c(\tilde{D} + \tilde{S}_c)$. Notice that both overfinancing and underfinancing are harmful to the firm's profitability, so both can be termed as risks. Second, as the crowdfunding capital supply is uncertain, the firm may set its financing goal either too low (underachieving) or too high (overstretching) due to an inaccurate estimation of the crowdfunding capital volume. Specifically, *underachieving goal setting* is the risk of setting a goal too low (which constrains the maximal amount of capital that can be raised from crowdfunding) such that the cheaper crowd funds remain less than fully utilized. Notice the underachieving goal-setting risk is exacerbated by high underfinancing risk but is alleviated by high overfinancing risk. *Overstretching goal setting* is the risk of setting a goal so high that the firm may raise more crowd funds than what is needed, leading to wasteful funds and over-payment of fees. Notice that overstretching goal-setting risk is exacerbated by high overfinancing risk but is alleviated by high underfinancing risk. The two types of risks are inter-related and affect the financing decision in two ways: the financing risk dictates how much capital in total needs to be financed, while the goal-setting risk determines the substitution between crowd funds and bank loans—crowd funds are cheaper but unstable while bank loan is expensive but consistent.

For the financing costs, let r denote the bank's loan interest rate for the firm, and f the platform fee rate of crowdfunding, $0 < f < r$. We assume the product is non-trivially profitable so that its profit margin ratio is greater than the fee rates, i.e. $f < r < \frac{p-c}{c}$, or otherwise production is never profitable.

At the second stage, both the crowdfunding demand $\tilde{S}_c$ and the post-campaign retail market demand $\tilde{D}$ are revealed to the firm as s_c and d, and the available capital from crowdfunding (i.e., $\min\{F_c, ps_c\}$) and F_b have also been determined. The total production cost to satisfy both crowdfunding and post-campaign retail demands is $c(d + s_c)$. The firm chooses the production quantity Q to maximize profit:

$$\max_Q \Pi(F_c, F_b) = (p-c)Q - f\min\{F_c, ps_c\} - rF_b, \tag{1}$$

$$\text{s.t.} \begin{cases} cQ \leq \min\{F_c, ps_c\} + F_b, \\ \\ Q \leq d + s_c. \end{cases}$$

In the objective function, $(p-c)Q$ is the net sales profit, and $f\min\{F_c, ps_c\}$ and rF_b are the financing costs for crowdfunding and bank loans, respectively. There are two constraints. The first constraint is the financial constraint, i.e., the production cost cannot exceed the total available capital. The second constraint is the demand constraint, i.e., the firm cannot sell more than the revealed aggregate demand.

[16] In fact, the alternative All-Or-Nothing mechanism does not have a closed-form solution in our setting.

After solving the optimal production quantity Q^* in the second stage, the first-stage problem for the firm is to maximize expected profits by choosing (F_b, F_c):

$$\max_{(F_b,F_c)} \quad (p-c)E[Q^*] - fE[\min\{F_c, p\tilde{S}_c\}] - rF_b.$$

We want to give several clarifications before moving to the model analysis.

First, although we are ignoring production fixed costs in the model, adding fixed costs does not qualitatively change the optimization problem. Specifically, because we are considering the Keep-It-All mechanism without moral hazard, the entrepreneur always receives funds and will commit to production in the second stage. In other words, we consider the case where the entrepreneur cannot run away with funds (i.e., moral hazard) or privately determine to refund crowdfunding backers in a way that violates the platform's rule. In such cases, the inclusion of fixed costs is trivial because it does not qualitatively change the entrepreneur's financing strategy (as fixed costs are just a constant subtracted from the objective function and the financing constraint),[17] but instead affects whether the entrepreneur enters the market in the first place—if the fixed costs are so high that the expected profit is negative, the entrepreneur will abandon the project even before the financing stage, simply because the project is ex-ante not profitable.

Second, the model focuses on a specific project with exogenous capital supply and demand. Tacit in the assumptions is that the crowdfunding capital supply $p\tilde{S}_c$ and retail demand $\tilde{D}$ will be determined by the firm's status, the project quality, and the entrepreneur's social capital (Bi et al. 2017, Agrawal et al. 2015). High firm reputation, good crowdfunding campaign design, high quality, and good networking may result in an upward movement of the random variables $\tilde{S}_c$ and $\tilde{D}$ (such as higher S_c and D), which will affect the financing strategies as well as the optimal profit. We will conduct comparative-static analyses concerning S_c and D in the following sections.

Lastly, notice that the two second-stage constraints are affected by different factors. The demand constraint, $Q \le d + s_c$, is affected by the product's exogenous demands (e.g., firm status) and their realization under market randomness. The financial constraint, $cQ \le \min\{F_c, ps_c\} + F_b$, is instead affected by the firm's first-stage financing decisions. Although there is a time lag between crowdfunding and retail selling, we assume that the transition is rather fast (see Footnote 11) in the sense that the entrepreneur cannot contract with the bank to receive additional loans after observing the market demand.

4. Model Analysis

4.1. The Second-Stage Analysis for Production Decision

We now use backward induction to solve the proposed two-stage problem. In the second stage, the total production quantity Q is constrained in (1) by the available capital (the first constraint) and the market demand (the second constraint). Given F_c, F_b, the revealed s_c and d fixed, the objective function monotonically increases in Q. So the optimal production quantity is entirely determined by the two constraints: $Q^* = \min\{d + s_c, \frac{\min\{F_c, ps_c\}+F_b}{c}\}$.

Recall the capital raised in crowdfunding is $\min\{F_c, ps_c\}$, so the revealed demand in crowdfunding is $\frac{\min\{F_c, ps_c\}}{p}$, implying the production quantity for the post-campaign retail market is $Q^* - \frac{\min\{F_c, ps_c\}}{p}$.

[17] In particular, if we assume the fixed costs are $C > 0$, then the financial constraint becomes $cQ + C \le \min\{F_c, ps_c\} + F_b$ or $cQ \le \min\{F_c, ps_c\} + F_b - C$, and the objective function can be rewritten as $(p-c)Q - f\min\{F_c, ps_c\} - r(F_b - C) - (1+r)C$. Then the optimization problem is the same if we replace F_b by $F_b - C$, while the constant $(1+r)C$ does not affect the optimal solution (Q^*, F_c^*, F_b^*). Therefore, adding fixed costs does not qualitatively change our results beyond slight modifications of the thresholds derived for F_b.

To see that crowdfunding backers are both buyers and capital providers, notice that producing for the crowdfunding demand costs c times the revealed crowdfunding demand, $\frac{c}{p}\min\{F_c, ps_c\}$, so that the rest of the crowdfunding capital and the bank loan, $\frac{p-c}{p}\min\{F_c, ps_c\} + F_b$, can be used to produce for the post-campaign retail market. Now that we have solved the optimal production quantity Q^*, in the following subsection, we analyze the firm's optimal financing strategy in the first stage.

4.2. The First-Stage Analysis for Financing Decision

In the first stage, both crowdfunding and post-market demand, $\tilde{S}_c$ and $\tilde{D}$, are uncertain, and the optimal production quantity Q^* is solved above as a function of the realized values s_c and d. The firm makes its financing decision (F_c, F_b) facing the capital supply uncertainty and demand uncertainty. Plugging Q^* into the first-stage profit maximization, the firm's objective function becomes:

$$\max_{F_c \geq 0, F_b \geq 0} (p-c)E[\min\{\tilde{D} + \tilde{S}_c, \frac{\min\{F_c, p\tilde{S}_c\} + F_b}{c}\}] - fE[\min\{F_c, p\tilde{S}_c\}] - rF_b. \tag{2}$$

The firm needs to choose its financing strategy cautiously to balance the risk of overfinancing, thereby paying unnecessary financing costs for capital that cannot be put into production, and the risk of underfinancing, thereby being unable to realize the profitability of the total market demand. Two major questions regarding the firm's financing strategy are: at what level the firm should set its crowdfunding goal, and whether the firm should resort to the more expensive bank loan for a higher capital allowance. It turns out that four alternative strategies can be optimal depending on the parameter values. We first give an overview of the four alternative strategies.

Single-source financing strategies via crowdfunding only: SA and SP. Single source means the firm sets a positive crowdfunding goal without getting loans from the bank. The intuition for using crowdfunding only lies in the fact that the financing cost of crowdfunding is always lower than bank capital (i.e. $f < r$), so when crowdfunding capital is likely to be able to cover the production cost (especially when the post-market demand is low), financing through crowdfunding only is the optimal strategy. Under the single-source financing category, there are two alternative strategies corresponding to two different goal-setting schemes depending on how expensive the crowdfunding capital is: i) *single-source financing via aggressive crowdfunding (SA)*, that is, the firm goes aggressive in goal setting by letting $F_c = pS_c$, which is the maximal possible value of the crowdfunding capital supply (recall S_c is the upper bound of $\tilde{S}_c$'s support),[18] and ii) *single-source financing via prudent crowdfunding (SP)*, that is, the firm goes prudent in goal setting by setting $F_c < pS_c$, which reflects a moderate crowdfunding goal.

Dual-source financing strategies via crowdfunding and bank: DA and DP. Since fulfilling the total market demand is always profitable as long as the capital constraint allows the firm to do so, the firm should start getting loans from the bank when it expects crowdfunding to be unable to meet the capital requirement, especially when the post-campaign retail demand is high. However, because part of the demand exists in the crowdfunding market and because the crowdfunding capital is cheaper, the firm still gives priority to crowdfunding and sees it as the preferred capital source (so single-source financing via bank is never optimal). Similarly, depending on the crowdfunding fee f there are two goal-setting schemes for the firm, either aggressive or prudent in setting F_c, while the amount of bank loans is determined by the firm's remaining capital needs. They are, i) *dual-source financing via aggressive crowdfunding and bank (DA)*, that is, the firm sets the crowdfunding goal as the upper bound of the crowdfunding capital

[18] In fact, setting F_c at any level above pS_c is mathematically equivalent, because the crowdfunding capital never exceeds pS_c. Here we let $F_c = pS_c$ and name it as the aggressive crowdfunding scheme, while in practice it means the firm sets the goal sufficiently high to capture all potential crowdfunding capital supply.

supply pS_c, and ii) *dual-source financing with prudent crowdfunding and bank financing (DP)*, that is, the firm sets a moderate crowdfunding goal between $(0, pS_c)$.

Given the distributions of the two random variables $\tilde{S}_c$ and $\tilde{D}$, the optimal crowdfunding goal and bank loan can be solved by using the Lagrange method and the Karush-Kuhn-Tucker (KKT) conditions for the optimization problem (2), which gives rise to the four types of financing strategies mentioned above, i.e., SA, SP, DA and DP. Specifically, it turns out that whether the firm should resort to bank loan depends on the ratio of $\tilde{D}$ and $\tilde{S}_c$'s upper bounds, i.e., $\frac{D}{S_c}$, which reflects the relative magnitude ratio of the post-campaign retail demand to the crowdfunding demand.[19]

To simplify notation, let $\rho \equiv \frac{D}{S_c}$ denote the *magnitude ratio* between two markets. We show (all proofs are included in the Appendix) that there exists a cutoff threshold $\hat{\rho}$ such that when $\rho > \hat{\rho}$, it is optimal for the firm to get bank loan in addition to the crowdfunding capital (i.e., dual-source financing), and when $\rho < \hat{\rho}$, the firm should use crowdfunding only (i.e., single-source financing). As for the crowdfunding goal-setting scheme, whether the firm should set an aggressive goal or a prudent goal depends on the crowdfunding cost f. We show that there exists a cutoff threshold $\hat{f}$ that is affected by the bank loan interest, the product's profit margin, etc., such that aggressive goal setting is optimal if and only if $f < \hat{f}$. The optimal financing strategy is summarized in the following Proposition 1.

Proposition 1. ***(Optimal Financing Strategy)****. Define two thresholds $\hat{f}$ and $\hat{\rho}$ as shown below.*[20] *Then the firm has the following four optimal financing strategies.*

$$\hat{\rho} = \begin{cases} \frac{(p-c)(p-c-ck)}{2c(p-c-cr)(1-k)}, & f < \frac{c-p+2cr}{c}, \\ \rho_1, & f > \frac{c-p+2cr}{c}. \end{cases}$$

$$\hat{f} = \begin{cases} \frac{(p-c)(cD(1-k)-(p-c-ck)S_c)}{c^2(1-k)D}, & f < \frac{c-p+2cr}{c}, \\ \frac{2c^2D(1-k)r-(p-c)(p-c-ck)S_c}{2c^2(1-k)D}, & f > \frac{c-p+2cr}{c}. \end{cases}$$

- *SA Strategy (Single-source financing via aggressive crowdfunding):*

$$F_c^{SA} = pS_c \text{ and } F_b^{SA} = 0, \quad \text{if } \rho < \hat{\rho} \text{ and } f < \hat{f}.$$

- *SP Strategy (Single-source financing via prudent crowdfunding):*

$$F_c^{SP} = \frac{cp(1+k)}{2p-c(1+k)}S_c + \frac{2cp(1-k)}{2p-c(1+k)}(1 - \frac{cf}{p-c})D < pS_c \text{ and } F_b^{SP} = 0, \quad \text{if } \rho < \hat{\rho} \text{ and } f > \hat{f}.$$

- *DA Strategy (Dual-source financing via aggressive crowdfunding and bank):*

$$F_c^{DA} = pS_c \text{ and } F_b^{DA} = \frac{c^2}{p-c}(1-k)(\frac{p-c}{c} - r)D + \frac{c(1+k)-p}{2}S_c, \quad \text{if } \rho > \hat{\rho} \text{ and } f < \hat{f}.$$

[19] $\tilde{S}_c$ and $\tilde{D}$'s expectations are $\mathbb{E}(\tilde{S}_c) = \frac{S_c}{2}$ and $\mathbb{E}(\tilde{D}) = \mathbb{E}(\mathbb{E}(\tilde{D}|s_c)) = \frac{(1-k)D+kS_c}{2}$. Then $\frac{\mathbb{E}(\tilde{D})}{\mathbb{E}(\tilde{S}_c)} = (1-k)\frac{D}{S_c} + k$, so one can say that $\frac{D}{S_c}$ reflects the relative magnitude ratio of the post-campaign retail market demand to the crowdfunding demand given any correlation proxy parameter k.

[20] $\rho_1 = \frac{(p-c)[4p(r-f)(p-c-ck)-c(1+k)^2(p-c-cr)]}{4p(1-k)(p-c-cf)^2} + \frac{(p-c)(2p-c-ck)\sqrt{4p(r-f)^2(p-c-ck)+(1+k)^2(p-c-cr)^2}}{4p(1-k)(p-c-cf)^2}$.

- *DP Strategy (Dual-source financing via prudent crowdfunding and bank):*[21]

$$F_c^{DP} = F_{c1} < pS_c \text{ and } F_b^{DP} = \frac{c^2}{p-c}(1-k)(\frac{p-c}{c} - r)D + \frac{c(1+k)}{2}S_c - (1 - \frac{F_{c1}}{2pS_c})F_{c1},$$

$$\text{if } \rho > \hat{\rho} \text{ and } f > \hat{f}.$$

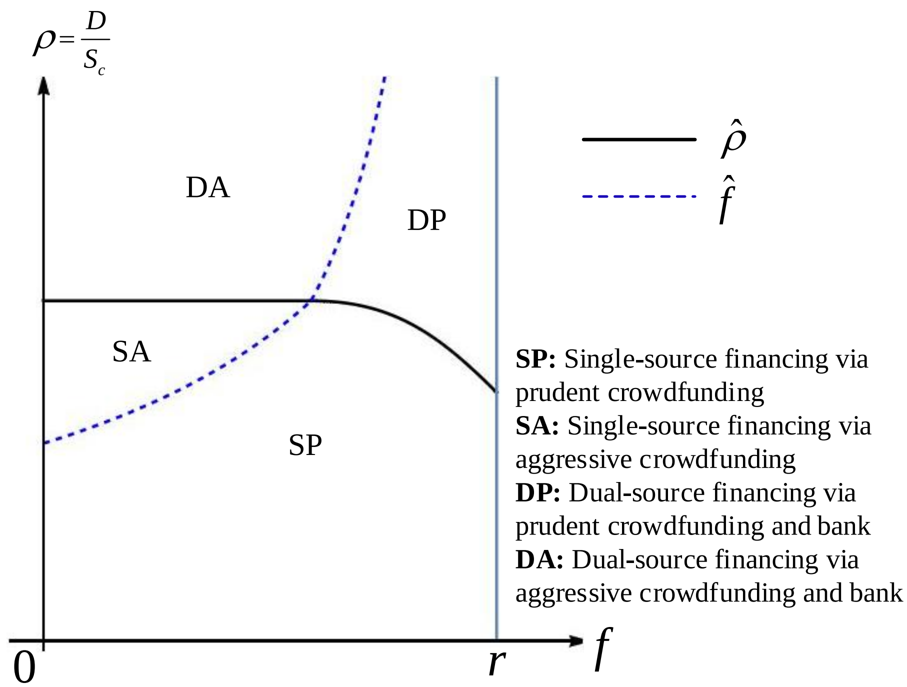

Figure 1: Optimal financing strategy for different parameters.

Proposition 1 gives a clear-cut partition of the parameter space for each of the four financing strategies to be optimal, as illustrated in Figure 1. The vertical axis is the magnitude ratio $\rho = \frac{D}{S_c}$ and the horizontal axis is the crowdfunding fee f. The solid (dotted) line is the threshold $\hat{\rho}$ ($\hat{f}$) that separates dual-source strategies from single-source strategies (aggressive goal from prudent goal strategies).

The strategy's transitioning at the thresholds is related to the goal-setting risk and the financing risk involved in the capital supply and demand uncertainties. Intuitively, the magnitude ratio $\rho = \frac{D}{S_c}$ reflects the size of the post-campaign retail market compared to the crowdfunding demand, thus affecting the degree of necessity to get external bank loan besides running crowdfunding campaigns. The crowdfunding goal-setting scheme depends on its cost f, as a higher crowdfunding fee exacerbates the financial loss associated with overfinancing risk when the post-market demand is revealed to be low. Also, notice that the two thresholds $\hat{\rho}$ and $\hat{f}$ are dynamically related: When the crowdfunding fee f becomes higher, $\hat{\rho}$ is lower since the optimal strategy transitions from single-source financing to dual-source financing faster as crowdfunding loses part of its cost advantage to the bank loan; And when the magnitude ratio ρ is higher, $\hat{f}$ increases, since the optimal strategy transitions from aggressive crowdfunding to prudent crowdfunding more slowly, as high post-campaign retail demand calls for more capital and the cheaper crowdfunding capital is prioritized in the ease of overfinancing risk.

[21]We have $F_{c1} = \frac{S_c(p-c)}{2}\sqrt{\frac{8c^2p(1-k)(r-f)D}{(p-c)^3S_c} + \frac{c^2(1+k)^2}{(p-c)^2}} + \frac{c(1+k)}{2}S_c$ as given in (A.10) in Appendix.

Regarding the transition between single-source and dual-source financing, notice that when $\frac{D}{S_c}$ increases, either D increases or S_c decreases. As D increases, $\tilde{D}$ increases stochastically, so there is higher risk of underfinancing, i.e., $\text{Prob}\{\min\{F_c, p\tilde{S}_c\} + F_b < c(\tilde{D} + \tilde{S}_c)\}$ is higher as the right-hand-side capital needed increases, and we have shown in the second-stage analysis that underfinancing hurts profitability because production is always profitable up to the point of revealed total demand. Therefore, underfinancing risk calls for more capital and brings the transition from single-source to dual-source financing.[22] As S_c decreases, the dynamic change of risks is more subtle: On one hand, the capital raised in crowdfunding $\min\{F_c, p\tilde{S}_c\}$ decreases, but on the other, both demands $\tilde{S}_c$ and $\tilde{D}$ (which is uniform conditional on s_c) decrease stochastically so the capital needed is also lower. So it is not clear how $\text{Prob}\{\min\{F_c, p\tilde{S}_c\} + F_b < c(\tilde{D} + \tilde{S}_c)\}$ changes with S_c. It can be verified that when $F_c = pS_c$ (e.g., in strategy SA), $\text{Prob}\{\min\{F_c, p\tilde{S}_c\} + F_b < c(\tilde{D} + \tilde{S}_c)\}$ decreases in S_c, so underfinancing risk is higher when S_c decreases, and meanwhile the risk of overstretching goal setting becomes higher as there is less capital available from crowdfunding (though this risk is alleviated by the underfinancing risk), which suggests substituting F_b for part of F_c. Therefore, when S_c is low enough it becomes optimal to use the expensive bank loan, i.e., SA $\rightarrow$ DA. The case of SP is less straightforward, because $\text{Prob}\{\min\{F_c, p\tilde{S}_c\} + F_b < c(\tilde{D} + \tilde{S}_c)\}$ does not have a clear-cut relationship with S_c, i.e., the underfinancing risk is not always higher for lower S_c, but the overstretching goal-setting risk still calls for partial substitution of F_b for F_c, and our result shows that single-source financing again transitions to dual-source financing when S_c is sufficiently low, while the trajectory (whether SP $\rightarrow$ DP, or SP $\rightarrow$ SA $\rightarrow$ DA) depends on f.

Likewise, the optimality between aggressive and prudent crowdfunding schemes depends on the crowdfunding service fee f. In Equation (2), it is clear that f only appears in the crowdfunding capital cost $fE[\min\{F_c, p\tilde{S}_c\}]$, thus having a direct impact on the decision variable F_c, but not on F_b. Since we assume $f < \frac{p-c}{c}$ always holds, it is always profitable to acquire capital in crowdfunding even if f increases, as long as the capital can be put into production. However, under the capital supply and demand uncertainties the probability of overfinancing is always positive,[23] hence the economic cost of overfinancing (i.e., wasteful capital) increases with f. Therefore, when f increases to a crucial point, it is optimal to decrease F_c as a way to reduce the overfinancing risk, which results in the transition from aggressive crowdfunding to prudent crowdfunding.

In the following paper, we will discuss some important comparative statics of the current model. We first look at comparative statics for the important market parameters S_c and D, which reflect the market size of crowdfunding and the retail market, as well as the bank financing cost r, which is very volatile in real markets contingent on the firm-bank relationship and firm's status. We discuss how these parameters affect the firm's optimal financing strategy and the optimal profit in the next subsection 4.3.

Following that, we delegate the next whole section to the discussion of uncertainty. Section 5.1 compares the current model with a hypothetical, complete information model, where the firm knows the true crowdfunding demand and post-campaign retail demand perfectly, for a full understanding of how uncertainty has affected the firm's financing strategies in our setup. Section 5.2 looks at how parameter k,

[22] More specifically, for low financing cost f, crowdfunding is prioritized so the optimal strategy transitions from SP $\rightarrow$ SA first, and when the crowdfunding goal is at its maximal (aggressive crowdfunding), then from SA $\rightarrow$ DA. For higher f, the economic cost of overstretching goal (leading to overfinancing) is high, so SP $\rightarrow$ DP first, and if D continues to increase which alleviates the overfinancing risk, the optimal strategy starts to transition from DP $\rightarrow$ DA.

[23] Note that the event that the market demand is completely disrupted, i.e., $\tilde{S}_c' \leq \epsilon_1$ and $\tilde{D}' \leq \epsilon_2$ for arbitrary small ϵ_1 and ϵ_2, always has positive probability under the uniform distribution specification. So the probability of overfinancing is always positive.

which is a proxy for the correlation between the crowdfunding demand and post-campaign retail demand, affects the optimal financing strategy and profit.

4.3. Comparative statics of Market Parameters

Proposition 1 fully characterizes F_c and F_b and the optimal parameter space for each strategy, followed by some analysis on the transition of the optimality of the four alternative strategies as the magnitude ratio $\frac{D}{S_c}$ and the crowdfunding fee f changes. Now for comparative statics, we will focus on three crucial parameters that are related to the product market and the financing market: D, which reflects the size of the retail market that is independent of the crowdfunding market's influence, S_c, which primarily reflects the size of the crowdfunding market but also affects the size of the retail market (as $\tilde{D}$ is conditional on $\tilde{S}_c$'s realization), and r, which is usually negotiated between the firm and the bank and thus has high volatility across firms.

In the following two Propositions, we show that the impact of $D\&S_c$ on the crowdfunding goal F_c is always positive, but the impact of $D\&S_c$ on the bank loan F_b depends on other parameters.

Proposition 2. ***(Impacts of retail market demand on financing strategies)***. *When D increases:*

(1) F_c^j increases for $j = SP, DP$, and remains the same for $j = SA, DA$.

(2) F_b^{DA} increases. F_b^{DP} increases in general, but decreases when $f\&D$ are sufficiently low and r is sufficiently high.

Proposition 3. ***(Impacts of crowdfunding demand on financing strategies)***. *When S_c increases:*

(1) F_c^j increases for $j = SA, SP, DA, DP$.

(2) F_b^{DP} decreases. F_b^{DA} decreases in general, but increases if $c > \frac{p}{1+k}$.

Let us first discuss the impact of $D\&S_c$ on F_c^j, i.e., Proposition 2(1) and 3(1). When D increases, the underfinancing risk increases. Since crowdfunding capital is cheaper, the optimal F_c^j increases in D whenever possible, i.e., in prudent strategies $j = SP, DP$, and remains at its maximal value in aggressive strategies $j = SA, DA$. When S_c increases, on one hand, crowdfunding can supply more of its cheap capital, calling for increasing F_c, but on the other hand, the total market demand $\tilde{S}_c + \tilde{D}$ inflates quickly, bringing about overfinancing risks and calling for decreasing either F_c or F_b. Overall, the cost advantage of crowdfunding dominates, so F_c^j increases in S_c for $j = SA, SP, DA, DP$.

As for F_b^j in Proposition 2(2) and 3(2), recall that we have given the expression of F_b^j for $j = DA, DP$ in Proposition 1, which can be summarized as

$$F_b^j = \frac{c^2}{p-c}(1-k)(\frac{p-c}{c} - r)D + \frac{c}{2}(1+k)S_c + (-F_c^j)(1 - \frac{F_c^j}{2pS_c}), \quad j = DA, DP,$$

where F_c^j is the corresponding optimal F_c for each strategy (one can substitute $F_c^{DA} = pS_c$ to get the expression given in Proposition 1). The three terms in F_b^j indicate that D and S_c affect F_b^j both directly, through the first and second term and indirectly through F_c^j. Because both $\frac{c^2}{p-c}(1-k)(\frac{p-c}{c} - r)$ and $\frac{c}{2}(1+k)$ are positive, F_b^j increases in D and S_c from their direct impacts as market demand expands. However, because the third term $(-F_c^j)(1 - \frac{F_c^j}{2pS_c})$ always decreases in F_c^j for any $F_c^j \leq pS_c$, and because higher $D\&S_c$ always lead to higher F_c^j as just discussed above, it means the indirect impact through F_c^j

is always negative in a form of partial substitution of F_b^j by F_c^j. Roughly speaking, D has an overall positive impact on F_b^j due to the pure market expansion effect, while S_c has an overall negative impact on F_b^j due to the primary expansion of the crowdfunding market and the cost advantage of crowdfunding capital, though both impacts are subject to parameter exceptions.

Specifically, when D increases, in $j = DA$, because $F_c^{DA} = pS_c$ cannot further increase, F_b^{DA} always increases in D. In $j = DP$, the positive direct impact of market expansion (in the first term of F_b^j) outweighs the negative indirect impact of crowdfunding's substitution (in the third term of F_b^j) in most cases, except when the retail market D is small and crowdfunding's cost advantage is significant, i.e., f sufficiently low and r sufficiently high, as indicated in Proposition 2(2). As for the impact of S_c, note that a higher S_c primarily means a higher market size for crowdfunding, though the retail market is also positively affected. This means crowdfunding can provide more capital at a low cost, which makes F_c^j increase quite fast in S_c, and eventually results in the negative indirect impact of crowdfunding's substitution dominating in most cases. However, in $j = DA$ where $F_c^{DA} = pS_c$ is already at its maximum and is thus limited in its growth speed with S_c, so if $c > \frac{p}{1+k}$, then the direct impact in the second term of F_b^j, $\frac{c}{2}(1+k)S_c$, dominates, which makes F_b^{DA} increase in S_c as indicated in Proposition 3(2).

The key takeaway of Proposition 2 and 3 can be demonstrated as the following. Crowdfunding as the cost-saving capital source always sees an increase in its goal when the market expands, no matter whether the expansion comes from the crowdfunding market or the retail market. But for the bank loan, considering the substitution effect of crowdfunding, the retail market expansion favors increasing bank loans (except when the retail market is very small and crowdfunding's cost advantage is very significant), while the crowdfunding market expansion calls for reducing bank loan (except when the production cost is too high that crowdfunding capital alone cannot keep up with the cost inflation).

Apart from D and S_c, how do financial costs f and r affect F_c^j and F_b^j? By directly differentiating their expressions, one can see that whenever $F_c^j < pS_c$ in prudent crowdfunding, F_c^j decreases in f and increases in r, and whenever $F_b^j > 0$ in dual-source financing, F_b^j increases in f and decreases in r. This is a direct implication of the maximization problem (2) as f and r only affect the costs of F_c and F_b.

The more interesting question to ask is how financial costs affect the thresholds $\hat{\rho}$ and $\hat{f}$, thus shifting the four strategies over the parameter space in terms of their optimality. Since f is now part of the parameter space, we consider the impact of the bank loan interest rate r. The results are given in Proposition 4 and illustrated in Figure 2.

Proposition 4. ***(Impacts of bank loan interest rate on thresholds)***. *When r increases:*

(1) The magnitude ratio threshold $\hat{\rho}$ increases.

(2) The crowdfunding fee threshold $\hat{f}$ increases between DA and DP, and remains the same between SA and SP.

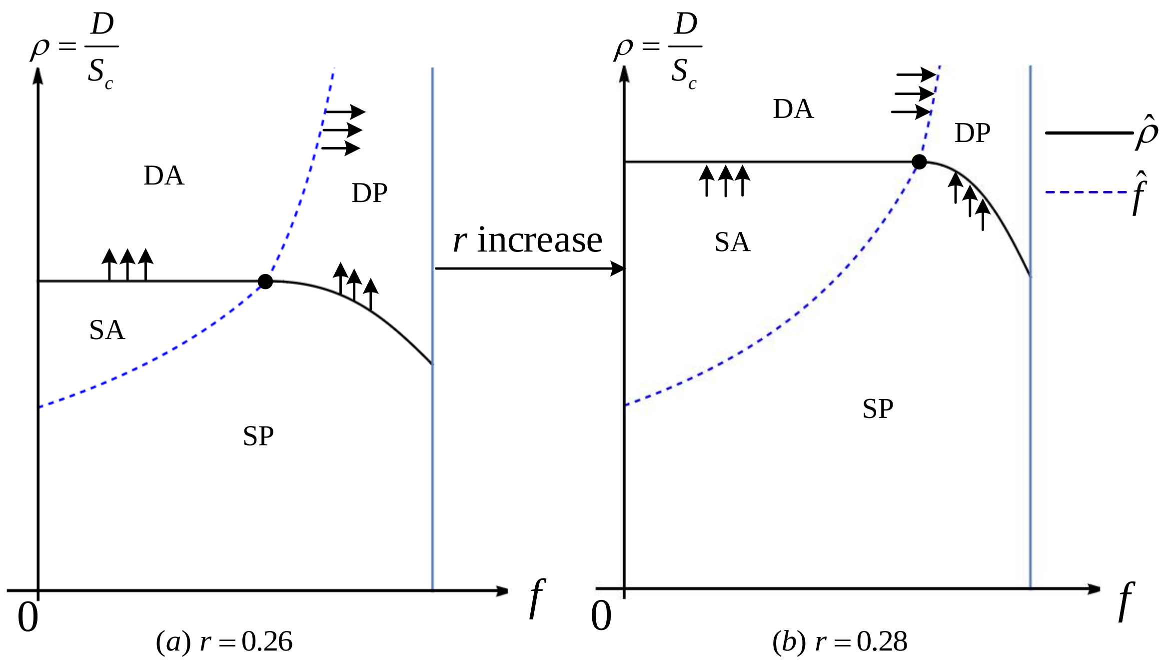

Figure 2: The optimal financing strategies when the financing cost of bank loans increases.

Recall that the transitioning from single-source financing to dual-source financing happens when $\frac{D}{S_c}$ increases, as discussed after Proposition 1, as a result of higher underfinancing risk and/or higher overstretching goal-setting risk. Therefore, this transition to dual-source financing will be discouraged by higher bank loan interest r, which makes overfinancing more costly (thus a higher level of underfinancing risk becomes acceptable) and inflates the cost advantage of crowdfunding (thus a slightly overstretching goal becomes tolerable). So, the threshold $\hat{\rho}$ shifts up as r increases. Similarly, the transition from aggressive crowdfunding to prudent crowdfunding happens when the crowdfunding cost f increases. From SA to SP, no bank loan is involved, so that r does not affect the transitioning and $\hat{f}$ remains the same. But from DA to DP, the crowdfunding's cost advantage becomes more significant as r increases, so the transition from aggressive to prudent is slowed down, resulting in a right shifting of $\hat{f}$ in this region. In general, a higher bank loan r makes single-source financing and aggressive crowdfunding more favorable, so the thresholds shift in the upper right direction. In a nutshell, a higher bank loan r favors single-source strategies over dual ones, and favors aggressive crowdfunding goal-setting over prudent ones.

Lastly, let us look at the comparative static for the profit function. The results are summarized in the following Proposition.

Proposition 5. ***(Parameter impacts on the optimal profit)***. *Let Π^j denote the optimal profit.*

(1) Π^j decreases in r for $j = DA, DP$, unaffected by r for $j = SA, SP$.

(2) Π^j always increases in D for $j = SA, SP, DA, DP$.

(3) Π^j increases in S_c for $j = SA, DA$.

For Π^j in SP and DP, $\frac{\partial \Pi^j}{\partial S_c}$ involves high order terms of S_c and is analytically intractable. So we simulated sufficient repetitions (see Proof of Proposition 5 for simulation procedure details) and from simulation, we conclude that there exists some $\bar{S}_c$ such that for $j = SP, DP$, Π^j increases in S_c when $S_c < \bar{S}_c$, decreases in S_c when $S_c > \bar{S}_c$, while $\bar{S}_c$ increases in D, as shown in the following Figure.

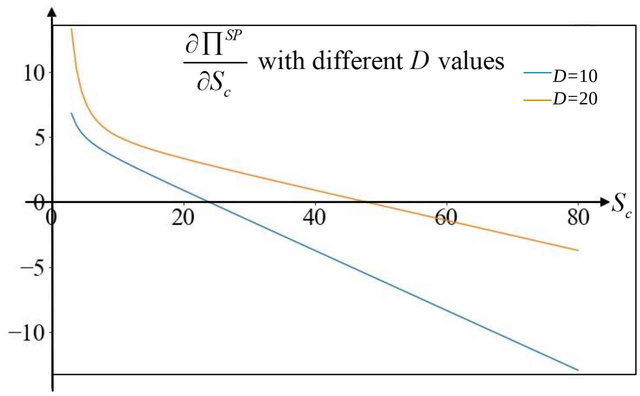

Figure 3: $\frac{\partial \Pi^{SP}}{\partial S_c}$ shifts to the right when D increases, where $p = 10$, $c = 4$, $r = 0.25$, $f = 0.05$, $k = 0.5$.

The comparative statics of Π^j in r are straightforward. Since r enters Π only in the cost term of (2), by Envelop Theorem, the optimal Π^j must decrease in r whenever $F_b > 0$ (i.e., in DA and DP). And for the same reason, Π^j decreases in f and increases in p, as a direct implication of the Envelop Theorem.

The comparative statics of Π^j in D is similar to r, although we cannot apply the Envelop Theorem directly because of the expectation operator. However, as D enters Π only in the revenue term, and higher D means the expansion of the retail market, which means the entrepreneur can sell more products ceteris paribus, so we can prove that the optimal Π^j must increase in D.

The comparative statics of Π^j in S_c is more nuanced, as S_c appears in both the revenue term and the crowdfunding cost term. When S_c increases, both the crowdfunding market and the retail market expand, meaning higher profitability, but the overfinancing risk also increases, as the firm may raise too much capital from crowdfunding. As stated in (3), we prove that in aggressive crowdfunding, Π^j increases in S_c because the market expansion outweighs the overfinancing risk. In fact, the low overfinancing risk is the reason for aggressive crowdfunding strategies' optimality in the first place for this parameter range. In prudent crowdfunding SP and DP, we find in simulation (details contained in the Proof) that Π^j increases in S_c when S_c is small but decreases in S_c when S_c exceeds a cutoff threshold $\bar{S}_c$, indicating that overfinancing risk will outweigh the market expansion benefit once S_c grows too large. Moreover, the cutoff $\bar{S}_c$ increases in D, meaning that the domination of overfinancing risk is mediated by higher retail market D. Thus the retail market demand serves to mediate the negative impact of overfinancing risk on optimal profit. Therefore, market expansion, in general, increases the optimal profit, but when the crowdfunding market size grows too big, the optimal profit may fall due to the overfinancing risk, because all funds collected through crowdfunding are subject to fees.

5. Extensive Analysis on Uncertainty

In this section, we conduct two important comparative statics analyses regarding the demand/capital supply uncertainties faced by the firm: One is a stylized, qualitative comparison of the information structure, and the other is a comparative statics analysis for the correlation parameter k. Specifically, in Subsection 5.1 we compare the optimal financing strategy between the above analyzed uncertain model (with random $\tilde{S}_c\&\tilde{D}$), and a hypothetical complete information model where the uncertainty over

$\tilde{S}_c\&\tilde{D}$ is completely resolved. Although the complete information scenario is highly hypothetical, it is an important baseline to help understand how adding uncertainty affects the firm's financing strategy, especially in the context where the firm can use the crowdfunding profit margin to produce for retail market demand. So this first extensive analysis compares two models with and without uncertainty, in an absence of market correlation. And following the logic we conduct the standard comparative statics for k in Subsection 5.2, to show the impact of market correlation.

For ease of presentation, we use abbreviations for each model under examination. The baseline model is called *correlated uncertainty model (CUM)*, where (i) both markets are uncertain, and (ii) two markets are correlated, i.e., $k > 0$. The hypothetical model is called *complete information model (CIM)*, where (i) both markets are certain, and (ii) two markets are uncorrelated. To separate the impact of k, which will be discussed in Subsection 5.2, we let $k = 0$ in *CUM* in Subsection 5.1 and call it *uncorrelated uncertainty model (UUM)*, where (i) both markets are uncertain, and (ii) two markets are uncorrelated, i.e., $k = 0$.

5.1. Comparison between CIM and UUM

The complete information model CIM is a highly hypothetical model since in reality, crowdfunding and retail markets are full of uncertainty. As a matter of fact, market uncertainty is unequivocally the key focus of this paper. However, in order to understand the precise role played by demand uncertainties in the firm's financing problem, we need to contrast the baseline model using the counterfactual CIM. By contrasting the two models, we show that it is demand uncertainties, even in an absence of market correlation, that (i) give rise to SA and DP strategies, which are absent in the complete information case, and (ii) make f a key parameter in determining which strategy is optimal and which trajectory of capital source the firm should choose for additional capital needs (i.e., crowdfunding first, or bank first) when the retail market grows in its relative size. The analysis thus has important managerial implications for firms. It justifies the combinatory use of prudent crowdfunding goal-setting and positive bank loans and shows that crowdfunding is only prioritized for additional capital needs (as the retail market expands) when its cost advantage is significant.

In the complete information model (CIM), the firm knows the product demand for both the crowdfunding and the post-campaign retail market. Let s_c^{CI} denote the crowdfunding demand and d^{CI} the post-campaign retail market demand, which are two constants, then the firm's objective function can be written as:

$$\max_{F_c \geq 0, F_b \geq 0} (p-c)\min\{d^{CI} + s_c^{CI}, \frac{\min\{F_c, ps_c^{CI}\} + F_b}{c}\} - f\min\{F_c, ps_c^{CI}\} - rF_b.$$

The expectation operator appearing in (2) is dropped for CIM. Differentiating the objective function and by standard calculation, we solve the optimal financing strategy for CIM as given in Lemma 1.

Lemma 1. *There are two optimal financing strategies in CIM.*

(1) When $\frac{d^{CI}}{s_c^{CI}} < \frac{p-c}{c}$, SP is optimal, where $F_b^{SP-CI} = 0$ and $F_c^{SP-CI} = c(d^{CI} + s_c^{CI})$.

(2) When $\frac{d^{CI}}{s_c^{CI}} > \frac{p-c}{c}$, DA is optimal, where $F_b^{DA-CI} = c(d^{CI} + (1-p)s_c^{CI})$ and $F_c^{DA-CI} = ps_c^{CI}$.

Recall that in the baseline CUM, crowdfunding has a cost advantage but is unstable, leading to potential underfinancing or overfinancing risks. However, with complete information all types of risk vanish, so crowdfunding is always prioritized due to its lower cost (i.e., $f < r$). Therefore, when crowdfunding capital supply ps_c^{CI} is sufficient to cover the production cost, that is, $cd^{CI} < (p-c)s_c^{CI}$ as in Lemma

1(1), it is optimal to use crowdfunding only, while the crowdfunding goal equals the total capital needed, which is the SP strategy. If the crowdfunding capital supply alone cannot cover the production cost, i.e., $cd^{CI} > (p-c)s_c^{CI}$ as in Lemma 1(2), the crowdfunding goal is set at maximum ps_c^{CI} and the lacking capital is obtained by bank loan, which is the DA strategy.

Comparing Lemma 1 (CIM) with Proposition 1 (CUM), we see that adding uncertainty to two markets gives rise to SA and DP strategies. In fact, SA and DP strategies' optimality comes from uncertainties. In Lemma 1, SA strategy is optimal only when $\frac{d^{CI}}{s_c^{CI}} = \frac{p-c}{c}$, which is transient. But in Proposition 1, SA strategy is optimal for a range of $(f, \frac{D}{S_c})$ with low f and moderate $\frac{D}{S_c}$. This suggests that without demand uncertainty, the firm resorts to the bank loan immediately after exhausting the crowdfunding capital. But with demand uncertainty, the firm should refrain from doing so until the retail market grows past a certain size. This is to avoid the overfinancing risk in case retail demand $\tilde{D}$ has a low realization, given that crowdfunding (bank) is relatively cheap (expensive). Similarly, in Lemma 1 DP strategy is never optimal, while in Proposition 1 DP strategy is optimal for a range of $(f, \frac{D}{S_c})$ with high f and high $\frac{D}{S_c}$. This is to avoid underfinancing risk when the retail demand is high and crowdfunding is relatively expensive, so the bank loan is utilized as a buffer.

In other words, it is the two markets' demand uncertainties that give rise to the complex trade-offs in crowdfunding capital (cheap vs. uncertain), even in the absence of the two markets' correlation. Without demand uncertainties, there is no such trade-off, and crowdfunding is always prioritized as long as it has the cost advantage ($f < r$), while with demand uncertainties, crowdfunding loses its absolute priority, and now it matters how much the cost advantage (f vs. r) is. This gives rise to the next Lemma 2.

Lemma 2. *Crowdfunding cost f affects the optimal financing strategy through demand uncertainties.*

- *In CIM, the optimal financing strategy is determined by the magnitude ratio $\frac{d^{CI}}{s_c^{CI}}$ only.*
- *In UUM, the optimal financing strategy is determined by both the magnitude ratio and f.*

Lastly, let us examine an important issue closely related to Lemma 2—the trajectory of capital source when the retail market grows in its relative size—and see how demand uncertainties weigh in on this trajectory. Formally, we want to answer the question: Where should the firm obtain additional funds when the magnitude ratio increases, with and without demand uncertainties? The next Proposition follows immediately from Lemma 1 and Proposition 1 (or Figure 1), since UUM is a special case of the baseline CUM by letting $k = 0$ and has the same four optimal strategies.

Proposition 6. ***(Financing strategy transition for CIM and UUM).*** *When the magnitude ratio increases, the optimal financing strategy transitions in the following way.*

(1) In CIM, the optimal strategy transitions from $SP \to DA$ for any f.

(2) In UUM, the optimal strategy transitions from

i) $SP \to SA \to DA$, when $f < \frac{2rc-(p-c)}{c}$;

ii) $SP \to DP \to DA$, when $f > \frac{2rc-(p-c)}{c}$.

When the magnitude ratio increases ($\frac{d^{CI}}{s_c^{CI}}$ in CIM and $\frac{D}{S_c}$ in UUM), retail market demand grows in its relative size, calling for more capital. In Lemma 1 we have $F_c^{SP-CI} = c(d^{CI} + s_c^{CI})$, which means in CIM the firm first adds F_c until it hits the maximum of crowdfunding capital supply, then the firm adds F_b. Crowdfunding is used first and up to exhaustion before using the bank loan, so the trajectory is $SP \to DA$. The transition is immediate once the crowdfunding goal is at maximum.

In UUM, this trajectory is affected by f due to demand uncertainties as demonstrated in Lemma 2. For low f, similar to CIM, crowdfunding is utilized first and then the bank loan, but unlike CIM, there is a medium range of $\frac{D}{S_c}$ where SA is optimal. In other words, as the retail market grows in its relative size, the firm does not immediately resort to the bank loan when the crowdfunding goal reaches the maximum but stays in SA (single-source aggressive) until the retail market size is sufficiently large. For high f, on the opposite, the firm resorts to bank loans first before going aggressive in crowdfunding, so $SP \to DP \to DA$. This is contrary to the intuition that crowdfunding should always be prioritized in response to higher retail demand due to its cost advantage. For $f > \frac{2rc-(p-c)}{c}$, it is optimal for the firm to resort to bank loans first before going aggressive in crowdfunding. Note here that f is the key to determining the optimality of different trajectories, only when demand uncertainties are in play.

5.2. Comparative statics for market correlation in CUM

In this Subsection, we conduct comparative statics for the market correlation parameter k in the baseline model CUM. Recall that the retail market demand $\tilde{D}$ follows a uniform distribution conditional on the realized value s_c, i.e., $\tilde{D}|s_c \sim U(ks_c, (1-k)D+ks_c)$, where $0 \leq k \leq 1$. If the crowdfunding demand is realized at a higher value, the retail market demand is also more likely to have a higher realization, and vice versa. Formally, we show that the correlation coefficient between $\tilde{D}$ and $\tilde{S}_c$ has a complicated expression as shown in the next Lemma 3. It is hard to directly analyze the correlation, but as it increases in k, we can take k as a proxy for the correlation.

Lemma 3. *In CUM,* $Corr[\tilde{S}_c, \tilde{D}] = \left((\frac{D}{S_c})^2(\frac{1}{k}-1)^2+1\right)^{-\frac{1}{2}}$*, which increases in* k*.*

When k increases, the two markets become more closely related, which may appear to favor crowdfunding. However, as we have mentioned in the Introduction and model setup, the correlation between crowdfunding and the retail market is a double-edged sword that may intensify overfinancing or underfinancing risks, depending on the profit margin $p-c$ and market sizes.

Proposition 7. ***(Impact of market correlation on financing strategies)****. As* k *increases,*

(1) when $c < \frac{p}{2}$*,* $\hat{\rho}$ *increases and* $\hat{f}$ *decreases,* F_c^j *decreases for* $j = SP, DP$*, and* F_b^j *increases for* $j = DA, DP$ *if and only if* D *is sufficiently small.*

(2) when $c > \frac{p}{2}$*,* $\hat{\rho}$ *decreases and* $\hat{f}$ *increases,* F_c^j *increases for* $j = SP, DP$*, and* F_b^j *increases for* $j = DA, DP$ *if and only if* D *is sufficiently small.*[24]

The threshold movement results for $\hat{\rho}$ and $\hat{f}$ in Proposition 7 are illustrated in Figure 4. The movement of the two thresholds depends on the relative magnitude of the production cost c to p. In both cases, increasing the market correlation does not necessarily make crowdfunding more favorable to the bank loan: When c is low, single-source financing (SA and SP) becomes optimal for a larger parameter range,

[24] As shown in the proof, the cutoff value of D in DA strategy is $\frac{(p-c)S_c}{2(p-c-cr)}$ for both $c < \frac{p}{2}$ and $c > \frac{p}{2}$, while the cutoff value of D in DP strategy is $\frac{(p-c)S_c}{2(p-c-cr)}$ only when $c < \frac{p}{2}$, and admits a different, intractable value $\hat{D}$ when $c > \frac{p}{2}$.

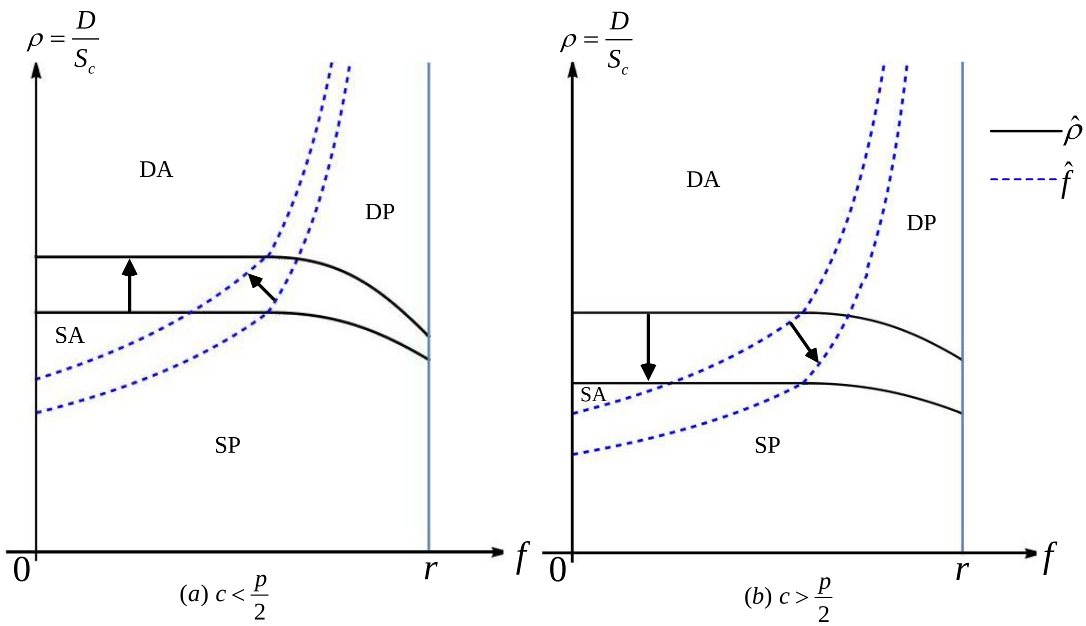

Figure 4: Optimal financing strategy for different parameters.

but so does prudent crowdfunding financing (SP and DP); When c is high, aggressive crowdfunding financing (SA and DA) becomes optimal for a larger parameter range, but so does dual-source financing (DA and DP). One might expect that as k increases, crowdfunding better reflects the retail demand so that single-source financing (crowdfunding only) and aggressive goal setting should both be optimal for a larger parameter space. But the Proposition shows that the two schemes go in opposite directions: when the shifting calls for a larger space of single-source financing, it also calls for a larger space of prudent goal-setting, and vice versa.

The intuition is related to overfinancing and underfinancing risks through crowdfunding profit margin, thus depending on the production costs. With low production costs, the more correlated the two markets are, the more likely that crowdfunding alone can supply the needed capital for the post-campaign retail market through the profit margin $p-c$ (calling for single-source), but the high profit margin also intensifies overfinancing risk when the correlation increases[25] (calling for prudent crowdfunding), so the optimal strategy shifts towards single-source and prudent goal-setting. Indeed, Proposition 7 also confirms that the crowdfunding goal, whenever prudent, decreases in k when $c < \frac{p}{2}$. Similarly, with high production costs, the correlation exacerbates underfinancing risk, so the optimal strategy shifts towards dual-source financing and aggressive goal-setting, and even in the prudent strategy spaces (for SP and DP) the crowdfunding goal increases in k to overcome the underfinancing risk. The comparative static for F_b is more straightforward, as the bank loan, once determined, does not fluctuate as the crowdfunding capital supply does. The bank loan increases in k whenever the post-campaign retail market D is below some threshold. This is because when D is sufficiently small, higher k means that the post-campaign market demand is less likely to be affected by D (thus less likely to be such small), which calls for increasing bank loan. The above discussion hence highlights the importance of incorporating financing risks into

[25]Consider the extreme case when the retail demand equals the crowdfunding demand but $p > 2c$, then part of the crowdfunding capital, specifically $p - 2c$ from each backer, is always not used in production and wasted.

the analysis of optimal financing strategies, especially for the crowdfunding goal-setting strategy as the market correlation has complex implications for the capital availability and sufficiency in the crowdfunding market. Next, we give the comparative statics with respect to k for the optimal profits.

Proposition 8. ***(Impact of market correlation on optimal profit)****. There exists a threshold $\bar{D}$ such that for $j = SA, DA$, Π^j increases in k if and only if $k < \frac{3c-p}{c}$ and $D < \bar{D}$.*[26]

Therefore in $j = SA, DA$, with D below the cutoff $\bar{D}$, Π^j increases in k for $k < \frac{3c-p}{c}$ and then decreases in k, but if $D > \bar{D}$, Π^j monotonically decreases in k. For Π^j in SP and DP, $\frac{\partial \Pi^j}{\partial k}$ involves high-order terms and is analytically intractable, so we used simulation (details contained in Proof) for computation and concluded that Π^j has a general concave pattern in k just as in the SA, DA strategies, and the increasing interval of k will vanish in a similar way for sufficiently large or small D. That is, Π^{SP} monotonically decreases in k if D is sufficiently small, while Π^{DP} monotonically decreases in k if D is sufficiently small or sufficiently large. Otherwise Π^{SP} and Π^{DP} increases in k first before decreasing in k. The underlying intuition is related to the financing risks.

Note that when k increases, retail market demand is more correlated with crowdfunding demand, and its impact on optimal profit is twofold: crowdfunding capital can better match retail market demand which, however, may intensify overfinancing or underfinancing risk depending on the value of D. So the optimal profit shows a concave pattern as it increases in k first before decreasing. The twofold impacts are affected by D in the following way. When D is very high, the independent component of the retail market demand is sizable, so higher k intensifies underfinancing risk, which is exacerbated in the SA and DA strategies where aggressive goal setting is already leveraged as a measure to combat underfinancing risk. Therefore, in SA and DA, when D is above the cutoff value given in Proposition 8, the optimal profit monotonically decreases in k. When D is very low, on the other hand, higher k intensifies overfinancing risk which is already prominent in the single-source prudent strategy, SP, so in this case the optimal profit monotonically decreases in k. Since DP strategy represents a more subtle response to the financing risks, prudent in crowdfunding to combat overfinancing and dual-source to combat underfinancing, D being either too high or too low would intensify each risk and thus cause the optimal profit to decrease monotonically in k.

The comparative analysis for market correlation again shows that the financing decision is entwined with uncertainty and financing risks, the latter being affected by not only the market demand and capital costs, but also the production cost or profit margin. And high market correlation between crowdfunding and retail is not always beneficial. The firm must understand the interplay of different risks in its full span to be able to correctly and maximally utilize low-cost crowdfunding capital.

6. Conclusion

Financing through crowdfunding is growing rapidly with the development of information technology, especially for start-up firms which are still establishing its market position and uncertain of the market demand as crowdfunding provides a credible consumer survey about the market demand. Compared to traditional financing sources, crowdfunding capital is less costly as it lessens informational asymmetry for potential consumers so that these early buyers/investors do not require risk compensation in return, which is in contrast to the bank capital. However, the amount of capital raised from crowdfunding

[26] See Proof of Proposition 8 in Appendix for the expression of $\bar{D}$.

is affected by many fluctuating factors and the problem of capital supply uncertainty is prominent. Demand uncertainty is another major challenge for such businesses as it is directly related to profits, especially in recent days with the acceleration of new product development which led to high variation in demand forecasting. Considering crowdfunding supply uncertainty and demand uncertainty, we analyze the financing strategy of a capital constraint firm when crowdfunding interacts with traditional bank loan. The firm faces the trade off between the financing cost and the uncertainty of capital supply in choosing the optimal financing strategy.

We establish a model where a firm raises production capital from both crowdfunding and the bank to meet the demand in both crowdfunding and post-campaign retail markets. The results show that the firm can choose single-source financing with crowdfunding only or dual-source financing with crowdfunding and bank loan depending on the uncertainty ratio between the post-campaign retail demand and the crowdfunding demand. Since capital supply uncertainty is unique to the crowdfunding mechanism, our paper provides some implications for entrepreneurs when they utilize the crowdfunding platform as a capital supplier in addition to traditional financing sources. Specifically, the major implications are as follows:

• As a financing source, crowdfunding substitutes bank loan if the uncertainty ratio is lower than the threshold, and supplements bank loan when the uncertainty ratio is higher than the threshold.

Compared with a bank loan, crowdfunding has advantages in reducing financing costs and testing the market demand. But the capital supply of crowdfunding is uncertain and thus the threshold of the uncertainty ratio is used to balance the trade-off to determine whether crowdfunding substitutes or supplements bank loans.

• Whether the crowdfunding goal is prudent or aggressive in dual-source financing strategies is not only cost-based but uncertainty-related.

It is easy to overlook how capital supply and demand uncertainties weigh in firms' financing strategy, with the latter usually entailing sole cost consideration. We show that the financing-cost threshold is related to uncertainty: the crowdfunding goal should still be prudent rather than aggressive if the financing cost exceeds the threshold, even though the financing cost of crowdfunding is lower than the bank loan in an absolute sense.

• Even though the demand testing value of crowdfunding (i.e., the correlation between the crowdfunding market demand and the post-campaign retail market demand) is seen as an advantage of crowdfunding, we find that the increase of correlation does not necessarily make crowdfunding capital more favorable to the bank loan or increases the profit. The production cost and market demand need to be co-examined with consideration of the overfinancing and underfinancing risk.

Lastly, we want to point out that our model has some limitations which can be improved by future research. One is that the crowd fundraising process is largely simplified such that the heterogeneity of consumer behavior is not considered. That is, this paper does not model the buyer/investor-side decisions in an endogenous way but rather abstract them in the market demand. Second, we impose uniform distribution on the two markets' demand for traceability. It may be a beneficial direction to study the optimal financing problem with a wider range of distributions and some new implications on the optimal financing strategy may emerge from there.

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Appendix A.

Proof of Proposition 1.

Recall that $\tilde{S}_c$ and $\tilde{D}$ follow two-dimensional uniform distribution, in which $\tilde{S}_c \in [0, S_c]$ and $\tilde{D} \in [0, D]$. With the uniform assumption, we can write out the expectations, $E[\min\{c(\tilde{D}+\tilde{S}_c), \min\{F_c, p\tilde{S}_c\}+F_b\}]$ and $E[\min\{F_c, p\tilde{S}_c\}]$, as the following equations (A.1) and (A.2).

$$\begin{aligned} E[\min\{c(\tilde{D}+\tilde{S}_c), \min\{F_c, p\tilde{S}_c\}+F_b\}] = \\ & \int_0^{\frac{F_c}{p}} \frac{1}{(1-k)DS_c}\Big(\int_{k\tilde{S}_c}^{\frac{p\tilde{S}_c+F_b}{c}-\tilde{S}_c} c(\tilde{D}+\tilde{S}_c)d\tilde{D} \\ & + \int_{\frac{p\tilde{S}_c+F_b}{c}-\tilde{S}_c}^{(1-k)D+k\tilde{S}_c} (F_b+p\tilde{S}_c)d\tilde{D}\Big)d\tilde{S}_c \\ & + \int_{\frac{F_c}{p}}^{S_c} \frac{1}{(1-k)DS_c}\Big(\int_{k\tilde{S}_c}^{\frac{F_c+F_b}{c}-\tilde{S}_c} c(\tilde{D}+\tilde{S}_c)d\tilde{D} \\ & + \int_{\frac{F_c+F_b}{c}-\tilde{S}_c}^{(1-k)D+K\tilde{S}_c} (F_b+F_c)d\tilde{D}\Big)d\tilde{S}_c \end{aligned} \tag{A.1}$$

$$E[\min\{F_c, p\tilde{S}_c\}] = \int_0^{\frac{F_c}{p}} p\tilde{S}_c\frac{1}{S_c}d\tilde{S}_c + \int_{\frac{F_c}{p}}^{S_c} F_c\frac{1}{S_c}d\tilde{S}_c. \tag{A.2}$$

Firstly, by calculating equations (A.1) and (A.2), we get

$$\begin{aligned} E[\min\{c(\tilde{D}+\tilde{S}_c), \min\{F_c, p\tilde{S}_c\}+F_b\}] =& \frac{p^2S_c(c^2(1+k)^2S_c^2+3F_b(2cD(-1+k)-c(1+k)S_c+F_b))}{6cD(-1+k)p^2S_c} \\ & - \frac{3p^2S_c(c(2D-2Dk+S_c+kS_c)-2F_b)F_c+3p(c(D-Dk)+pS_c-F_b)F_c^2}{6cD(-1+k)p^2S_c} \\ & + \frac{(c+ck-2p)F_c^3}{6cD(-1+k)p^2S_c} \end{aligned} \tag{A.3}$$

and

$$E[min(F_c, pS_c)] = \frac{F_c^2}{2pS_c} + \frac{F_c(S_c-\frac{F_c}{p})}{S_c}. \tag{A.4}$$

Plugging equations (A.3) and (A.4) in to (2), we can rewrite the objective function of the firm as the following (A.5).

$$\begin{aligned} \max_{F_c\geq 0, F_b\geq 0} \Pi(F_c, F_b) =& \frac{p-c}{c}\Big(\frac{p^2S_c(c^2(1+k)^2S_c^2+3F_b(2cD(-1+k)-c(1+k)S_c+F_b))}{6cD(-1+k)p^2S_c}\Big) \\ & - \frac{p-c}{c}\Big(\frac{3p^2S_c(c(2D-2Dk+S_c+kS_c)-2F_b)F_c+3p(c(D-Dk)+pS_c-F_b)F_c^2}{6cD(-1+k)p^2S_c}\Big) \\ & + \frac{p-c}{c}\Big(\frac{(c+ck-2p)F_c^3}{6cD(-1+k)p^2S_c}\Big) - f\Big(\frac{F_c^2}{2pS_c} + \frac{F_c(S_c-\frac{F_c}{p})}{S_c}\Big) - rF_b. \end{aligned} \tag{A.5}$$

To get the optimal solution, we rewrite our optimization problem above as follows the following (A.6).

$$\begin{aligned} &\min -\Pi(F_c, F_b), \\ &s.t. - F_b \le 0; -F_c \le 0; F_c \le pS_c. \end{aligned} \tag{A.6}$$

To solve the above optimization problem (A.6), we need to construct a Lagrange function as the following (A.7).

$$L(F_c, F_b, u_1, u_2, u_3) = -\Pi(F_c, F_b) - u_1 F_c - u_2 F_b + u_3(F_c - pS_c). \tag{A.7}$$

Based on the equations (A.6) and (A.7), the Karush-Kuhn-Tucker (KKT) conditions that used to solve the above optimization problem is illustrated as the following (A.8).

$$\begin{aligned} &(1)\ \frac{\partial L}{\partial F_c} = 0, \frac{\partial L}{\partial F_b} = 0. \\ &(2)\ u_1 \ge 0, u_2 \ge 0, u_3 \ge 0, F_c \ge 0, F_b \ge 0, F_c \le pS_c. \\ &(3)\ u_1 F_c = 0, u_2 F_b = 0, u_3(F_c - pS_c) = 0. \end{aligned} \tag{A.8}$$

According to the KKT conditions demonstrated in the (A.8), the following cases are illustrated to show the optimal solutions.

$$\begin{aligned} &1.u_1 = 0, u_2 = 0, u_3 = 0, F_c \ge 0, F_b \ge 0, F_c \le pS_c. \\ &2.u_1 = 0, u_2 = 0, u_3 > 0, F_c \ge 0, F_b \ge 0, F_c = pS_c. \\ &3.u_1 > 0, u_2 = 0, u_3 = 0, F_c = 0, F_b \ge 0, F_c < pS_c. \\ &4.u_1 = 0, u_2 > 0, u_3 = 0, F_c \ge 0, F_b = 0, F_c \le pS_c. \\ &5.u_1 > 0, u_2 > 0, u_3 = 0, F_c = 0, F_b = 0, F_c \le pS_c. \\ &6.u_1 > 0, u_2 = 0, u_3 > 0, F_c = 0, F_b \ge 0, F_c = pS_c. \\ &7.u_1 = 0, u_2 > 0, u_3 > 0, F_c \ge 0, F_b = 0, F_c = pS_c. \\ &8.u_1 > 0, u_2 > 0, u_3 > 0, F_c = 0, F_b = 0, F_c = pS_c. \end{aligned}$$

Now we are going to solve each of these eight cases as follows.

1. u_1=0, u_2=0, u_3=0, $F_c \ge 0$,$F_b \ge 0$,$F_c \le pS_c$.

In this case, the condition $u_1 F_c = 0, u_2 F_b = 0, u_3(F_c - pS_c) = 0$ is always met, so we just consider the following four cases $1.1 - 1.4$ for the different values of F_c and F_b.

1.1 $F_c > 0$,$F_b > 0$,$F_c < pS_c$.

The KKT conditions for the optimal solution including: (1) $\frac{\partial L}{\partial F_c} = -\frac{\partial \Pi(F_c,F_b)}{\partial F_c} = 0$, $\frac{\partial L}{\partial F_b} = -\frac{\partial \Pi(F_c,F_b)}{\partial F_b} = 0$, (2) $F_c > 0$,$F_b > 0$,$F_c < pS_c$.

The first-order conditions of $\frac{\partial \Pi(F_c,F_b)}{\partial F_c}$ and $\frac{\partial \Pi(F_c,F_b)}{\partial F_b}$ are illustrated in the following (A.9).

$$\begin{aligned} \frac{\partial \Pi(F_c, F_b)}{\partial F_c} &= -\frac{(pS_c - F_c)(cp(2D(-1+k)(c+cf-p) - (1+k)(c-p)S_c) - (c-p)(c(1+k)F_c - 2p(F_b+F_c)))}{2c^2 D(-1+k)p^2 S_c}, \\ \frac{\partial \Pi(F_c, F_b)}{\partial F_b} &= \frac{pS_c(-2cD(-1+k)(c-p+cr) + c(1+k)(c-p)S_c) + 2p(-c+p)S_c F_b + (c-p)F_c(-2pS_c + F_c)}{2c^2 D(-1+k)pS_c}. \end{aligned} \tag{A.9}$$

Therefore, the optimal solution can be obtained by solving the following simultaneous equations:

$$\frac{\partial L}{\partial F_c} = -\frac{\partial \Pi(F_c, F_b)}{\partial F_c} = 0,$$
$$\frac{\partial L}{\partial F_b} = -\frac{\partial \Pi(F_c, F_b)}{\partial F_b} = 0.$$

By solving the above equations, we get

$$F_c^* = F_{c1} = \frac{S_c(p-c)}{2}\sqrt{\frac{8c^2p(1-k)(r-f)D}{(p-c)^3 S_c} + \frac{c^2(1+k)^2}{(p-c)^2}} + \frac{c(1+k)}{2}S_c. \tag{A.10}$$

$$F_c^* = F_{c2} = \frac{S_c(c-p)}{2}\sqrt{\frac{8c^2p(1-k)(r-f)D}{(p-c)^3 S_c} + \frac{c^2(1+k)^2}{(p-c)^2}} + \frac{c(1+k)}{2}S_c.$$

$$F_b^* = \frac{c^2}{p-c}(1-k)(\frac{p-c}{c} - r)D + \frac{c}{2}(1+k)S_c - F_c(1 - \frac{F_c}{2pS_c}).$$

Combining the feasible conditions in our model: $0 < f < r, S_c > 0, D > 0, p - c > cf, p - c > cr, 0 < k < 1$, we get $F_{c1} > 0, F_{c2} < 0$. So $F_c^* = F_{c1}$. Recall the optimal condition that $F_c < pS_c$, therefore, by solving the inequality $F_{c1} < pS_c$, we get the condition

$$D < \frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}.$$

Furthermore, taking $F_c^* = F_{c1}$ into the expression of F_b^*, and let $F_b^* > 0$ with the condition $D < \frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}$, we get the optimal conditions for this case (i.e., $F_b^* > 0, F_c^* < pS_c$) as the following equation.

(1) $0 < c \leq \frac{p}{3}, 0 < r < 1, 0 < f < r, 0 < k < 1, D_1 < D < \frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}.$

(2) $\frac{p}{3} < c \leq \frac{p}{2}, 0 < r \leq \frac{-c+p}{2c}, 0 < f < r, 0 < k < 1, D_1 < D < \frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}.$

(3) $\frac{p}{3} < c \leq \frac{p}{2}, \frac{-c+p}{2c} < r < 1, \frac{c-p+2cr}{c} < f < r, 0 < k < 1, D_1 < D < \frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}.$

(4) $\frac{p}{2} < c < p, 0 < r \leq \frac{-c+p}{2c}, 0 < f < r, 0 < k < 1, D_1 < D < \frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}.$

(5) $\frac{p}{2} < c < p, \frac{-c+p}{2c} < r < 1, \frac{c-p+2cr}{c} < f < r, 0 < k < 1, D_1 < D < \frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}.$

The expression of D_1 is shown as the following equation.

$$D_1 = \frac{(p-c)[4p(r-f)(p-c-ck) - c(1+k)^2(p-c-cr)]}{4p(1-k)(p-c-cf)^2}S_c + \frac{(p-c)(2p-c-ck\sqrt{4p(r-f)^2(p-c-ck) + (1+k)^2(p-c-cr)^2})}{4p(1-k)(p-c-cf)^2}S_c.$$

1.2 u_1=0, u_2=0, u_3=0, $F_c \geq 0, F_b = 0, F_c \leq pS_c$.

In this case, since the optimal value of bank loan $F_b = 0$ cannot met the first order condition of F_b: $\frac{\partial L}{\partial F_b} = -\frac{\partial \Pi(F_c, F_b)}{\partial F_b} = 0$, so this case is infeasible.

1.3 u_1=0, u_2=0, u_3=0, $F_c = 0, F_b > 0$.

In this case, since the optimal value of bank loan $F_c = 0$ cannot met the first order condition of F_b: $\frac{\partial L}{\partial F_c} = -\frac{\partial \Pi(F_c,F_b)}{\partial F_c} = 0$, so this case is infeasible.

1.4 u_1=0, u_2=0, u_3=0, $F_c = pS_c$,$F_b > 0$.

In this case, $F_c^* = pS_c$, recall the case 1.1, we get the condition for $F_c^* = pS_c$ is $D > \frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}$. Then taking $F_c^* = pS_c$ into the first order condition's solution of F_b, that is $F_b^* = \frac{c^2}{p-c}(1-k)(\frac{p-c}{c} - r)D + \frac{c}{2}(1+k)S_c - F_c(1-\frac{F_c}{2pS_c})$.

Therefore, we need to keep $F_b^* > 0$ with the condition $D > \frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}$, solving the inequality by mathmatica, we get the following conditions for $F_c^* = pS_c$, and $F_b^* = \frac{c^2}{p-c}(1-k)(\frac{p-c}{c} - r)D + \frac{c}{2}(1+k)S_c - F_c(1-\frac{F_c}{2pS_c}) > 0$.

(1) $0 < c \le \dfrac{p}{3}, 0 < r < 1, 0 < f < r, 0 < k < 1, D > \dfrac{(p-c)(p-c-ck)S_c}{2c(p-c-cr)(1-k)}$.

(2) $\dfrac{p}{3} < c \le \dfrac{p}{2}, 0 < r \le \dfrac{-c+p}{2c}, 0 < k < 1, D > \dfrac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}$.

(3) $\dfrac{p}{3} < c \le \dfrac{p}{2}, \dfrac{-c+p}{2c} < r < 1, 0 < f < \dfrac{c-p+2cr}{c}, 0 < k < 1, D > \dfrac{(p-c)(p-c-ck)S_c}{2c(p-c-cr)(1-k)}$.

(4) $\dfrac{p}{3} < c \le \dfrac{p}{2}, \dfrac{-c+p}{2c} < r < 1, f > \dfrac{c-p+2cr}{c}, 0 < k < 1, D > \dfrac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}$.

(5) $\dfrac{p}{2} < c < p, 0 < r \le \dfrac{-c+p}{2c}, 0 < k < \dfrac{p-c}{c}, D > \dfrac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}$.

(6) $\dfrac{p}{2} < c < p, \dfrac{-c+p}{2c} < r < 1, 0 < f < \dfrac{c-p+2cr}{c}, 0 < k < \dfrac{p-c}{c}, D > \dfrac{(p-c)(p-c-ck)S_c}{2c(p-c-cr)(1-k)}$.

(7) $\dfrac{p}{2} < c < p, \dfrac{-c+p}{2c} < r < 1, f > \dfrac{c-p+2cr}{c}, 0 < k < \dfrac{p-c}{c}, D > \dfrac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}$.

(8) $\dfrac{p}{2} < c < p, k > \dfrac{p-c}{c}$.

2. u_1=0, u_2=0, $u_3 > 0$, $F_c = pS_c$,$F_b > 0$.

In this case, the optimal conditions are: (1) $\frac{\partial L}{\partial F_c} = -\frac{\partial \Pi(F_c,F_b)}{\partial F_c} = 0, \frac{\partial L}{\partial F_b} = -\frac{\partial \Pi(F_c,F_b)}{\partial F_b} = 0$, (2) $F_c > 0$,$F_b > 0$,$F_c < pS_c$, and (3) $u_3 = \frac{\partial \Pi(F_c,F_b)}{\partial F_c} > 0$.

First, taking taking $F_c^* = pS_c$ into the first order condition's solution of F_b, that is $F_b^* = \frac{c^2}{p-c}(1-k)(\frac{p-c}{c} - r)D + \frac{c}{2}(1+k)S_c - F_c(1-\frac{F_c}{2pS_c})$. However, taking the above optimal solutions $F_c^* = pS_c$ and F_b^* into the first order condition $\frac{\partial \Pi(F_c,F_b)}{\partial F_c}$, we get $\frac{\partial \Pi(F_c,F_b)}{\partial F_c} < 0$ always holds. Therefore, the optimal condition $u_3 = \frac{\partial \Pi(F_c,F_b)}{\partial F_c} > 0$ cannot be satisfied, which means this case is not the optimal for the optimization problem.

3. $u_1 > 0$, u_2=0, $u_3 > 0$, $F_c = 0$,$F_b > 0$.

In this case, the optimal conditions are: (1) $\frac{\partial L}{\partial F_c} = -\frac{\partial \Pi(F_c,F_b)}{\partial F_c} = 0, \frac{\partial L}{\partial F_b} = -\frac{\partial \Pi(F_c,F_b)}{\partial F_b} = 0$, (2) $F_c > 0$,$F_b > 0$,$F_c < pS_c$, and (3) $u_1 = -\frac{\partial \Pi(F_c,F_b)}{\partial F_c} > 0$.

First, taking taking $F_c^* = 0$ into the first order condition's solution of F_b, that is $F_b^* = \frac{c^2}{p-c}(1-k)(\frac{p-c}{c} - r)D + \frac{c}{2}(1+k)S_c - F_c(1-\frac{F_c}{2pS_c})$. However, taking the above optimal solutions $F_c^* = 0$ and F_b^* into the first order condition $\frac{\partial \Pi(F_c,F_b)}{\partial F_c}$, we get $-\frac{\partial \Pi(F_c,F_b)}{\partial F_c} < 0$ always holds. Therefore, the optimal condition $u_1 = -\frac{\partial \Pi(F_c,F_b)}{\partial F_c} > 0$ cannot be satisfied, which means this case is not the optimal for the optimization problem.

4. u_1=0, $u_2 > 0$, u_3=0, $F_c > 0$,$F_b = 0$, $F_c \le pS_c$.

In this case, the optimal conditions are: (1) $\frac{\partial L}{\partial F_c} = -\frac{\partial \Pi(F_c,F_b)}{\partial F_c} = 0, \frac{\partial L}{\partial F_b} = -\frac{\partial \Pi(F_c,F_b)}{\partial F_b} = 0$, (2) $F_c > 0$,$F_b > 0$,$F_c < pS_c$, and (3) $u_2 = -\frac{\partial \Pi(F_c,F_b)}{\partial F_b} > 0$.

First, by solving the first order condition of F_c: $\frac{\partial L}{\partial F_c}=-\frac{\partial \Pi(F_c,F_b)}{\partial F_c}=0$ with the condition u_1=0, u_3=0, and $F_b=0$, we get $F_c=pS_c$ or $F_c=\frac{2cp(1-k)}{2p-c-ck}(1-\frac{cf}{p-c})D+\frac{cp(1+k)}{2p-c-ck}S_c$. So there are two cases that should be analyzed as followings:

1) $F_c=\frac{2cp(1-k)}{2p-c-ck}(1-\frac{cf}{p-c})D+\frac{cp(1+k)}{2p-c-ck}S_c<pS_c$, $F_b=0$; 2) $F_c=\frac{2cp(1-k)}{2p-c-ck}(1-\frac{cf}{p-c})D+\frac{cp(1+k)}{2p-c-ck}S_c>pS_c$, $F_b=0$.

1) $F_c=\frac{2cp(1-k)}{2p-c-ck}(1-\frac{cf}{p-c})D+\frac{cp(1+k)}{2p-c-ck}S_c<pS_c$, $F_b=0$.

First, by solving the inequality $\frac{2cp(1-k)}{2p-c-ck}(1-\frac{cf}{p-c})D+\frac{cp(1+k)}{2p-c-ck}S_c<pS_c$, we get$D<\frac{(p-c)(p-c-cK)S_c}{c(p-c-cf)(1-k)}$.

Second, taking $F_c=\frac{2cp(1-k)}{2p-c-ck}(1-\frac{cf}{p-c})D+\frac{cp(1+k)}{2p-c-ck}S_c+\frac{2p}{c+ck-2p}F_b<pS_c$, $F_b=0$ and $D<\frac{(p-c)(p-c-ck)S_c}{c(p-c-cf)(1-k)}$ into the optimal condition $u_2=-\frac{\partial \Pi(F_c,F_b)}{\partial F_b}$ and let $u_2>0$, we get

(1) $0<c\leq \frac{p}{3}, 0<r<1, 0<f<r, 0<k<1, 0<D<D_1$.

(2) $\frac{p}{3}<c\leq \frac{p}{2}, 0<r\leq \frac{-c+p}{2c}, 0<f<r, 0<k<1, 0<D<D_1$.

(3) $\frac{p}{3}<c\leq \frac{p}{2}, \frac{-c+p}{2c}<r<1, 0<f<\frac{c-p+2cr}{c}, 0<k<1, 0<D<\frac{(p-c)(p-c-ck)S_c}{c(p-c-cf)(1-k)}$.

(4) $\frac{p}{3}<c\leq \frac{p}{2}, \frac{-c+p}{2c}<r<1, \frac{c-p+2cr}{c}<f<r, 0<k<1, 0<D<D_1$.

(5) $\frac{p}{2}<c<p, 0<r\leq \frac{-c+p}{2c}, 0<f<r, 0<k<1, 0<D<D_1$.

(6) $\frac{p}{2}<c<p, \frac{-c+p}{2c}<r<1, 0<f<\frac{c-p+2cr}{c}, 0<k<1, 0<D<\frac{(p-c)(p-c-ck)S_c}{c(p-c-cf)(1-k)}$.

(7) $\frac{p}{2}<c<p, \frac{-c+p}{2c}<r<1, \frac{c-p+2cr}{c}<f<r, 0<k<1, 0<D<D_1$.

2) $F_c=\frac{2cp(1-k)}{2p-c-ck}(1-\frac{cf}{p-c})D+\frac{cp(1+k)}{2p-c-ck}S_c>pS_c$, $F_b=0$.

First, by solving the inequality $\frac{2cp(1-k)}{2p-c-ck}(1-\frac{cf}{p-c})D+\frac{cp(1+k)}{2p-c-ck}S_c>pS_c$, we get $D>\frac{(p-c)(p-c-ck)S_c}{c(p-c-cf)(1-k)}$.

Second, taking $F_c=pS_c$, $F_b=0$ and $D<\frac{(p-c)(p-c-ck)S_c}{c(p-c-cf)(1-k)}$ into the optimal condition $u_2=-\frac{\partial \Pi(F_c,F_b)}{\partial F_b}$ and let $u_2>0$, we get the following conditions as

(1)$\frac{p}{3}<c\leq \frac{p}{2}, \frac{-c+p}{2c}<r<1, 0<f<\frac{c-p+2cr}{c}, 0<k<1, \frac{(p-c)(p-c-ck)S_c}{c(p-c-cf)(1-k)}<D<\frac{(p-c)(p-c-ck)S_c}{2c(p-c-cr)(1-k)}$.

(2)$\frac{p}{2}<c<p, \frac{-c+p}{2c}<r<1, 0<f<\frac{c-p+2cr}{c}, 0<k<1, \frac{(p-c)(p-c-ck)S_c}{c(p-c-cf)(1-k)}<D<\frac{(p-c)(p-c-ck)S_c}{2c(p-c-cr)(1-k)}$.

5. $u_1>0$, $u_2>0$, u_3=0, $F_c=0$,$F_b=0$, $F_c<pS_c$.

In this case, $F_c=0$,$F_b=0$, and the optimal conditions are (1) $\frac{\partial L}{\partial F_c}=-\frac{\partial \Pi(F_c,F_b)}{\partial F_c}=0$, $\frac{\partial L}{\partial F_b}=-\frac{\partial \Pi(F_c,F_b)}{\partial F_b}=0$, (2)$u_1=-\frac{\partial \Pi(F_c,F_b)}{\partial F_c}>0$ and $u_2=-\frac{\partial \Pi(F_c,F_b)}{\partial F_b}>0$.

Taking $F_c=0$,$F_b=0$ into the condition $u_1=-\frac{\partial \Pi(F_c,F_b)}{\partial F_c}$ and $u_2=-\frac{\partial \Pi(F_c,F_b)}{\partial F_b}$, we get $u_1<0, u_2<0$. Therefore, this case is not the optimal solution.

6. $u_1>0$, u_2=0, $u_3>0$, $F_c=0$,$F_b>0$, $F_c=pS_c$.

Since $F_c=0$, and $F_c=pS_c$ cannot satisfied at the same time, so we give this case up.

7. u_1=0, $u_2>0$, $u_3>0$, $F_c>0$,$F_b=0$, $F_c=pS_c$.

In this case, $F_b=0$, $F_c=pS_c$, and the optimal conditions are (1) $\frac{\partial L}{\partial F_c}=-\frac{\partial \Pi(F_c,F_b)}{\partial F_c}=0$, $\frac{\partial L}{\partial F_b}=-\frac{\partial \Pi(F_c,F_b)}{\partial F_b}=0$, (2)$u_2=-\frac{\partial \Pi(F_c,F_b)}{\partial F_b}>0$ and $u_3=\frac{\partial \Pi(F_c,F_b)}{\partial F_c}>0$.

Taking $F_b=0$, $F_c=pS_c$ into the values of $u_2=-\frac{\partial \Pi(F_c,F_b)}{\partial F_b}$ and $u_3=\frac{\partial \Pi(F_c,F_b)}{\partial F_c}$, we get $u_3<0$, therefore this case is not the optimal solution.

8. $u_1>0$, $u_2>0$, $u_3>0$, $F_c=0$,$F_b=0$, $F_c=pS_c$.

Since $F_c = 0$, and $F_c = pS_c$ cannot satisfied at the same time, so we give this case up.

Based on the above 8 cases, we can get all the solutions as follows.

Solution 1: Inner solution (DP strategy), that is, $0 < F_c^* < pS_c$, and $F_b^* > 0$

$$F_c^* = F_{c1} = \frac{S_c(p-c)}{2}\sqrt{\frac{8c^2p(1-k)(r-f)D}{(p-c)^3S_c} + \frac{c^2(1+k)^2}{(p-c)^2}} + \frac{c(1+k)}{2}S_c.$$

$$F_b^* = \frac{c^2}{p-c}(1-k)(\frac{p-c}{c} - r)D + \frac{c}{2}(1+k)S_c - F_c(1 - \frac{F_c}{2pS_c}).$$

With the following conditions satisfied:

(1) $0 < c \leq \frac{p}{3}, 0 < r < 1, 0 < f < r, 0 < k < 1, D_1 < D < \frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}$.

(2) $\frac{p}{3} < c \leq \frac{p}{2}, 0 < r \leq \frac{-c+p}{2c}, 0 < f < r, 0 < k < 1, D_1 < D < \frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}$.

(3) $\frac{p}{3} < c \leq \frac{p}{2}, \frac{-c+p}{2c} < r < 1, \frac{c-p+2cr}{c} < f < r, 0 < k < 1, D_1 < D < \frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}$.

(4) $\frac{p}{2} < c < p, 0 < r \leq \frac{-c+p}{2c}, 0 < f < r, 0 < k < 1, D_1 < D < \frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}$.

(5) $\frac{p}{2} < c < p, \frac{-c+p}{2c} < r < 1, \frac{c-p+2cr}{c} < f < r, 0 < k < 1, D_1 < D < \frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}$.

Solution 2: Corner solution (DA strategy), that is, $F_c^* = pS_c$, $F_b^* = \frac{c^2}{p-c}(1-k)(\frac{p-c}{c} - r)D + \frac{c}{2}(1+k)S_c - F_c(1 - \frac{F_c}{2pS_c}) > 0$.

With the following conditions satisfied:

(1) $0 < c \leq \frac{p}{3}, 0 < r < 1, 0 < f < r, 0 < k < 1, D > \frac{(p-c)(p-c-ck)S_c}{2c(p-c-cr)(1-k)}$.

(2) $\frac{p}{3} < c \leq \frac{p}{2}, 0 < r \leq \frac{-c+p}{2c}, 0 < k < 1, D > \frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}$.

(3) $\frac{p}{3} < c \leq \frac{p}{2}, \frac{-c+p}{2c} < r < 1, 0 < f < \frac{c-p+2cr}{c}, 0 < k < 1, D > \frac{(p-c)(p-c-ck)S_c}{2c(p-c-cr)(1-k)}$.

(4) $\frac{p}{3} < c \leq \frac{p}{2}, \frac{-c+p}{2c} < r < 1, f > \frac{c-p+2cr}{c}, 0 < k < 1, D > \frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}$.

(5) $\frac{p}{2} < c < p, 0 < r \leq \frac{-c+p}{2c}, 0 < k < \frac{p-c}{c}, D > \frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}$.

(6) $\frac{p}{2} < c < p, \frac{-c+p}{2c} < r < 1, 0 < f < \frac{c-p+2cr}{c}, 0 < k < \frac{p-c}{c}, D > \frac{(p-c)(p-c-ck)S_c}{2c(p-c-cr)(1-k)}$.

(7) $\frac{p}{2} < c < p, \frac{-c+p}{2c} < r < 1, f > \frac{c-p+2cr}{c}, 0 < k < \frac{p-c}{c}, D > \frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}$.

(8) $\frac{p}{2} < c < p, k > \frac{p-c}{c}$.

Solution 3: Corner solution (SP strategy), that is, $F_c^* = \frac{2cp(1-k)}{2p-c-ck}(1 - \frac{cf}{p-c})D + \frac{cp(1+k)}{2p-c-ck}S_c < pS_c$,

$F_b^* = 0$. With the following conditions satisfied:

(1) $0 < c \leq \frac{p}{3}, 0 < r < 1, 0 < f < r, 0 < k < 1, 0 < D < D_1$.

(2) $\frac{p}{3} < c \leq \frac{p}{2}, 0 < r \leq \frac{-c+p}{2c}, 0 < f < r, 0 < k < 1, 0 < D < D_1$.

(3) $\frac{p}{3} < c \leq \frac{p}{2}, \frac{-c+p}{2c} < r < 1, 0 < f < \frac{c-p+2cr}{c}, 0 < k < 1, 0 < D < \frac{(p-c)(p-c-ck)S_c}{c(p-c-cf)(1-k)}$.

(4) $\frac{p}{3} < c \leq \frac{p}{2}, \frac{-c+p}{2c} < r < 1, \frac{c-p+2cr}{c} < f < r, 0 < k < 1, 0 < D < D_1$.

(5) $\frac{p}{2} < c < p, 0 < r \leq \frac{-c+p}{2c}, 0 < f < r, 0 < k < 1, 0 < D < D_1$.

(6) $\frac{p}{2} < c < p, \frac{-c+p}{2c} < r < 1, 0 < f < \frac{c-p+2cr}{c}, 0 < k < 1, 0 < D < \frac{(p-c)(p-c-ck)S_c}{c(p-c-cf)(1-k)}$.

(7) $\frac{p}{2} < c < p, \frac{-c+p}{2c} < r < 1, \frac{c-p+2cr}{c} < f < r, 0 < k < 1, 0 < D < D_1$.

Solution 4: Corner solution (SA strategy), that is,$F_c^* = pS_c$, $F_b^* = 0$. With the following conditions satisfied:

(1)$\frac{p}{3} < c \leq \frac{p}{2}, \frac{-c+p}{2c} < r < 1, 0 < f < \frac{c-p+2cr}{c}, 0 < k < 1, \frac{(p-c)(p-c-ck)S_c}{c(p-c-cf)(1-k)} < D < \frac{(p-c)(p-c-ck)S_c}{2c(p-c-cr)(1-k)}$.

(2)$\frac{p}{2} < c < p, \frac{-c+p}{2c} < r < 1, 0 < f < \frac{c-p+2cr}{c}, 0 < k < 1, \frac{(p-c)(p-c-ck)S_c}{c(p-c-cf)(1-k)} < D < \frac{(p-c)(p-c-ck)S_c}{2c(p-c-cr)(1-k)}$.

Check the range of the parameter and let $\frac{D}{S_c}$ as ρ, we can rewrite the above four optimal solutions as follows:

1. When $f < \frac{c-p+2cr}{c}$, the optimal solution is
(i) SP strategy with $\rho < \frac{(p-c)(p-c-ck)}{c(p-c-cf)(1-k)}$.
(ii) SA strategy with $\frac{(p-c)(p-c-ck)}{c(p-c-cf)(1-k)} < \rho < \frac{(p-c)(p-c-ck)}{2c(p-c-cr)(1-k)}$.
(iii) DA strategy with $\rho > \frac{(p-c)(p-c-ck)}{2c(p-c-cr)(1-k)}$.
2. When $f > \frac{c-p+2cr}{c}$, the optimal solution is
(i) SP strategy with $\rho < \rho_1$ [27].
(ii) DP strategy with $\rho_1 < \rho < \frac{(p-c)(p-c-ck)}{2c^2(r-f)(1-k)}$.
(iii) DA strategy with $\rho > \frac{(p-c)(p-c-ck)}{2c^2(r-f)(1-k)}$.

To simplify notation and combine the above four financing strategy, let denote $\hat{\rho}$ and $\hat{f}$ representing two thresholds for ρ and f as the following equations.

$$\hat{\rho} = \begin{cases} \frac{(p-c)(p-c-ck)}{2c(p-c-cr)(1-k)}, & f < \frac{c-p+2cr}{c}, \\ \rho_1, & f > \frac{c-p+2cr}{c}. \end{cases}$$

$$\hat{f} = \begin{cases} \frac{(p-c)(cD(1-k)-(p-c-ck)S_c)}{c^2(1-k)D}, & f < \frac{c-p+2cr}{c}, \\ \frac{2c^2D(1-k)r-(p-c)(p-c-ck)S_c}{2c^2(1-k)D}, & f > \frac{c-p+2cr}{c}. \end{cases}$$

Based on the expressions of $\hat{\rho}$ and $\hat{f}$, we can get the optimal conditions and the optimal financing strategies.

(1) When $\rho < \hat{\rho}$ and $f < \hat{f}$, SA Strategy (Single-source financing via aggressive crowdfunding) is the optimal financing strategy and $F_c^{SA} = pS_c, F_b^{SA} = 0$.

[27] $\rho_1 = \frac{(p-c)[4p(r-f)(p-c-ck)-c(1+k)^2(p-c-cr)]}{4p(1-k)(p-c-cf)^2} + \frac{(p-c)(2p-c-ck)\sqrt{4p(r-f)^2(p-c-ck)+(1+k)^2(p-c-cr)^2}}{4p(1-k)(p-c-cf)^2}$.

(2) When $\rho < \hat{\rho}$ and $f > \hat{f}$, SP Strategy (Single-source financing via prudent crowdfunding) is the optimal financing strategy and $F_c^{SP} = \frac{cp(1+k)}{2p-c(1+k)}S_c + \frac{2cp(1-k)}{2p-c(1+k)}(1-\frac{cf}{p-c})D < pS_c$ and $F_b^{SP} = 0$.

(3) When $\rho > \hat{\rho}$ and $f < \hat{f}$, DA Strategy (Dual-source financing via aggressive crowdfunding and bank) is the optimal financing strategy and $F_c^{DA} = pS_c$ and $F_b^{DA} = \frac{c^2}{p-c}(1-k)(\frac{p-c}{c} - r)D + \frac{c(1+k)-p}{2}S_c$.

(4) When $\rho > \hat{\rho}$ and $f > \hat{f}$, DP Strategy (Dual-source financing via prudent crowdfunding and bank) is the optimal financing strategy and $F_c^{DP} = F_{c1} < pS_c$ and $F_b^{DP} = \frac{c^2}{p-c}(1-k)(\frac{p-c}{c} - r)D + \frac{c(1+k)}{2}S_c - (1 - \frac{F_{c1}}{2pS_c})F_{c1}$.

Proposition 1 is proved. □

Proof of Proposition 2.

From Proposition 1, we can get the four financing strategies and comparative statics as follows:

1. SA Strategy :

$$F_c^{SA} = pS_c \text{ and } F_b^{SA} = 0 \quad \Rightarrow \quad \frac{\partial F_c}{\partial D} = 0, \frac{\partial F_b}{\partial D} = 0.$$

2. SP Strategy:

$$F_c^{SP} = \frac{cp(1+k)}{2p-c(1+k)}S_c + \frac{2cp(1-k)}{2p-c(1+k)}(1-\frac{cf}{p-c})D \text{ and } F_b^{SP} = 0.$$

$$\Rightarrow \frac{\partial F_c}{\partial D} = \frac{2c(-1+K)(c+cf-p)p}{(c+ck-2p)(c-p)} > 0, \frac{\partial F_b}{\partial D} = 0.$$

3. DA Strategy:

$$F_c^{DA} = pS_c \text{ and } F_b^{DA} = \frac{2cD(1-k)(p-c-cr)}{2(p-c)} + \frac{c(1+k)S_c}{2} - \frac{pS_c}{2}.$$

$$\Rightarrow \frac{\partial F_c}{\partial D} = 0, \frac{\partial F_b}{\partial D} = \frac{c(1-K)(p-c-cr)}{p-c} > 0.$$

4. DP Strategy:

$$F_c^{DP} = F_{c1} < pS_c \text{ and } F_b^{DP} = \frac{2cD(1-k)(p-c-cr)}{2(p-c)} + \frac{c(1+k)S_c}{2} - (1 - \frac{F_{c1}}{2pS_c})F_{c1},$$

where F_c^{DP} is given in (A.10) earlier. We get:

$$\frac{\partial F_{c1}}{\partial D} = \frac{2c^2(-1+k)(f-r)}{(c-p)^2 p\sqrt{\frac{c^2(-8D(-1+k)p(f-r)+(1+k)^2(c-p)S_c)}{(c-p)^3p^4S_c}}} > 0,$$

and

$$\frac{\partial F_b}{\partial D} = \frac{c(p-c-cf)}{p-c} + \frac{c(p-c)(2p-c)(r-f)}{c-p}\sqrt{\frac{S}{(p-c)((p-c)S+8p(r-f)D}}$$

Let $\frac{\partial F_b}{\partial D} > 0$, we get the following conditions

$$\begin{aligned}
&(1)\quad r < \frac{p-c}{2p-c}, \text{ or} \\
&(2)\quad r > \frac{p-c}{2p-c}, f > \frac{p-c+cr-2pr}{2c-2p}, \text{ or} \\
&(3)\quad r > \frac{p-c}{2p-c}, f < \frac{p-c+cr-2pr}{2c-2p}, D > \frac{(p-c)((1+2f)(c-p)+(2p-c)r)(p(2f-2r-1)+c(1+r))S}{8(p-cf-c)^2p(r-f)}.
\end{aligned} \tag{A.11}$$

Proposition 2 is proved. □

Proof of the Proposition 3.

From Proposition 1, we can get the four financing strategies and comparative statics as follows:
1. SA Strategy :

$$F_c^{SA} = pS_c \text{ and } F_b^{SA} = 0 \quad \Rightarrow \quad \frac{\partial F_c}{\partial S_c} = p > 0, \frac{\partial F_b}{\partial S_c} = 0.$$

2. SP Strategy:

$$F_c^{SP} = \frac{cp(1+k)}{2p-c(1+k)}S_c + \frac{2cp(1-k)}{2p-c(1+k)}(1-\frac{cf}{p-c})D \text{ and } F_b^{SP} = 0$$

$$\Rightarrow \frac{\partial F_c}{\partial S_c} = \frac{2c(-1+k)(c+cf-p)p}{(c+ck-2p)(c-p)} > 0, \frac{\partial F_b}{\partial S_c} = 0.$$

3. DA Strategy:

$$F_c^{DA} = pS_c \text{ and } F_b^{DA} = \frac{2cD(1-k)(p-c-cr)}{2(p-c)} + \frac{c(1+k)S_c}{2} - \frac{pS_c}{2},$$

$$\Rightarrow \frac{\partial F_c}{\partial S_c} = p > 0, \frac{\partial F_b}{\partial S_c} = \frac{c-p}{2} < 0.$$

4. DP Strategy:

$$F_c^{DP} = F_{c1} < pS_c \text{ and } F_b^{DP} = \frac{2cD(1-k)(p-c-cr)}{2(p-c)} + \frac{c(1+k)S_c}{2} - (1-\frac{F_{c1}}{2pS_c})F_{c1},$$

where F_c^{DP} is given in (A.10) earlier. We can get

$$\frac{\partial F_{c1}}{\partial S_c} = \frac{c(c(p-c)S_c + 4cp(r-f)D + (2p-c)p^3 S_c\sqrt{\frac{c^2(p-c)((p-c)S_c+8p(r-f)D)}{(2p-c)^2p^6S_c)}}}{2(2p-c)p^3S_c\sqrt{\frac{c^2(p-c)((p-c)S_c+8p(r-f)D)}{(2p-c)^2p^6S_c}}} > 0,$$

and

$$\frac{\partial F_b}{\partial S_c} = \frac{c^2(4p(f-r)D + S_c(c-p+p^3\sqrt{\frac{c^2(p-c)((p-c)S_c+8p(r-f)D)}{(2p-c)^2p^6S_c}}))}{4p^4S_c\sqrt{\frac{c^2(p-c)((p-c)S_c+8p(r-f)D)}{(2p-c)^2p^6S_c}}}.$$

Let $\frac{\partial F_b}{\partial S_c} > 0$, and we get

$$c > \frac{p}{2} \text{ and } k > \frac{p-c}{c}.$$

Recall we had the assumptions $c < p$, and $0 \le k \le 1$, so we can rewrite the condition for $\frac{\partial F_b}{\partial S_c} > 0$ as $c > \frac{p}{1+k}$.

Proposition 3 is proved. □

Proof of Proposition 4.

The formula of $\hat{\rho}$ and $\hat{f}$ in Proposition 1 are as follows:

$$\hat{\rho} = \begin{cases} \frac{(p-c)(p-c-ck)}{2c(p-c-cr)(1-k)}, & f < \frac{c-p+2cr}{c}, \\ \rho_1, & f > \frac{c-p+2cr}{c}. \end{cases}$$

$$\hat{f}=\begin{cases}\frac{(p-c)(cD(1-k)-(p-c-ck)S_c)}{c^2(1-k)D}, & f<\frac{c-p+2cr}{c},\\ \frac{2c^2D(1-k)r-(p-c)(p-c-ck)S_c}{2c^2(1-k)D}, & f>\frac{c-p+2cr}{c}.\end{cases}$$

First, when $f<\frac{c-p+2cr}{c}$, $\hat{\rho}=\frac{(p-c)(p-c-ck)}{2c(p-c-cr)(1-k)}$ and $\hat{f}=\frac{(p-c)(cD(1-k)-(p-c-ck)S_c)}{c^2(1-k)D}$. By taking the derivative, we can get $\frac{\partial\hat{\rho}}{\partial r}=\frac{(p-c)(p-c-ck)S_c}{2(1-k)(p-c-cr)^2}>0$, and $\frac{\partial\hat{f}}{\partial r}=0$.

Second, when $f>\frac{c-p+2cr}{c}$, $\hat{\rho}=\rho_1$ and $\hat{f}=\frac{2c^2D(1-k)r-(p-c)(p-c-ck)S_c}{2c^2D(1-k)}$. By taking the derivative, we can get $\frac{\partial\hat{\rho}}{\partial r}=\frac{(2p-c-ck)(p-c)[2p-c-ck-\frac{c(1+k)^2(p-c-cr)-4p(r-f)(p-c-ck)}{\sqrt{4p(p-c-ck)(r-f)^2+(1+k)^2(p-c-cr)^2}}]}{4(p-c-cf)^2p(1-k)}>0$, and $\frac{\partial\hat{f}}{\partial r}=1>0$.

To conclude, when r increases, we get the following two cases:

(1) The threshold for the magnitude ratio $\hat{\rho}$ increases.

(2) The threshold for the crowdfunding fee $\hat{f}$ increases when $f>\frac{c-p+2cr}{c}$, that is, $\hat{f}$ increases between DA and DP. Otherwise, $\hat{f}$ remains the same between SA and SP.

Proposition 4 is proved. □

Proof of Proposition 5.

From Proposition 1, we can get the profit function as illustrated in the following equation (A.12).

$$\begin{aligned}\Pi(F_c,F_b)=&\frac{p-c}{c}(\frac{p^2S_c(c^2(1+k)^2S_c^2+3F_b(2cD(-1+k)-c(1+k)S_c+F_b))}{6cD(-1+k)p^2S_c})\\&-\frac{p-c}{c}(\frac{3p^2S_c(c(2D-2Dk+S_c+kS_c)-2F_b)F_c+3p(c(D-Dk)+pS_c-F_b)F_c^2}{6cD(-1+k)p^2S_c})\\&+\frac{p-c}{c}(\frac{(c+ck-2p)F_c^3}{6cD(-1+k)p^2S_c})-f(\frac{F_c^2}{2pS_c}+\frac{F_c(S_c-\frac{F_c}{p})}{S_c})-rF_b.\end{aligned}\tag{A.12}$$

First, we give the detailed calculation of $\frac{\partial\Pi(F_c,F_b)}{\partial r}$.

Based on Proposition 1, the four financing strategies as follows:

1. SA Strategy :

$$F_c^{SA}=pS_c \text{ and } F_b^{SA}=0 \quad\Rightarrow\quad \frac{\partial\Pi(F_c,F_b)}{\partial r}=0.$$

2. SP Strategy:

$$F_c^{SP}=\frac{cp(1+k)}{2p-c(1+k)}S_c+\frac{2cp(1-k)}{2p-c(1+k)}(1-\frac{cf}{p-c})D \text{ and } F_b^{SP}=0$$

$$\Rightarrow\frac{\partial\Pi(F_c,F_b)}{\partial r}=0.$$

3. DA Strategy:

$$F_c^{DA}=pS_c \text{ and } F_b^{DA}=\frac{2cD(1-k)(p-c-cr)}{2(p-c)}+\frac{c(1+k)S_c}{2}-\frac{pS_c}{2},$$

$$\Rightarrow\frac{\partial\Pi(F_c,F_b)}{\partial r}=\frac{cD(1-k)(p-c-cr)}{c-p}-\frac{(c+ck-p)S_c}{2}.$$

Let $\frac{\partial\Pi(F_c,F_b)}{\partial r}=\frac{cD(1-k)(p-c-cr)}{c-p}-\frac{(c+ck-p)S_c}{2}>0$, we can get the condition $D<\frac{(p-c)(p-c-ck)S_c}{2c(1-k)(p-c-cr)}$.

Recall the optimal condition for the DA strategy from the proposition 1, that is $D > \frac{(p-c)(p-c-ck)S_c}{2c(1-k)(p-c-cr)}$. Therefore, the we can get the $\frac{\partial \Pi(F_c,F_b)}{\partial r} < 0$ within the DA strategy.

4. DP Strategy:

$$F_c^{DP} = F_{c1} < pS_c \text{ and } F_b^{DP} = \frac{2cD(1-k)(p-c-cr)}{2(p-c)} + \frac{c(1+k)S_c}{2} - (1-\frac{F_{c1}}{2pS_c})F_{c1},$$

and the first-order condition of $\Pi(F_c, F_b)$ on r as follows:

$$\begin{aligned}\frac{\partial \Pi(F_c,F_b)}{\partial r} =& \frac{(c+ck-2p)(c-p)^2 S_c \sqrt{\frac{c^2(8D(1-k)p(r-f)+(1+k)^2(p-c)S_c)}{(p-c)^3 S_c}}}{4(c-p)p} \\ &+ \frac{c^3(1+k)S_c(-1-k) + cp^2(-4D(-1+k)) + c^2p(4D(1+f)(-1+k) + S_c - (2+k)S_c(-k))}{4(c-p)p}.\end{aligned}$$

Let $\frac{\partial \Pi(F_c,F_b)}{\partial r} > 0$, we get $D < D_1$. Recall the optimal conditions for DP strategy that is $D_1 < D < \frac{p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}$ (refer to the Proposition 1 for detail), we get $\frac{\partial \Pi(F_c,F_b)}{\partial r} < 0$ always holds in DP strategy.

Second, we give the detailed calculation of $\frac{\partial \Pi(F_c,F_b)}{\partial D}$. Based on Proposition 1, the four financing strategies as follows:

1. SA Strategy :

$$F_c^{SA} = pS_c \text{ and } F_b^{SA} = 0 \quad \Rightarrow \quad \frac{\partial \Pi(F_c,F_b)}{\partial D} = \frac{(p-c)(c+ck-p)^2 S_c^2}{6c^2D^2(1-k)} > 0.$$

2. SP Strategy:

$$F_c^{SP} = \frac{cp(1+k)}{2p-c(1+k)}S_c + \frac{2cp(1-k)}{2p-c(1+k)}(1-\frac{cf}{p-c})D \text{ and } F_b^{SP} = 0$$

$$\Rightarrow \frac{\partial \Pi(F_c,F_b)}{\partial D} = \frac{(p-c)(p^2S_c^3c^2(1+k)^2 - 3p^2S_c^2c(1+k)F_c + 3p^2S_cF_c^2 + (c+ck-2p)F_c^3)}{6c^2D^2(1-k)p^2S_c}.$$

Since $6c^2D^2(1-k)p^2S_c > 0$ and $p - c > 0$, we just need to verify the formulation $p^2S_c^3c^2(1+k)^2 - 3p^2S_c^2c(1+k)F_c + 3p^2S_cF_c^2 + (c+ck-2p)F_c^3 > 0$ to get the condition of $\frac{\partial \Pi(F_c,F_b)}{\partial D} > 0$.

And the formulation $p^2S_c^3c^2(1+k)^2 - 3p^2S_c^2c(1+k)F_c + 3p^2S_cF_c^2 + (c+ck-2p)F_c^3 > 0$ always holds for any values of $0 < F_c < pS_c$. Therefore, we can get $\frac{\partial \Pi(F_c,F_b)}{\partial D} > 0$ always holds in the SP strategy.

3. DA Strategy:

$$F_c^{DA} = pS_c \text{ and } F_b^{DA} = \frac{2cD(1-k)(p-c-cr)}{2(p-c)} + \frac{c(1+k)S_c}{2} - \frac{pS_c}{2},$$

$$\Rightarrow \frac{\partial \Pi(F_c,F_b)}{\partial D} = \frac{(1-k)(c-p+cr)^2}{2(p-c)} + \frac{(p-c)(c+ck-p)^2S_c^2}{24c^2D^2(1-k)} > 0.$$

4. DP Strategy:

$$F_c^{DP} = F_{c1} < pS_c \text{ and } F_b^{DP} = \frac{2cD(1-k)(p-c-cr)}{2(p-c)} + \frac{c(1+k)S_c}{2} - (1-\frac{F_{c1}}{2pS_c})F_{c1},$$

and the first-order condition of $\Pi(F_c, F_b)$ on D as follows:

$$\begin{aligned}\frac{\partial\Pi(F_c,F_b)}{\partial D} =& \frac{(p-c)p^2S_c(c^2(1+k)^2S_c^2+3F_b(-c(1+k)S_c+F_b))}{6c^2D^2(1-k)p^2S_c}\\ &+\frac{(p-c)(-3p^2S_c(c(1+k)S_c-2F_b)F_c+3p(pS_c-F_b)F_c^2+(c+ck-2p)F_c^3)}{6c^2D^2(1-k)p^2S_c}.\end{aligned}$$

Since $6c^2D^2(1-k)p^2S_c>0$ and $p-c>0$, we just need to verify the formulation $p^2S_c(c^2(1+k)^2S_c^2+3F_b(-c(1+k)S_c+F_b)-3p^2S_c(c(1+k)S_c-2F_b)F_c+3p(pS_c-F_b)F_c^2+(c+ck-2p)F_c^3>0$ to get the condition of $\frac{\partial\Pi(F_c,F_b)}{\partial D}>0$.

And the formulation $p^2S_c(c^2(1+k)^2S_c^2+3F_b(-c(1+k)S_c+F_b)-3p^2S_c(c(1+k)S_c-2F_b)F_c+3p(pS_c-F_b)F_c^2+(c+ck-2p)F_c^3>0$ always holds for any values of $0<F_c<pS_c$ and $F_b>0$. Therefore, we can get $\frac{\partial\Pi(F_c,F_b)}{\partial D}>0$ always holds in the DP strategy.

Third, we give the calculation of $\frac{\partial\Pi(F_c,F_b)}{\partial S_c}$. Based on Proposition 1, the four financing strategies are as follows:

1. SA Strategy :

$$F_c^{SA}=pS_c \text{ and } F_b^{SA}=0 \quad\Rightarrow\quad \frac{\partial\Pi(F_c,F_b)}{\partial S_c}=\frac{3c(P-c-cf)p-\frac{2(p-c)(c+ck-p)^2S_c}{D(1-k)}}{6c^2}.$$

Let $\frac{\partial\Pi(F_c,F_b)}{\partial S_c}>0$, we get the condition $D>\frac{2(p-c)(p-c-ck)^2S_c}{3c(1-k)(p-c-cf)p}$.

Recall the optimal condition of the SA strategy, that is $\frac{(p-c)(p-c-ck)S_c}{c(p-c-cf)(1-k)}<D<\frac{(p-c)(p-c-ck)S_c}{2c(p-c-cr)(1-k)}$.

One can verify (through Mathematica) that $\frac{2(p-c)(p-c-ck)^2S_c}{3c(1-k)(p-c-cf)p}<\frac{(p-c)(p-c-ck)S_c}{c(p-c-cf)(1-k)}$ always holds given the parameter range, which means $\frac{\partial\Pi(F_c,F_b)}{\partial S_c}>0$ always holds for the SA strategy.

2. DA Strategy:

$$F_c^{DA}=pS_c \text{ and } F_b^{DA}=\frac{2cD(1-k)(p-c-cr)}{2(p-c)}+\frac{c(1+k)S_c}{2}-\frac{pS_c}{2},$$

$$\Rightarrow \frac{\partial\Pi(F_c,F_b)}{\partial S_c}=\frac{-c(1+k)(1+r)+p(1-f+k+r)}{2}-\frac{(c-p)(c+ck-p)^2S_c}{12c^2D(-1+k)}.$$

Let $\frac{\partial\Pi(F_c,F_b)}{\partial S_c}>0$, we get the condition $D>\frac{(p-c)(p-c-ck)^2S_c}{6c^2(1-k)((1-f)p-c(1+k)(1+r)+p(k+r))}$.

Recall the optimal condition of DA strategy, that is $D>max\{\frac{(p-c)(p-c-ck)S_c}{2c(p-c-cr)(1-k)},\frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}\}$.

One can verify (through Mathematica) that

$$\frac{(p-c)(p-c-ck)^2S_c}{6c^2(1-k)((1-f)p-c(1+k)(1+r)+p(k+r))}<min\{\frac{(p-c)(p-c-ck)S_c}{2c(p-c-cr)(1-k)},\frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}\}$$

always holds given the parameter range, which means $\frac{\partial\Pi(F_c,F_b)}{\partial S_c}>0$ always holds for the DA strategy.

For SP and DP, $\frac{\partial\Pi(F_c,F_b)}{\partial S_c}$ involves high order terms of S_c as shown below, so we use simulation to compute the derivatives and see how $\Pi(F_c,F_b)$ changes with S_c.

$$\begin{aligned}\frac{\partial\Pi^{SP}}{\partial S_c} =& \frac{-2(1+k)^2(p-c)S_c}{6D(1-k)}-\frac{c(2p-c-ck)(p-c)p(\frac{2D(1-k)(p-c-cf)}{(2p-c-ck)(p-c)}-\frac{(1+k)S_c}{1+k-2p})^3}{6D(1-k)S_c^2}\\ &+\frac{3cd(1-k)(p-c-cf)p(\frac{(1+k)S_c}{1+k-2p}-\frac{2D(1-k)(p-c-cf)}{(2p-c-ck)(p-c)})^2}{6D(1-k)S_c^2}\\ &-\frac{3(1+k)p(\frac{2D(1-k)(p-c-cf)}{c+ck-2p}+\frac{(1+k)(p-c)S_c}{1+k-2p})}{6D(1-k)}.\end{aligned}$$

$$\frac{\partial \Pi^{DP}}{\partial S_c} = \frac{6cD(1-k)(p-c-cf)p(c(1+k-M)-Mp)^2 + 16c^2(1+k)^2(p-c)p^2 S_c}{48c^2D(1-k)p^2}$$
$$+ \frac{12c(1+k)(p-c)p^2(c(1+k-M)+Mp)S_c}{48c^2D(1-k)p^2} - \frac{(2p-c-ck)(p-c)(c(1+k-M)+Mp)^3 S_c}{48c^2D(1-k)p^2}$$
$$- \frac{3(-8cd(1-k)p(p-c-cr) - 4c(1+k)(p-c)pS_c)(c(1+k)p - 1/4(c(1+k-M)+Mp)^2)}{48c^2D(1-k)p^2}$$
$$- \frac{3((c-p)(c(1+k-M)+(-4+M)p)(c(1+k-M)+Mp)S_c)(c(1+k)p - 1/4(c(1+k-M)+Mp)^2)}{48c^2D(1-k)p^2},$$

with

$$M = \sqrt{\frac{8c^2p(1-k)(r-f)D}{(p-c)^3 S_c} + \frac{c^2(1+k)^2}{(p-c)^2}}.$$

Simulation Computational Framework. The simulation was executed using Python, leveraging the symbolic mathematics library SymPy for the definition and manipulation of the mathematical expression, and Matplotlib for visualization. The specific Python environment included SymPy for symbolic mathematics operations and NumPy for numerical computations to ensure high precision in mathematical analysis. We can provide the code if needed.

Simulation Procedure. Step 1. Randomly choose the parameters values of p, c, r, f and k that meet the constraints: $p-c > cr$, $p-c > cf$, $p-c > ck$, $S_c > 0$, $D > 0$.

Step 2. Calculate the value of $\frac{\partial \Pi(F_c,F_b)}{\partial S_c}$ with the above parameters' values.

Step 3. Repeat steps 1 and 2, and we obtain a total of 320,000 simulation results. By analyzing these data, we concluded that Π^j increases in S_c first before decreasing in S_c, while the cutoff value of S_c increases in D, for $j = SP, DP$. We then selected representative parameter values and visualized the results as shown in Figure 3 and Figure A.1. □

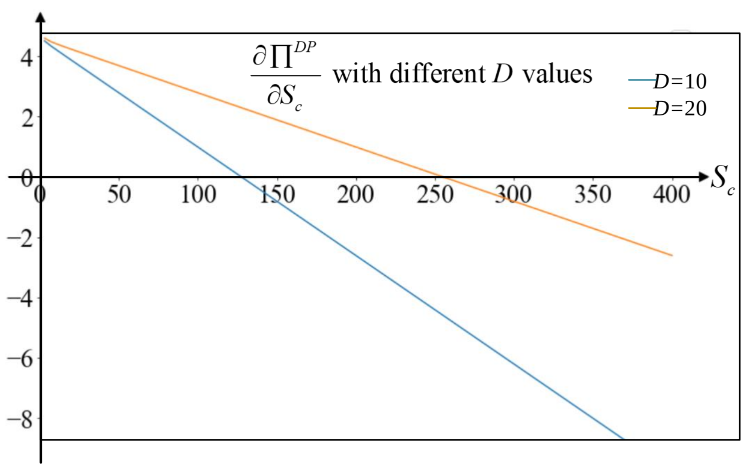

Figure A.1: $\frac{\partial \Pi^{DP}}{\partial S_c}$ shifts to the right when D increases, where $p = 10$, $c = 4$, $r = 0.25$, $f = 0.05$, $k = 0.5$.

Proof of Lemma 1.

In the maximization problem for CIM, the profit function decreases with F_b when $d^{CI} + s_c^{CI} < \frac{\min\{F_c, ps_c^{CI}\}+F_b}{c}$, or $F_b > c(d^{CI}+s_c^{CI}) - \min\{F_c, ps_c^{CI}\}$. That is, $d^{CI}+s_c^{CI} < \frac{\min\{F_c,ps_c^{CI}\}+F_b}{c}$, $\frac{\partial \Pi(F_c,F_b)}{\partial F_b} =$

$-r < 0$. The profit function increases with F_b when $F_b < c(d^{CI} + s_c^{CI}) - \min\{F_c, ps_c^{CI}\}$. That is, when $d^{CI} + s_c^{CI} > \frac{\min\{F_c, ps_c^{CI}\}+F_b}{c}$, $\frac{\partial \Pi(F_c,F_b)}{\partial F_b} = \frac{p-c}{c} - r > 0$. Thus, $F_b^* = c(d^{CI} + s_c^{CI}) - \min\{F_c, ps_c^{CI}\}$ when it is positive, and $F_b^* = 0$ when $c(d^{CI} + s_c^{CI}) - \min\{F_c, ps_c^{CI}\} < 0$.

For the crowdfunding goal, plugging $F_b^* = c(d^{CI}+s_c^{CI}) - \min\{F_c, ps_c^{CI}\}$ into the profit function, we can get $\frac{\partial \Pi(F_c,F_b)}{\partial \min\{F_c, ps_c^{CI}\}} = r - f > 0$, therefore, we get $F_c^* = ps_c^{CI}$. Otherwise, if $c(d^{CI}+s_c^{CI}) - \min\{F_c, ps_c^{CI}\} < 0$, or $\min\{F_c, ps_c^{CI}\} > c(d^{CI} + s_c^{CI})$, we have $F_b^* = 0$. And we have $\frac{\partial \Pi(F_c,F_b)}{\partial \min\{F_c, ps_c^{CI}\}} = -f < 0$, thus $F_c^* = c(d^{CI} + s_c^{CI})$ which must be lower than ps_c^{CI}, i.e., $\frac{d^{CI}}{s_c^{CI}} < \frac{p-c}{c}$. To conclude, there are two cases here. First, when $\frac{d^{CI}}{s_c^{CI}} < \frac{p-c}{c}$, we get $F_b* = 0, F_c^* = c(d^{CI} + s_c^{CI})$, i.e., SP strategy. Second, when $\frac{d^{CI}}{s_c^{CI}} > \frac{p-c}{c}$, we get $F_b^* = c(d^{CI} + s_c^{CI}) - ps_c^{CI}, F_c^* = ps_c^{CI}$, i.e., DA strategy. □

Proof of Lemma 3.

Based on the distribution of $\tilde{D}$ and $\tilde{S_c}$, that is, $\tilde{D}|s_c \sim U[ks_c, (1-k)D + ks_c]$ and $\tilde{S_c} \sim U[0, S_c]$, we can get the following equations.

$$E[\tilde{D}] = \int_0^{kS_c} \frac{1}{(1-k)DkS_c}\tilde{D}\tilde{D}d\tilde{D} + \int_{kS_c}^{(1-k)D} \frac{1}{(1-k)D}\tilde{D}d\tilde{D} + \int_{(1-k)D}^{(1-k)D+kS_c} \left(\frac{1}{(1-k)D} + \frac{1}{kS_c} - \frac{\tilde{D}}{k(1-k)DS_c}\right)\tilde{D}d\tilde{D} = \frac{(1-k)D + kS_c}{2},$$

$$E[\tilde{D}\tilde{D}] = \int_0^{kS_c} \frac{1}{(1-k)DkS_c}\tilde{D}\tilde{D}\tilde{D}d\tilde{D} + \int_{kS_c}^{(1-k)D} \frac{1}{(1-k)D}\tilde{D}\tilde{D}d\tilde{D} + \int_{(1-k)D}^{(1-k)D+kS_c} \left(\frac{1}{(1-k)D} + \frac{1}{kS_c} - \frac{\tilde{D}}{k(1-k)DS_c}\right)\tilde{D}\tilde{D}d\tilde{D}$$
$$= \frac{2D^2(1-k)^2 + kS_c(3D(1-k)+2kS_c)}{6},$$

$$E[\tilde{S_c}] = \frac{S_c}{2},$$
$$Var(\tilde{S_c}) = \frac{S_c^2}{12},$$
$$Var(\tilde{D}) = \frac{S_c^2}{12},$$
$$E[\tilde{S_c}\tilde{D}] = E[\tilde{D}\tilde{D}] - E[\tilde{D}]^2 = \frac{D^2(1-k)^2 + k^2S_c^2}{12},$$
$$cov(\tilde{S_c}, \tilde{D}) = E[\tilde{S_c}\tilde{D}] - E[\tilde{S_c}]E[\tilde{D}] = \frac{kS_c^2}{12}.$$

Therefore, the correlation coefficient between $\tilde{D}$ and $\tilde{S_c}$ equals $\frac{cov(\tilde{S_c},\tilde{D})}{\sqrt{Var(\tilde{S_c})}\sqrt{Var(\tilde{D})}} = \frac{kS_c}{\sqrt{(1-k)^2D^2+k^2S_c^2}} = \left((\frac{D}{S_c})^2(\frac{1}{k} - 1)^2 + 1\right)^{-\frac{1}{2}}$. □

Proof of Proposition 7.

The formula of $\hat{\rho}$ and $\hat{f}$ in Proposition 1 are as follows:

$$\hat{\rho} = \begin{cases} \frac{(p-c)(p-c-ck)}{2c(p-c-cr)(1-k)}, & f < \frac{c-p+2cr}{c}, \\ \rho_1, & f > \frac{c-p+2cr}{c}. \end{cases} \tag{A.13}$$

$$\hat{f} = \begin{cases} \frac{(p-c)(cD(1-k)-(p-c-ck)S_c)}{c^2(1-k)D}, & f < \frac{c-p+2cr}{c}, \\ \frac{2c^2D(1-k)r-(p-c)(p-c-ck)S_c}{2c^2D(1-k)}, & f > \frac{c-p+2cr}{c}. \end{cases} \tag{A.14}$$

Based on the expressions of $\hat{\rho}$ and $\hat{f}$, there are two cases here:

(1) When $f < \frac{c-p+2cr}{c}$,
$\hat{\rho} = \frac{(p-c)(p-c-ck)}{2c(p-c-cr)(1-k)} \Rightarrow \frac{\partial\hat{\rho}}{\partial k} = -\frac{(c-p)(2c-p)}{2c(-1+k)^2(c+cr-p)} > 0 \Rightarrow c < \frac{p}{2}$.
$\hat{f} = \frac{(p-c)(cD(1-k)-(p-c-ck)S_c)}{c^2(1-k)D} \Rightarrow \frac{\partial\hat{f}}{\partial k} = -\frac{(cp)(2cp)S_c}{c^2D(-1+k)^2} > 0 \Rightarrow c > \frac{p}{2}$.
(2) When $f > \frac{c-p+2cr}{c}$,
$\hat{\rho} = \rho_1 \Rightarrow \frac{\partial\hat{\rho}}{\partial k} > 0 \Rightarrow c < \frac{p}{2}$.
$\hat{f} = \frac{2c^2D(1-k)r-(p-c)(p-c-ck)S_c}{2c^2(1-k)D} \Rightarrow \frac{\partial\hat{f}}{\partial k} = -\frac{(2c^2-3cp+p^2)S_c}{2c^2D(-1+k)^2} > 0 \Rightarrow c > \frac{p}{2}$.

The derivative simplification can be verified through Mathematica. To conclude, we have the following two cases:

(1) When $c < \frac{p}{2}$, $\frac{\partial\hat{\rho}}{\partial k} > 0$, $\frac{\partial\hat{f}}{\partial k} < 0$.
(2) When $c > \frac{p}{2}$, $\frac{\partial\hat{\rho}}{\partial k} < 0$, $\frac{\partial\hat{f}}{\partial k} > 0$.

Furthermore, for $\frac{\partial F_c}{\partial k}$ and $\frac{\partial F_b}{\partial k}$, we have the following three cases:

1. SP Strategy:

$$F_c^{SP} = \frac{cp(1+k)}{2p-c(1+k)}S_c + \frac{2cp(1-k)}{2p-c(1+k)}(1-\frac{cf}{p-c})D \text{ and } F_b^{SP} = 0.$$

$$\Rightarrow \frac{\partial F_c}{\partial k} = \frac{2cp(2cD(1+f)+p(-2D+S_c))}{(c+ck-2p)^2}, \frac{\partial F_b}{\partial k} = 0$$

Let $\frac{\partial F_c}{\partial k} = \frac{2cp(2cD(1+f)+p(-2D+S_c))}{(c+ck-2p)^2} > 0$, we get the condition $c > \frac{p}{2}$. Therefore, within the SP strategy, we get

(1) $\frac{\partial F_c}{\partial k} > 0$ with $c > \frac{p}{2}$,
(2) $\frac{\partial F_c}{\partial k} < 0$ with $c < \frac{p}{2}$,
(3) and $\frac{\partial F_b}{\partial k} = 0$.

2. DA Strategy:

$$F_c^{DA} = pS_c \text{ and } F_b^{DA} = \frac{2cD(1-k)(p-c-cr)}{2(p-c)} + \frac{c(1+k)S_c}{2} - \frac{pS_c}{2},$$

$$\Rightarrow \frac{\partial F_c}{\partial k} = 0, \frac{\partial F_b}{\partial k} = \frac{1}{2}c(D(-2-\frac{2cr}{c-p})+S_c)$$

Let $\frac{\partial F_b}{\partial k} = \frac{1}{2}c(D(-2-\frac{2cr}{c-p})+S_c) > 0$, we get the condition as the following equation (A.15).

$$D < \frac{(p-c)S_c}{2(p-c-cr)}. \tag{A.15}$$

Recall that in the proof of Proposition 1, we have concluded that D must satisfy the following equation

(A.16).

$$
\begin{aligned}
&(1)\ 0<c\le\frac{p}{3},0<r<1,0<f<r,0<k<1,D>\frac{(p-c)(p-c-ck)S_c}{2c(p-c-cr)(1-k)}.\\
&(2)\ \frac{p}{3}<c\le\frac{p}{2},0<r\le\frac{-c+p}{2c},0<k<1,D>\frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}.\\
&(3)\ \frac{p}{3}<c\le\frac{p}{2},\frac{-c+p}{2c}<r<1,0<f<\frac{c-p+2cr}{c},0<k<1,D>\frac{(p-c)(p-c-ck)S_c}{2c(p-c-cr)(1-k)}.\\
&(4)\ \frac{p}{3}<c\le\frac{p}{2},\frac{-c+p}{2c}<r<1,f>\frac{c-p+2cr}{c},0<k<1,D>\frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}.\\
&(5)\ \frac{p}{2}<c<p,0<r\le\frac{-c+p}{2c},0<k<\frac{p-c}{c},D>\frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}.\\
&(6)\ \frac{p}{2}<c<p,\frac{-c+p}{2c}<r<1,0<f<\frac{c-p+2cr}{c},0<k<\frac{p-c}{c},D>\frac{(p-c)(p-c-ck)S_c}{2c(p-c-cr)(1-k)}.\\
&(7)\ \frac{p}{2}<c<p,\frac{-c+p}{2c}<r<1,f>\frac{c-p+2cr}{c},0<k<\frac{p-c}{c},D>\frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}.\\
&(8)\ \frac{p}{2}<c<p,k>\frac{p-c}{c}.
\end{aligned}
\tag{A.16}
$$

Comparing the condition of D as illustrated in equation (A.15) with the conditions of D in the DA strategy (i.e., the equation (A.16)), we get the following cases for the value of $\frac{\partial F_b}{\partial k}$.

(1) $c<\frac{p}{3}$: $\frac{\partial F_b}{\partial k}<0$.

Specifically, the condition of DA strategy $D>\frac{(p-c)(p-c-ck)S_c}{2c(p-c-cr)(1-k)}$ makes the condition for $\frac{\partial F_b}{\partial k}>0$ (i.e., $D<\frac{(p-c)S_c}{2(p-c-cr)}$) invalid.

(2) $\frac{p}{3}<c<\frac{p}{2}$, $0<r<\frac{p-c}{2c}$: $\frac{\partial F_b}{\partial k}<0$.

Specifically, the condition of DA strategy $D>\frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}$ makes the condition for $\frac{\partial F_b}{\partial k}>0$ (i.e., $D<\frac{(p-c)S_c}{2(p-c-cr)}$) invalid.

(3) $\frac{p}{3}<c<\frac{p}{2}$, $r>\frac{p-c}{2c}$, $0<f<\frac{c-p+2cr}{c}$: $\frac{\partial F_b}{\partial k}<0$.

Specifically, the condition of DA strategy $D>\frac{(p-c)(p-c-ck)S_c}{2c(p-c-cr)(1-k)}$ makes the condition for $\frac{\partial F_b}{\partial k}>0$ (i.e., $D<\frac{(p-c)S_c}{2(p-c-cr)}$) invalid.

(4) $\frac{p}{3}<c<\frac{p}{2}$, $r>\frac{p-c}{2c}$, $f>\frac{c-p+2cr}{c}$: $\frac{\partial F_b}{\partial k}<0$.

Specifically, the condition of DA strategy $D>\frac{(p-c)(p-c-ck)S_c}{2c^2(r-f)(1-k)}$ makes the condition for $\frac{\partial F_b}{\partial k}>0$ (i.e., $D<\frac{(p-c)S_c}{2(p-c-cr)}$) invalid.

(5) $c>\frac{p}{2}$, apart from the original constraints of the DA strategy, we get $\frac{\partial F_b}{\partial k}>0$ if and only if $D<\frac{(p-c)S_c}{2(p-c-cr)}$.

Based on the analysis of the above five scenarios, we can draw the following conclusions, that is, no matter $c<\frac{p}{2}$ or $c>\frac{p}{2}$, it is always true that $\frac{\partial F_b}{\partial k}>0$ if and only if $D<\frac{(p-c)S}{2(p-c-cr)}$ for DA strategy. And it is obvious that $\frac{\partial F_c}{\partial k}=0$.

3. DP Strategy:

$$F_c^{DP}=F_{c1}<pS_c \text{ and } F_b^{DP}=\frac{2cD(1-k)(p-c-cr)}{2(p-c)}+\frac{c(1+k)S_c}{2}-(1-\frac{F_{c1}}{2pS_c})F_{c1},$$

$$\Rightarrow\frac{\partial F_c}{\partial k}=\frac{1}{2}c(S_c-\frac{4Dp(r-f)+(1+k)(p-c)S_c}{(p-c)\sqrt{\frac{8D(1-k)p(r-f)+(1+k)^2(c-p)S_c}{(p-c)S_c}}}),\frac{\partial F_b}{\partial k}=\frac{c}{2}(D(-2-\frac{2cr}{c-p})+S_c)+\frac{\partial F_b}{\partial F_c}\frac{\partial F_c}{\partial k}.$$

First, for $\frac{\partial F_c}{\partial k}$, we have verified that $\frac{\partial F_c}{\partial k}>0$ if and only if $c>\frac{p}{2}$.

Second, for $\frac{\partial F_b}{\partial k}=\frac{c}{2}(D(-2-\frac{2cr}{c-p})+S_c)+\frac{\partial F_b}{\partial F_c}\frac{\partial F_c}{\partial k}$, we define the following three parts to analyze $\frac{\partial F_b}{\partial k}$.

Part1: $\frac{c}{2}(D(-2-\frac{2cr}{c-p})+S_c)$;
Part2: $\frac{\partial F_b}{\partial F_c}=-1+\frac{F_c}{pS_c}$;
Part3: $\frac{\partial F_c}{\partial k}=\frac{1}{2}c(S_c-\frac{4Dp(r-f)+(1+k)(p-c)S_c}{(p-c)\sqrt{\frac{8D(1-k)p(r-f)+(1+k)^2(c-p)S_c}{(p-c)S_c}}})$.

Since $F_c<pS_c$, we get $\frac{\partial F_b}{\partial F_c}=-1+\frac{F_c}{pS_c}<0$, so Part2 is always negative. Then we get the following four cases:

Case 1: Part1 (positive) + Part2 (negative) Part3 (positive): uncertain;

Case 2: Part1 (positive) + Part2 (negative) Part3 (negative), $\frac{\partial F_b}{\partial k}>0$;

Case 3: Part1 (negative)+ Part2 (negative) Part3 (positive), $\frac{\partial F_b}{\partial k}<0$;

Case 4: Part1 (negative) + Part2 (negative) Part3 (negative): uncertain.

Notice here that Cases 2 and 3 have a deterministic sign for $\frac{\partial F_b}{\partial k}$, while Cases 1 and 4 need further analysis. Let us first write down the known conditions to sign Part1 and Part3.

- Case 1 ($\frac{\partial F_b}{\partial k}>0$ if and only if $D<\hat{D}$, explained below):

  Part1 (positive): $\frac{c}{2}(D(-2-\frac{2cr}{c-p})+S_c)>0$, that is, $D<\frac{(p-c)S_c}{2(p-c-cr)}$.

  Part3 (positive): $\frac{\partial F_c}{\partial k}>0$, that is, $c>\frac{p}{2}$.

- Case 2 ($\frac{\partial F_b}{\partial k}>0$ always holds):

  Part1 (positive): $\frac{c}{2}(D(-2-\frac{2cr}{c-p})+S_c)>0$, that is, $D<\frac{(p-c)S_c}{2(p-c-cr)}$.

  Part3 (negative): $\frac{\partial F_c}{\partial k}<0$, that is, $c<\frac{p}{2}$.

- Case 3 ($\frac{\partial F_b}{\partial k}<0$ always holds):

  Part1 (negative): $\frac{c}{2}(D(-2-\frac{2cr}{c-p})+S_c)<0$, that is, $D>\frac{(p-c)S_c}{2(p-c-cr)}$.

  Part3 (positive): $\frac{\partial F_c}{\partial k}>0$, that is, $c>\frac{p}{2}$.

- Case 4 ($\frac{\partial F_b}{\partial k}<0$ is implied by the following constraints, explained below):

  Part1 (negative): $\frac{c}{2}(D(-2-\frac{2cr}{c-p})+S_c)<0$, that is, $D>\frac{(p-c)S_c}{2(p-c-cr)}$.

  Part3 (negative): $\frac{\partial F_c}{\partial k}<0$, that is, $c<\frac{p}{2}$.

It is easy to verify that in Case 4, $D>\frac{(p-c)S_c}{2(p-c-cr)}$ and $c<\frac{p}{2}$ combined with the constraints for DP strategy given in Proposition 1, directly imply that Part1 + Part2 * Part3 is negative without additional conditions, so $\frac{\partial F_b}{\partial k}<0$ always holds in Case 4.

However, Case 1 turns out to involve high-order conditions and does not admit analytical expressions, so we use simulation to compute and verify the sign of $\frac{\partial F_b}{\partial k}$. By analyzing 144,000 simulated results, we find that for Case 1, there is a threshold $\hat{D}$ for D such that $\frac{\partial F_b}{\partial k}>0$ if and only if $D<\hat{D}$. This result is similar to that for the DA strategy, with a different cutoff value for D.

Therefore, no matter $c<\frac{p}{2}$ or $c>\frac{p}{2}$, it is always true that $\frac{\partial F_b}{\partial k}>0$ if and only if D is below some cutoff value, that is, sufficiently small, while for $\frac{\partial F_c}{\partial k}$, we have proved that $\frac{\partial F_c}{\partial k}>0$ if and only if $c>\frac{p}{2}$. Proposition 7 is proved.□

Proof of Proposition 8.

Based on Proposition 1, the four financing strategies are as follows:

1. SA Strategy :

$$F_c^{SA} = pS_c \text{ and } F_b^{SA} = 0 \quad \Rightarrow \quad \frac{\partial \Pi(F_c, F_b)}{\partial k} = \frac{(p-c)(c+ck-p)(c(-3+k)+p)S_c^2}{6c^2 D(-1+k)^2}.$$

Combining the parameter constraints within SA strategy and the value of $\frac{\partial \Pi(F_c,F_b)}{\partial k}$, we get the followings cases.

(1) $c < \frac{p}{3}$: $\frac{\partial \Pi(F_c,F_b)}{\partial k} < 0$.
(2) $\frac{p}{3} < c < \frac{p}{2}$, $k < \frac{3c-p}{c}$: $\frac{\partial \Pi(F_c,F_b)}{\partial k} > 0$.
(3) $\frac{p}{3} < c < \frac{p}{2}$, $k > \frac{3c-p}{c}$: $\frac{\partial \Pi(F_c,F_b)}{\partial k} < 0$.
(4) $c > \frac{p}{2}$: $\frac{\partial \Pi(F_c,F_b)}{\partial k} > 0$.

2. DA Strategy:

$$F_c^{DA} = pS_c \text{ and } F_b^{DA} = \frac{2cD(1-k)(p-c-cr)}{2(p-c)} + \frac{c(1+k)S_c}{2} - \frac{pS_c}{2},$$

$$\Rightarrow \frac{\partial \Pi(F_c, F_b)}{\partial k} = \frac{D(c-p+cr)^2}{2(c-p)} - \frac{(c-p+cr)S_c}{2} - \frac{(c-p)(c+ck-p)(c(-3+k)+p)S_c^2}{24c^2 D(-1+k)^2}.$$

Combining the parameter constraints within SA strategy and the value of $\frac{\partial \Pi(F_c,F_b)}{\partial k}$, we get a threshold

$$\bar{D} = \frac{(3c(p-c)(1-k) + (p-c)\sqrt{3(2ck-p)(2c(-2+k)+p)})S_c}{6c(1-k)(p-c-cr)}$$

and the following cases.

(1) $c < \frac{p}{3}$: $\frac{\partial \Pi(F_c,F_b)}{\partial k} < 0$.
(2) $\frac{p}{3} < c < \frac{p}{2}$, $k < \frac{3c-p}{c}$, $D < \bar{D}$: $\frac{\partial \Pi(F_c,F_b)}{\partial k} > 0$.
(3) $\frac{p}{3} < c < \frac{p}{2}$, $k < \frac{3c-p}{c}$, $D > \bar{D}$: $\frac{\partial \Pi(F_c,F_b)}{\partial k} < 0$.
(4) $\frac{p}{3} < c < \frac{p}{2}$, $k > \frac{3c-p}{c}$: $\frac{\partial \Pi(F_c,F_b)}{\partial k} < 0$.
(5) $c > \frac{p}{2}$, $D < \bar{D}$: $\frac{\partial \Pi(F_c,F_b)}{\partial k} > 0$.
(6) $c > \frac{p}{2}$, $D > \bar{D}$: $\frac{\partial \Pi(F_c,F_b)}{\partial k} < 0$.

Additionally, it can be verified that $D < \bar{D}$ always holds in the cases (2) and (4) for SA strategy with its parameter constraints. Also notice that $\frac{3c-p}{c} < 0 < k$ if $p > 3c$, $\frac{3c-p}{c} > 1 > k$ if $p < 2c$. Therefore, the conditions for $\frac{\partial \Pi(F_c,F_b)}{\partial k} > 0$ within SA and DA strategies can be concluded as $k < \frac{3c-p}{c}$ and $D < \bar{D}$.

For SP and DP, $\frac{\partial \Pi(F_c,F_b)}{\partial k}$ involves high order terms as shown below, so we use simulation to calculate the derivative and see how $\Pi(F_c, F_b)$ changes with k.

$$\frac{\partial \Pi^{SP}}{\partial k} = \frac{-c^2(1+k)^2(p-c)p^2S_c^3 + 3p^2S_c(2cD(1-k)(p-c-cf) + c(1+k)(p-c)S_c)N_1}{6c^2D(1-k)^2p^2S_c}$$
$$+ \frac{(2p-c-ck)(p-c)N_1^3 - 3pN_1^2(cD(1-k)(p-c-cf) + p(-c+p)S_c)}{6c^2D(1-k)^2p^2S_c}$$
$$+ \frac{(1-k)(-2c^2(1+k)(p-c)p^2S_c^3 - c(p-c)N_1^3 + 3cD(p-c-cf)pN_1^2)}{6c^2D(1-k)^2p^2S_c}$$
$$+ \frac{(1-k)(\frac{6cp^2S_c(2D(1-k)(p-c-cf)+(1+k)(p-c)S_c)(2cpD(1+k-2p)^2(c+cf-p)+c(c+ck-2p)^2p^2S_c)}{(1+k-2p)^2(c+ck-2p)^2})}{6c^2D(1-k)^2p^2S_c}$$
$$+ \frac{(1-k)(\frac{3(c-p)N_1^2(4cpD(1+k-2p)^2(c+cf-p)+2c(c+ck-2p)^2p^2S_c)}{(1+k-2p)^2(c+ck-2p)})}{6c^2D(1-k)^2p^2S_c}$$
$$- \frac{(1-k)(\frac{6pN_1(4cpD(1+k-2p)^2(c+cf-p)+2c(c+ck-2p)^2p^2S_c)(cD(-1+k)(c+cf-p)+p(-c+p)S_c)}{(1+k-2p)^2(c+ck-2p)^2})}{6c^2D(1-k)^2p^2S_c}$$
$$+ \frac{(1-k)(3cp^2S_cN_1(2cD(1+f) - cS_c + p(-2D+S_c))}{6c^2D(1-k)^2p^2S_c},$$

with

$$N_1 = \frac{2cpD(1-k)(p-c-cf)}{(2p-c-ck)(p-c)} + \frac{c(1+k)pS_c}{2p-1-k}.$$

$$\frac{\partial \Pi^{DP}}{\partial k} = \frac{-c^6(1+k-N_2)^2(-25+5k(-2+3k)-2N_2+6kN_2+3N_2^2)S_c^2 + 6cN_2^3(-8+3N_2)p^5S_c^2}{384c^2D(1-k)^2(c-p)p^2}$$
$$+ \frac{(8-3N_2)N_2^3p^6S_c^2 + 6c^5(1+k-N_2)pS_c(16D(-1+k)^2(f-r) + (-19+k(-17+k(7+5k)) + 17N_2)S_c}{384c^2D(1-k)^2(c-p)p^2}$$
$$+ \frac{6c^5(1+k-N)pS_c^2((10-7k)kN_2 + (5-3k)N_2^2 - 3N_2^3) + c^2p^4S_c^2(120N_2^3 - 45N_2^4)}{384c^2D(1-k)^2(c-p)p^2}$$
$$+ \frac{c^4p^2S_c^2(192D^2(-1+k)^2(1+r)^2 + 96D(-1+k)^2(-2-f(3+k-2N_2)+r+kr-2N_2r)S_c + (201+284k)}{384c^2D(1-k)^2(c-p)p^2}$$
$$+ \frac{c^4p^2S_c^2(14k^2-84k^3-15k^4+72(1+k)(-5+k^2)N_2 - 36(-3+k)(1+k)N_2^2 + 120N_2^3 - 45N_2^4)}{384c^2D(1-k)^2(c-p)p^2}$$
$$+ \frac{c^2p^4S_c^2(192D^2(-1+k)^2 - 192D(-1+k)^2S_c + (-16(-3+k)(1+k) + 24(-3+k)(1+k)N_2 - 6(-3+k)(1+k)N_2^2)}{384c^2D(1-k)^2(c-p)p^2}$$
$$\frac{4c^3p^3(-96D^2(-1+k)^2(1+r) - 24D(-1+k)^2(-4+f(-2+N_2)-N_2r)S_c + (8(1+k)(-5+k^2))S_c^2)}{384c^2D(1-k)^2(c-p)p^2}$$
$$+ \frac{4c^3p^3S_c^2(-6(1+k)(-11+k(2+k))N_2 + 6(-3+k)(1+k)N_2^2 - 40N_2^3 + 15N_2^4)}{384c^2D(1-k)^2(c-p)p^2},$$

with

$$N_2 = \sqrt{\frac{8c^2p(1-K)(r-f)D}{(p-c)^3S} + \frac{c^2(1+K)^2}{(p-c)^2}}.$$

The simulation procedure is the same as described in Proof of Proposition 5. We computed $\frac{\partial \Pi(F_c,F_b)}{\partial k}$ with different parameters for over 1048,000 (144,000) repetitions for the DP (SP) strategy. By analyzing these data, we concluded that in general Π^j increases in k first before decreasing in k just as in the aggressive-crowdfunding strategies, but for SP strategy, Π^{SP} monotonically decreases in k if D is sufficiently small, while for DP strategy, Π^{DP} monotonically decreases in k if D is sufficiently small or sufficiently large. The simulation code and results are available from the authors upon request.□