University-firm coordination and competition in basic research*

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Abstract

This paper considers four different scenarios for non-cooperative and coordinated basic research between a university and a firm: a one-stage game, a two-stage game with grants, a research cartel and a cartel with research-specialized university. The university and the firm conduct research in order to increase their probability of success. We compare the performance of the two-stage game with grants and the one-stage basic game. The former leads to a win-win outcome relative to the latter and the effective probability of successful research (a proxy for social welfare) is also higher. We also consider two models of basic research consortium, a research cartel and a scenario specializing research to the university but in the context of a partnership. Both coordinated scenarios may yield a higher total profit and higher probability of success for research than either of the non-cooperative scenarios. The analysis suggests a central role for monetary transfers from the firm to the university, both in the two-stage game with grants and in the two coordinated scenarios for basic research.

Keywords: basic research, university-firm relations, research grants, research consortium, uncertain research.

Jel codes: D21, L13, O31

*The authors are grateful to Sudipta Sarangi for helpful feedback on the subject of this paper.
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1 Introduction

The interplay between market structure and research and development (R&D) is a central issue in industrial organization and related fields going back all the way to at least Schumpeter's (1942) classical work. In particular, that old debate centered around the relationship between the level of competitiveness of an industry and the associated propensity for R&D, as well as other dimensions of market performance, in a dynamic perspective. In recent decades, that classical debate has added hybrid market structures to the set of possibilities by integrating different forms of coordination in R&D between firms. The appearance of so-called R&D research joint ventures (RJVs) followed the passage of the National Cooperative Research Act in the U.S. in 1984, which conditionally exempted cooperation in R&D from antitrust scrutiny. However, such consortia had existed decades prior to that date in Japan and Europe. Seminal work by Katz (1986) and d' Aspremont and Jacquemin (1988, 1990) on the theoretical analysis of the merits of R&D cartels led to a burgeoning strand of literature investigating various aspects of inter-firm coordination of process R&D amongst otherwise competing firms in product markets.[1]

More recently, a literature strand has also emerged to integrate explicitly the role that universities might play in this debate. Thus, another literature strand, mostly of an empirical and/or policy-oriented nature but also including some theoretical work, is specifically dedicated to university-firm R&D consortia (e.g., Poyago-Theotoky, Beath and Siegel, 2002; Hall, Link and Scott, 2003; Bercovitz and Feldman, 2007; Link and Scott, 2005, among others). Link and Scott (2005) establish empirically that larger consortia are more likely than smaller ones to include a university as a research partner "because larger ventures are less likely to expect substantial additional appropriability problems to result because of the addition of a university partner and because the larger ventures have both a lower marginal cost and a higher marginal value from university R&D contributions to the ventures' innovative output" (also see Kohn and Scott, 1982). Using OECD data, Cabon-Dhersin and Gibert (2019, 2020) report that governments in OECD countries allocate public R&D funding differently between the private and the public research sectors (the latter includes universities and public laboratories) with a greater proportion (at least two-thirds) going to the public sector than to the private sector. Hall et al. (2003) report that universities tend to be included in industrial projects that involve "new" science with the expectation that the university will provide otherwise scarce research insight that will play a key role in light of the complex nature of the research being undertaken, particularly with regard to the use of basic knowledge. Reinforcing this view, Bercovitz and Feldman

[1]A short list of articles of the broad R&D literature includes Kamien, Muller and Zang (1992), Poyago-Theotoky (1995), Hinloopen (2000a-b), Amir (2000), Amir et. al. (2003), Cabon-Dhersin and Lahmandi-Ayed (2019,2020), Gil-Moltó, Poyago-Theotoky, and Zikos (2011), Chalioti (2015, 2019) and Osório and Pinto (2020).

(2007) examine the link between firms' innovation strategies and the level of involvement with university-based research. They find that firms with internal R&D strategies leaning toward exploratory activities or facing potential conflicts over intellectual property allocate a greater share of their R&D resources to university research and develop deeper relationships with their university research partners (also see Jaffe, 1989). Many empirical studies have demonstrated that research conducted in public laboratories and universities provide significant benefits to private research (Cohen, Nelson, and Walsh, 2002; and Veugelers and Cassiman, 2005, among others).[2]

The theoretical literature on university-firm relationships explores various aspects of this important nexus that include several dimensions of research and educational quality along with other factors. To provide an evaluation of coordination schemes involving universities and firms, non-cooperative scenarios are also often considered, if only as a relevant benchmark. Cabon-Dhersin and Gibert (2019, 2020) propose models combining inter-firm spillovers and one-way university-to-firm knowledge transfer to shed light on the socially optimal funding of R&D and its relationship to spillovers, both in cooperative and noncooperative settings. Hatsor and Zilcha (2020) investigate efficient government subsidization of different classes of universities through student aid and selection. Lahmandi-Ayed, Lasram, and Laussel (2021) study a model of vertical successive monopolies where university graduates form an input to firms that produce and sell in a product market, but focuses on the value of education for the labor market. Del Rey (2001) considers similar issues, but includes research as one of the policy goals. Finally, Correa et al. (2020) consider a political economy approach.[3]

A key characteristic of this literature, in contrast to the afore-mentioned far more established literature on RJVs with its well-known two-stage game, is the absence of a canonical model to account for the complex and multi-faceted university-firm relationship. Recent work by Stenbacka and Tombak (2020) has proposed a simple model to capture the strategic interaction between a university and a monopoly firm leading to the determination of their policies concerning basic research.[4] A player's decision variable is its probability of success in the uncertain research process, which is tied to a research cost function with decreasing returns to scale. The university always incurs its research cost, but is rewarded by a fixed government grant only when it is the sole successful player in the binary research process, in which case it transmits its discovery at no charge to the firm for subsequent development as a monopo-

[2] On the other hand, in an empirical investigation of the issue of whether public R&D is a complement or substitute for private R&D, David, Hall, and Toole (2000) report that "the findings overall are ambivalent and the existing literature as a whole is subject to the criticism that the nature of the 'experiment(s)' that the investigators envisage is not adequately specified".

[3] Other recent studies on this emerging topic are Lu (2021) and Murra-Anton (2021).

[4] A related setting of a general public-private relationship under uncertainty is analyzed in Attanasi, My, Buso, and Stenger (2020).

list. The firm is rewarded through profits both when it itself succeeds and/or the university succeeds, but must sink their cost of research in all cases. Amir et al. (2021) establish that a Stackelberg equilibrium arises as part of an endogenous timing scheme and Pareto dominates the Nash equilibrium of the same research game.

With the latter model as our basic one-shot game of research, we introduce a new, two-stage (non-cooperative) scenario wherein the firm chooses whether or not to offer the university a research grant to subsidize part of the latter's activity and the size of such a grant. Subsequently in the second stage, the two players simultaneously decide on their research investments.

Our main results are as follows. Comparing the two non-cooperative scenarios, the two-stage game with grants leads to a win-win outcome relative to the one-stage basic game: Both the university and the firm earn higher expected profits and the effective probability of successful basic research (taken as a proxy for social welfare) is also higher. The university ends up conducting more research than in the basic static game while the firm ends up with less, but both players are better off. This provides a good rationale for the private sector funding for-profit research, as grants provide a natural monetary transfer scheme that ends up being mutually beneficial to both players in this otherwise fully non-cooperative setting.

In the second part of the paper, we investigate the properties of two coordinated scenarios for basic research between the firm and the university. The first scenario is a research cartel or Pareto-optimal solution, wherein the two entities choose their research investments jointly to maximize total profits, in line with the well-known R&D cartel introduced in the literature (Katz, 1986 and d' Aspremont and Jacquemin, 1988). The second scenario, called research-specialized university, is based instead on the premise of research specialization, as it assigns all research to the university while maintaining total profit as the joint objective. The firm is then a passive partner as far as research is concerned though a full knowledge transfer of any ensuing research success remains as part of the arrangement. Both scenarios are a priori natural for a research consortium between a firm and a university. As in the past literature, the cartel is both of theoretical and normative interest as a benchmark scenario, but also has some real-life relevance. The research-specialized university is a realistic, though clearly sub-optimal, possibility, the interest for which being clearly of a positive, and not a priori normative, nature.

Comparing the two different types of basic research scenarios, we find that a cartel always yields a higher probability of successful research than either of the non-cooperative scenarios. The research-specialization always yields a higher success probability than the basic non-cooperative case, and also vis a vis the research cartel if and only if both the research cost parameter and the relative size of the research's commercial value are sufficiently low. The remaining comparisons of this probability between these cases follows similar threshold rules associated with the cost parameter and some restrictions on the relative size of the research's commercial value versus the government's grant to the university. When considering the profit comparisons between the coordinated scenarios, similar multi-faceted outcomes emerge depending on different thresholds. In addition, the analysis suggests a critical role for monetary transfers from the firm to the university as a pre-condition to operationalize the coordination scenarios and prevent financial losses for the university.

We also conduct an extensive performance comparison between the two coordinated and the two non-cooperative scenarios (the one-stage game and the two-stage game with grants) in terms of each entity's propensity for R&D, profits, and the resulting probability of success (or sometimes called the propensity for innovation), with the latter construed as a proxy both for technological progress and, with some qualification, also for social welfare. We find that the following ranking of the probability of successful basic research holds: research cartel (highest), two-stage game with grant, and the basic one-stage game (lowest), while research-specialized university could be ranked anywhere but the lowest place depending on the cost parameter. Another implication of these results is that the non-cooperative scenarios and the cartel for basic research all suffer from social waste in the form of redundant duplication of research effort. By avoiding the latter drawback, the research-specialization case emerges as a surprisingly high performer overall, when one takes into account that it is a profit-suboptimal scenario by its very ad-hoc construction.

The remainder of this paper is organized as follows. Section 2 describes the basic static model (the non-cooperative case). Section 3 introduces and analyzes the the novel two-stage game with grants. Section 4 considers the two coordinated scenarios and provides a performance comparison between the four scenarios. Finally, Section 5 concludes.

2 The basic non-cooperative model

This section presents a non-cooperative model of competition in basic research between a subsidized university and a monopoly firm. In the baseline model, two entities simultaneously decide on their research investments, so that the outcome features a standard duopoly Nash equilibrium and has been studied in previous works (Stenbacka and Tombak, 2020, and Amir, Chalioti and Halmenschlager, 2021). We start with a review of this baseline model in the first subsection, followed by a discussion of the scope of the model.

2.1 The model

Two entities, a university (Player 1) whose basic research activity is subsidized by the government and a profit-maximizing firm (Player 2), engage in competition regarding their basic research investments. The research outcome for each entity is a binary random variable with outcomes called success or failure of the research effort. No intermediate outcome is possible. A success may be thought of as leading to the opening of a new market. Alternatively, it may lead to expanding an existing market, or lowering existing production costs by a given fixed amount.

The success probability depends positively on the player's investment, with decreasing returns to scale. As in Stenbacka and Tombak (2020), the research investment (or expenditure) and the success probability are linked by a purely quadratic function.[5] The university and the firm indirectly decide their investments by choosing their respective success probabilities, denoted by p and q, respectively.[6] The two players are assumed to be equally efficient in basic research. Thus the research expenditure and success probability have the following function form for the university and the firm, respectively

$$C(p) = \frac{c}{2}p^2 \text{ and } C(q) = \frac{c}{2}q^2.$$

The university focuses on conducting basic research and receives a reward $V > 0$ only if it is the only entity who has succeeded to deliver scientific results. Thus, there is a criterion of scientific novelty in place. V may be thought of as a grant from the government. According to this criterion, the reward V is provided to the university for successful basic research only if the firm's research is unsuccessful. In particular, V is provided by a policymaker who may for instance decide the welfare-maximizing level of funding for the university. The (government) policymaker is not involved in the university's research operations but provides the funding upon successful delivery of scientific results.

Both players are assumed to be risk neutral. Therefore, the university's ex ante expected payoff function is

$$\Pi_1(p, q) = p(1-q)V - \frac{c}{2}p^2,$$

where the subscript represents the university's identity as Player 1.

The firm, on the other hand, is assumed to be able to acquire the knowledge generated by successful basic research, irrespective of whether the success is owed to the firm, the university, or both. A success allows the firm to realize a commercial value represented by a fixed monopoly

[5] Amir, Chalioti and Halmenschlager (2021) adds a linear cost term, kp or kq ,in the function which leads to corner solutions to the problem, i.e., one or both of the players may stop doing research when the linear cost term is sufficiently high. To avoid complication, we assume $k = 0$ in this paper, but our insights should be able to carry over to the setting with nontrivial linear costs quite naturally.

[6] This formulation has been adopted in a number of different settings due to its overall convenience, including in particular the ease of integrating the important feature of uncertain R&D: See e.g., Katsoulacos and Ulph (1998) and Kitahara and Matsumura (2006).

profit of $\pi > 0$.[7] Therefore, the present model assumes a full (one-way) disclosure in basic research from the university to the firm upon research success.[8] Tacit in this assumption is that the university is required to disclose to the firm all the scientific results generated in its basic research process if successful, while the firm can fully appropriate its research and is legally allowed to keep such results concealed from the public, for instance, as commercial secrets.

Accordingly, the firm's expected payoff is

$$\Pi_2(p,q) = q\pi + p(1-q)\pi - \frac{c}{2}q^2. \tag{1}$$

Clearly, the second term of (1) captures the one-way knowledge transfer that happens when the university is successful in basic research while the firm is not.

In this game, to be referred to as Case N or non-cooperative scenario below, the firm and the university maximize own payoffs by choosing own success probabilities simultaneously.

Each player's reaction curve can be solved via its own first-order condition. The university and the firm's reaction curves are defined as usual, respectively by

$$r_1(q) = \underset{0\le p\le 1}{\operatorname{argmax}}\, \Pi_1(p,q) = \frac{V}{c}(1-q) \quad \text{and } r_2(p) = \underset{0\le q\le 1}{\operatorname{argmax}}\, \Pi_2(p,q) = \frac{\pi}{c}(1-p).$$

The intersection of the two downward-sloping reaction curves is the Nash equilibrium of the non-cooperative duopoly game. It can be expected that if the research cost is too low, the players may choose to excessively invest in basic research to lock in a success with certainty. To maintain the uncertainty of the outcomes, the following assumption guarantees that the solution to Case N is interior.

Assumption 1 *$c > 2V$ and $c > 2\pi$.*

Assumption 1 states that the quadratic cost coefficient c is greater than two times the value of research to both players. Under this assumption, solving for the intersection of the reaction curves yields a unique Nash equilibrium of the non-cooperative game, described in the following Lemma.

Lemma 1 *For the non-cooperative game (Case N), the Nash equilibrium strategies in basic research for the university and the firm are, respectively,*

$$p^* = \frac{(c-\pi)V}{c^2-\pi V} \quad \text{and} \quad q^* = \frac{(c-V)\pi}{c^2-\pi V}.$$

[7] As stated in Amir, Chalioti and Halmenschlager (2021), the innovation can be thought of as either a product innovation, which establishes a monopoly status of the firm in a new market, or as a process innovation, which lowers the firm's production cost and increase its profit by π.

[8] This may also be thought of as a one-way full spillover from the university to the firm, although this terminology differs from the more common use of this term in the literature.

This is thus the solution for Case N, which is just the result of Stenbacka and Tombak (2020) in Eq.(3) and (4), and Amir, Chalioti, and Halmenschlager (2021) in their Lemma 1 upon setting $k = 0$.

2.2 On the scope of the model

Here, we comment on the nature of the present model and argue in some detail that it is suitable for a number of important cases in real-life industrial research, though probably not in a universal sense. In particular, we elaborate on the tacit assumptions that justify such a model. Further justification of the model may be found in Stenbacka and Tombak (2020).

A key part of the model is the financing of research for the university. As in Stenbacka and Tombak (2020), the university will receive a government grant, $V > 0$, conditional on the requirement that the university is the only player to succeed in research, i.e., that the firm is unsuccessful. This assumption may be partly justified a priori on normative or theoretical grounds, although some of these also have some positive dimension. For the model under consideration here, the idea is to expect universities (and research institutes) to bear the risk associated with their own research and to investigate the implications of such a feature on the resulting research levels. In many countries, this is of course intended to apply a priori only for leading universities, as these are those that benefit from typically highly-selective outside subsidies. Since research conducted at such universities is a priori destined for academic publication in the form of high-quality research articles, this requirement amounts to rewarding only successes since originality and quality are pre-requisites for publication. Another related facet of this issue is that, as Friedman and Friedman (1990) famously argued, most scientists write research proposals based on results already derived for the most part, and as such always endeavor to remain one step ahead of their government sponsors.[9]

In terms of empirical evidence on research award selectivity, Arora and Gambardella (2005) have found that past productivity and other observable researcher characteristics such as university quality are correlated with grant award selection. Their data set includes a broad sample, so one would expect the results to be even sharper under strict selection criteria.

Similar research sponsorship is also a hallmark of several large private firms that are known

[9] In Wade (1980), Milton Friedman gives as example of such widespread behavior the famous physicist Leo Szilard, who typically submits proposals on the basis of finished research, so "he could use the [new grant] money for research whose outcome he could not predict". One may go further and say that the manner of operation of many sponsors (e.g., NSF or NIH in the USA, DFG in Germany, etc...) forces scientists to behave in this way, since proposals are judged on the basis of the following common criteria: already available results, promise of eventual success and publications from the previous grant. To be funded, proposals have to present convincing preliminary results that would sway reviewers into viewing the proposal as both low-risk and high-value. As proposals usually extend over three years, this also maximizes the probability that authors would have produced published or accepted articles (in good outlets) from the previous grant when applying for the subsequent grant.

to have produced leading-edge basic research that ended up generating new often highly profitable technologies. A case in point is Bell Labs, whose overall output includes the transistor, the cosmic microwave background, algorithms for quantum computing (Shor's factorization and Grover's database search). Although a private monopoly, Bell Labs was broadly perceived as being financed by "essentially a built-in 'R&D tax' on telephone service" (Georgescu, 2022). Other examples of this trend include IBM and more recently Yahoo, Google and Microsoft. An early study by Rosenberg (1990) reports two interesting facts for the present paper: The first is that a significant number of firms conduct basic research (for data on this point, also see Akcigit et al., 2021 and Higon, 2016). The second is that, though usually taken for granted, the distinction between basic and applied research is often quite blurred and difficult to ascertain.[10]

The present model does not consider university and firm research activities to be complementary, as is often done, but rather substitutes. Again, the claim is not that this is a universally valid feature, but nevertheless one that fits in the spirit of the foregoing discussion, as a fomalization of some of the settings mentioned in Rosenberg (1990). This feature is consistent with the decreasing returns assumption on research activity, which is quite a standard postulate.

Since the model may be reduced to a firm with access to two draws from a random research black box, one may ask why the firm does not run two distinct research labs. The answer is that this would require prohibitively high fixed costs, which are already sunk for the firm's existing lab, as well as for the university's. Finally, while the question of unequal research capabilities is relevant for this model, the case of symmetry is both a simplifying assumption and a central case that sheds light on the other two possible cases.

3 The two-stage game with a grant

In this subsection, we introduce a commonly used tool in real life for firms to try and influence the level of effective research conducted: A research grant from the firm to the university. This is such a familiar feature in the research landscape that a justification of the need for its investigation is hardly needed.

In this model, we add a stage before the two entities engage in basic research competition in the basic model, which allows the firm to pre-specify and commit to a contingent grant it would give to the university, with similar terms as in the government grant in the basic model, in support of its basic research. We compare the two models and show some appealing properties of the two-stage game with grants: both the university and the firm prefer the two-stage game in terms of their equilibrium payoffs and the probability for research success is strictly higher

[10]Another relevant characteristic of research-oriented firms is that they operate on the basis of high-powered incentives, minimal labor protection, and more direct monitoring of their research personnel.

in the two-stage game.

As one might easily suspect, this option may well turn out to be quite beneficial to the firm in light of the disclosure requirement imposed on the university. Thus the firm may offer to subsidize the basic research of the university via a grant of a certain amount, conditional on the university being the only entity to succeed ex post in the process of research. In other words, analogously to the government grant, the university continues to be obligated to cover its own risk stemming from a potential failure of its own research.[11] The university and the firm still conduct basic research activities independently and simultaneously.

While one may view such a grant as a form of partial coordination in research between the two entities, the setting actually remains fully non-cooperative. Indeed, we shall see that such a grant may well turn out to be self-serving, though also beneficial to the university.

We model the resulting economic environment as a two-stage game and refer to it as scenario or Case G (for grant). The formal timing of the two-stage game is described as follows.

Stage 1. The firm commits to giving the university a grant $0 < g < \pi$ in support of its basic research, but only if the university alone succeeds in basic research ex post.
Stage 2. The two players simultaneously choose their success probabilities under the grant.

The amount of the grant is assumed to be less than π, the total value of the innovation to the firm. Therefore, the university and firm's payoffs in the two-stage game with grant are, respectively,

$$\begin{cases} \overline{\Pi}_1(p,q,g) = p(1-q)(V+g) - \frac{c}{2}p^2 \\ \overline{\Pi}_2(p,q,g) = q\pi + p(1-q)(\pi - g) - \frac{c}{2}q^2. \end{cases} \tag{2}$$

To solve for the subgame perfect equilibrium of the two-stage game, we use backward induction and start with the second-stage research problem. For any given grant g, the reaction curves for the university and the firm at the second stage become

$$\begin{cases} \overline{r}_1(q) = \underset{0\le p\le 1}{\operatorname{argmax}}\, \overline{\Pi}_1(p,q,g) = \frac{(1-q)(V+g)}{c} \\ \overline{r}_2(p) = \underset{0\le q\le 1}{\operatorname{argmax}}\, \overline{\Pi}_2(p,q,g) = \frac{(1-p)\pi+pg}{c}. \end{cases}$$

It can be seen by inspection that the second-stage research game has strategic substitutes. Compared to the one-stage model without grant, both the university and the firm's reaction curves shift outwards (as g increases). In other words, the grant encourages both the firm and the university to conduct more research given a fixed research level by the other. However, because the reaction curves are both downward-sloping, it is not straightforward whether the

[11] The justification of this aspect of the model given earlier applied here as well, at least for some country-specific settings.

two outward shifts result in more or less research conducted by each player. This question can be settled after solving for the subgame perfect equilibrium of the game.

Combining the two players' reaction curves, we get the second-stage equilibrium strategies as functions of g,

$$\overline{p}(g) = \frac{(c-\pi)(g+V)}{c^2+(g-\pi)(g+V)} \quad \text{and} \quad \overline{q}(g) = \frac{c\pi+(g-\pi)(g+V)}{c^2+(g-\pi)(g+V)}.$$

The expressions immediately yield the equilibrium strategies of the one-stage game given in Lemma 1 upon setting $g = 0$.

Given the second-stage decision, the firm maximizes its payoff by choosing a grant g, $\max_{0\leq g<\pi} \overline{\Pi}_2(\overline{p}(g), \overline{q}(g), g)$. The game has a unique subgame perfect equilibrium, $(g^*, \overline{p}(g^*), \overline{q}(g^*))$, described in the following Lemma. All proofs for the lemmas and propositions in this paper can be found in the Appendix.

Lemma 2 *For the two-stage game with grants, the equilibrium grant chosen by the firm is*

$$g^* = \begin{cases} \frac{\pi-V}{2} & \text{if } \pi > V, \\ 0 & \text{if } \pi \leq V. \end{cases}$$

When $\pi > V$ and thus $g^* > 0$, the associated success probabilities of the university and the firm are, respectively[12]

$$\overline{p} = \frac{2(c-\pi)(\pi+V)}{4c^2-(\pi+V)^2} \quad \text{and} \quad \overline{q} = \frac{4c\pi-(\pi+V)^2}{4c^2-(\pi+V)^2}.$$

It is verified in the Appendix that both $\overline{p}$ and $\overline{q}$ are interior (probability) solutions lying in $(0,1)$. The Lemma holds that the firm provides a grant for the university only when its monopoly profit is higher than the government grant, $\pi > V$. Otherwise, the value of basic research to the firm is too low for it to commit to a supporting grant at any level. When $\pi > V$, the firm will promise to provide a grant of $\frac{\pi-V}{2}$ to the university if its own research is unsuccessful while the university's is successful, and thus disclosed to the firm. The higher the difference in π and V, the higher the grant the firm gives as supportive compensation, which seems to be quite intuitive.

Since in case $\pi < V$, equilibrium calls for $g^* = 0$, the game with grants reduces to the non-cooperative model (Case N), Thus private financing and public financing of the university's research act like partial substitutes, and free riding on the task of financing the university may well be a relevant issue. In other words, each entity will be inclined to think that its investment

[12]Note that $\overline{p} \equiv \overline{p}(g^*)$, and $\overline{q} \equiv \overline{q}(g^*)$. When there is no risk of confusion, we often use the name of a function, such as $\overline{p}(\cdot)$ or $\overline{\Pi}_1(\cdot,\cdot,\cdot)$, to denote the equilibrium value of that same variable.

in research will crowd out the other entity's investment. Though perhaps to be expected, such an outcome is clearly noteworthy. Indeed, although the government may a priori finance the university up to the value of the resulting social welfare, thus in excess of π,[13] *it may well act in a more restrained manner, in anticipation of the firm stepping in to fill the void, since after all, the firm is the immediate beneficiary of the university's research.*[14] *One may also expect the firm to attempt to signal that it will not finance the university, as a way to induce the government to set a high value for its grant V to the university in the first place.*

Thus, besides its intrinsic interest as a realistic scenario for the underlying interaction, the analysis of Case G also sheds new light on the properties of the basic model (Case N) where the parameter V is a priori exogenous. Further interactive study of the two Cases is probably warranted.

Henceforth any mention of Case G will presume tacitly that $\pi > V$ (else, simply refer to Case N). The following discussion will mainly revolve around the nice properties of the equilibrium with strictly positive grants. These interesting results will be presented and discussed first, before being summarized in Proposition 1.

Under the given assumptions, the firm always finds it optimal to commit to a positive grant $\frac{\pi-V}{2}$ in support of the university's basic research, instead of remaining fully independent in the basic research competition. The fact that the firm can easily induce the non-cooperative play by letting $g = 0$ implies that the firm strictly prefers its equilibrium payoff net of the grant. It will become clear that with positive grants, the university invests more in research than its non-cooperative (Case N) strategy while the firm invests less, i.e., $\overline{p} > p^*$ and $\overline{q} < q^*$, so the probability of full knowledge transfer, $p(1-q)$, increases in favor of the firm. This gain, combined with the cost savings in basic research as the firm conducts less researchs, will always outweigh the loss associated with the grant, and thus the expected payoff of the firm strictly increases against its non-cooperative payoff. Interestingly, it can be proved that the university also prefers the game with grants to Case N, as the grants fully compensate for its higher research costs induced by higher $\overline{p}$. Therefore, the grant design does bring about a win-win situation where, despite remaining non-cooperative in research competition, both of the two entities end up better off.

In fact, besides the increase in both players' payoffs, the two-stage game has another nice property regarding the speed of technological progress in basic research. It is well-known that in most strategic settings with imperfectly appropriable research, the existence of knowledge transfer is likely to render basic research under-provided. In such cases, the speed of techno-

[13] In fact, the government may go well beyond the firm's proft π, to take into account the social value of the research beyond the firm's industry.

[14] As an illustration, consider the common case of an industry with linear inverse demand $P(Q) = a - bQ$ and liear cost c, we have monopoly profit $\pi = (a-c)^2/4$ and social welfare $W = 3(a-c)^2/8b$, so that $W = 3\pi/2$.

logical progress can be seen as a good proxy for a social welfare analysis, especially when the market primitives required for a standard welfare analysis are unspecified and beyond the scope of the current work (as in Stenbacka and Tombak, 2020 and Amir, Chalioti and Halmenschlager, 2021).[15]

In a setup comprised of two entities engaging in basic research whose outcome is uncertain, a meaningful measure of technological progress is the overall probability P of research success, irrespective of whether it is accomplished by the university, the firm or both. This probability captures three mutually exclusive events: the university succeeds while the firm does not, the firm succeeds while the university does not, or both entities succeed. Adding up the three probabilities yields

$$P = p(1-q) + (1-p)q + pq.$$

To compare the pace of technological progress, let P^* and $\overline{P}$ denote the effective probabilities of success in the basic game (Case N) and the two-stage game with grants (Case G) , respectively. Thus,

$$P^* = p^* + q^* - p^* q^* \quad \text{and} \quad \overline{P} = \overline{p} + \overline{q} - \overline{p}\,\overline{q}.$$

This measure also reflects the fact that, while increasing each player's research investment will increase the overall probability of success, these two effects are discounted by the potential duplication in basic research, namely that as long as one player succeeds in research, the other player's success is simply redundant.

The profit and success probability comparisons between Cases N and G are as follows.

Proposition 1 *Assume $\pi > V$. Comparing the equilibrium properties of the non-cooperative game (Case N) and the two-stage game with grants (Case G), we have:*
(i) The university conducts more basic research in Case G while the firm conducts less:

$$\overline{p} > p^* \quad \textit{and} \quad \overline{q} < q^*.$$

(ii) Both the university and the firm have higher expected payoff in Case G:

$$\overline{\Pi}_1 > \Pi_1^* > 0 \quad \textit{and} \quad \overline{\Pi}_2 > \Pi_2^*.$$

[15] Indeed, in light of the many externalities associated with know-how, the idea that research and development (R&D) cannot be expected to be provided at adequate levels by the market mechanism is a broadly accepted conclusion in economics (see e.g., Spence, 1984, among others). The sizable gap between the private returns and the social returns to R&D is well-documented (e.g., Griliches, 1995; Bernstein and Nadiri, 1988). In particular, if one focuses solely on basic (or fundamental) research as an isolated component, then the public good aspect and appropriation problems become even more pronounced. As a consequence, the proposition that governments ought to subsidize research is often a foregone conclusion.

(iii) The probability of success is higher in Case G:

$$\overline{P} > P^*.$$

According to the Proposition, the comparison between the one-stage game with no grant and the two-stage game with a grant is quite unambiguous. The two-stage game leads to more research specialization by the university but still calls for the firm to conduct a reduced research investment. On balance, the resulting probability of success is higher, implying a faster pace of technological progress. When the latter is taken as proxy for social welfare, the two-stage game with grants emerges as a superior form of organization of research. Finally, both the firm and the university end up netting higher profits, which makes the two-stage game a win-win arrangement all around.

The superior performance of the game with a grant relative to the game without grant is somewhat reminiscent of that of the cartelized research joint venture in the standard two-period R&D game involving oligopolistic firms (Kamien et al., 1992 and Amir et al,, 2003). A key difference though is that the latter case involves a comparison between full coordination and the non-cooperative solution whereas the present comparison involves two different non-cooperative scenarios.

Since research grants from the private sector to the university sector are widespread in all advanced industrial nations for research in engineering, bio and medical sciences, and management, the real-world relevance of the above result is amply confirmed. A final justification for it is that, when establishing a fully or otherwise partially collusive relationship between a firm and a university is difficult, either due to legal reasons, conflict of interest, or high contracting costs, this model provides a practical way to partly circumvent the roadblocks for more involved coordination.

Nevertheless, the notion of university-firm R&D consortium or research joint ventures (RJVs), a relationship of a coordinated nature, is quite important and fairly widespread in the real world. It has been extensively documented and addressed in the literature (e.g., Poyago-Theotoky, Beath and Siegel, 2002, Bercovitz and Feldman, 2007, Link and Scott, 2005, among others).

In the next section, we attempt to address two possibilities for university-firm research coordination in the form of a consortium (or cartel) and make a complete comparison of the equilibrium properties for all four models.

4 Two coordinated scenarios for basic research

In this section, we consider two alternative models for research collaboration, or research consortium between a firm and a university. The two scenarios might be thought of as displaying a mix of an R&D cartel and a research joint venture (see e.g., Amir, 2000). The first scenario commits the two partners to a research cartel, i.e., to choosing their expenditures in basic research to maximize the sum of their payoff functions.[16] The second is based solely on the idea of full specialization in research by the university partner, with the firm subsequently obtaining the scientific results for free in case of success. As such, this consortium is not based on any sort of joint optimization, at least on a priori grounds. In both cases, once the research investments are decided, the conduct of basic research is assumed to proceed independently across the two entities' labs, just as in the non-cooperative scenarios.

We derive the probabilities of basic research for the two partners under each scenario. We also use the coordinated cases as a benchmark against which to evaluate the resulting probability for successful basic research of each of our non-cooperative scenarios in a comparative perspective. In addition, since it is widely believed that, due to multiple externalities, R&D is usually provided at less than its socially optimal level (e.g., Bernstein and Nadiri, 1988 and Griliches, 1995), we may again tacitly take the propensity for innovation as a proxy for a social welfare assessment.

4.1 The basic research cartel

As in the literature on research joint ventures,[17] the joint objective function of the research cartel, also called research consortium, is the sum of the two partners' payoff functions, or

$$\Pi^c(p,q) = p(1-q)(V+\pi) + q\pi - \frac{c}{2}(p^2+q^2), \tag{3}$$

where the superscript c stands for cartel or consortium. If the university alone succeeds in basic research, the consortium gets $V+\pi$ as its total payoff (the first term), with the firm profiting from a knowledge transfer. If ever the firm succeeds in basic research, only the firm gets π (the second term). As this scenario gives rise to a Pareto optimal solution, it will serve as a useful benchmark to compare the other scenarios to in terms of overall performance, propensity for

[16]The RJV part of such a consortium is reflected by the fact that one-way knowledge transfer from the university to the firm are mandated as part of the non-cooperative model and preserved for the consortium (see Kamien, Muller and Zang, 1992).

[17]See d' Aspremont and Jacquemin (1988), Kamien, Muller and Zang (1992) and Amir (2000).

innovation and profits. The first-order conditions are

$$\begin{aligned}(1-q)(V+\pi)-cp &= 0,\\ -p(V+\pi)-cq &= 0.\end{aligned}$$

To guarantee interiority of the optimal solution, we need to impose an additional assumption on c, π and V.

Assumption 2 $c\pi > (V+\pi)^2$.

Under the assumptions, the unique (Pareto-optimal) solution is

$$p^c = \frac{(c-\pi)(\pi+V)}{c^2-(\pi+V)^2} \quad \text{and} \quad q^c = \frac{c\pi-(\pi+V)^2}{c^2-(\pi+V)^2}.$$

Interestingly, despite the obvious duplication in research inherent in this set-up, Pareto optimality still calls for both partners to conduct research. The probability of successful basic research is similarly defined as $P^c = p^c + q^c - p^c q^c$. As for payoffs, because the two entities are assumed to conduct their research activities independently once the research investments are specified, we may naturally treat $\Pi_1^c = p^c(1-q^c)V - \frac{c}{2}(p^c)^2$ as the university's payoff under Case C, and $\Pi_2^c = p^c(1-q^c)\pi + q^c\pi - \frac{c}{2}(q^c)^2$ as the firm's. However, we point out here that this does not rule out the possibility of a financial transfer between the two parties, which may be essential to maintaining the relationship as mutually beneficial.

Proposition 2 *Assume $\pi > V$. Comparing the cartel (Case C) and the two-stage game with grants (Case G):*
(i) The university conducts more, while the firm conducts less, research under Case C than under under Case G:

$$p^c > \overline{p} \quad \textit{and} \quad q^c < \overline{q}.$$

(ii) The firm's payoff is higher under Case C, while the university's payoff is negative:

$$\Pi_1^c < 0 < \overline{\Pi}_1 \quad \textit{and} \quad \Pi_2^c > \overline{\Pi}_2.$$

(iii) The probability of success is higher under Case C:

$$P^c > \overline{P}.$$

It is expected that the research cartel will allocate more research activity to the university and less to the firm because of the effect of one-way knowledge transfer. Indeed, compared to the game with grants, which by Proposition 1 already tilts research responsibility towards the university relative to the non-cooperative case, the consortium (Case C) goes further. That is, $p^c > \overline{p} > p^*$ and $q^c < \overline{q} < q^*$. Therefore, we may infer that Pareto optimality in terms of joint payoffs would require higher, though less than full, specialization in the two parties' research activities.

While the firm benefits from higher knowledge transfer and lower research expenditure, reflected in $\Pi_2^c > \overline{\Pi}_2$, the university's profits go down below zero due to its expanded research activity. In fact, $\Pi_1^c < 0 < \Pi_2^* < \overline{\Pi}_2$. Without monetary transfers, the university would rather go fully non-cooperative than behave coordinatedly. Nevertheless, Pareto optimality ensures that $\Pi_1^c + \Pi_2^c > \overline{\Pi}_1 + \overline{\Pi}_2$, so there are obvious ways in which an appropriate financial transfer from the firm to the university can make both entities better off.

Not only does the consortium achieve Pareto optimality in terms of the joint payoff, it also gives rise to the highest speed of technological progress among the three models discussed so far, i.e., $P^c > \overline{P} > P^*$, though, as mentioned before, a certain level of duplication in research effort is still present in the cartel. In a nutshell, the cartel calls for a higher degree of specialization in research, and seems to be superior to the non-cooperative set-ups, both by the Pareto optimality criterion (as is obvious) and by the proxy welfare criterion.

4.2 The case of a research-specialized university

This scenario is predicated on exogenously-imposed research specialization at the outset: All research is left to the university alone and the firm, not to be concerned with conducting any research directly, would receive the full potential benefits of an research success whenever the university achieves it. As this is still a coordinated scenario, the university takes into account both payoffs (with equal weights) in its decision, upon setting $q = 0$. This will be referred to as Case S (for specialization) below.

Therefore, the joint objective function for the specialization case is given by (3) with q exogenously set equal to 0:

$$\Pi^s(p, 0) = p(V + \pi) - \frac{c}{2}p^2,$$

where s stands for the scenario of research specialization. Taking the first-order condition with respect to p yields $V + \pi - cp = 0$. Hence, the unique solution is given by

$$p^s = \frac{\pi + V}{c} \quad \text{and} \quad q^s = 0.$$

This is the optimal level of basic research of a university-firm specialized consortium that leaves all basic research to the university, when the latter assigns equal weight to the firm's expected profit due to the partnership as to its own profit and is willing (as is always the case here) to fully share the fruits of its research. This solution reflects the intrinsically asymmetric roles of the entities and a complete specialization in the area of research.[18] In this model, not assigning any research activity to the firm goes against a research production efficiency argument that, the cost of research being strictly convex for both entities, the consortium has a bias towards splitting a given research expenditure equally across the identical labs of the two research entities. The countervailing argument in favor of this specialization is the desire to avoid duplication of research.

An inspection of the university's objective function reveals that, for the university, fully specialized partnership is equivalent to a game with grant $g = \pi$, in conjunction with the acknowledgement that the firm will invest zero in research. Therefore, keeping in mind the strategic substitutes property of the basic game, it is quite intuitive that the university will engage itself in a higher level of research activity than in the two-stage game with a grant strictly less than π.

But does the university conduct more research than it does in the cartel case? The answer is yes. Notwithstanding the coordinated nature of the cartel, the two parties' research investments in the cartel consortium are, as in the non-cooperative games, strategic substitutes, reflected in the negative cross partial derivative of the objective function (3). Hence, the marginal revenue of the university's success probability, p, decreases in the firm's success probability q. If q is assigned a value of zero, the marginal revenue of p should be the highest, thus the university should be conducting more research in the case of full specialization than in the cartel. This assertion is confirmed and summarized in the following Proposition, along with the comparison of other equilibrium characteristics between the research specialization consortium, the research cartel and the two-stage game with grants. Here, the constants $c_1 - c_4$ appearing in the results below are threshhold cost parameters that will be defined precisely (implicitly as roots of some cubic or quadratic polynomial equations) and discussed in the Appendix.

Proposition 3 *Assume $\pi > V$. Comparing the research specialization case (Case S) with the research cartel (Case C) and the two-stage with a grant (Case G), we have:*
(i) The university conducts more research in Case S than in Case C and the firm conducts less

$$p^s > p^c > \overline{p} \quad \text{and} \quad q^s = 0 < q^c < \overline{q}.$$

(ii) The university's payoff is lower in Case S than in Case C. The firm's payoff in Case S

[18] It is worth recalling that, even in R&D cartels involving two symmetric firms, the optimal solution may well be asymmetric, an interesting fact that has been ignored in much of the literature on RJVs (see Salant and Shaffer, 1998 and Amir et al., 2003 for a discussion on this point).

may be higher or lower than in Case C depending on c but is always higher than in Case G: for some $c_1 > 0$

$$\Pi_1^s < \Pi_1^c < 0 < \overline{\Pi}_1 \quad \textit{and} \quad \begin{cases} \Pi_2^s > \Pi_2^c > \overline{\Pi}_2 \ \textit{when}\ c < c_1, \\ \Pi_2^c > \Pi_2^s > \overline{\Pi}_2 \ \textit{when}\ c > c_1. \end{cases}$$

(iii) For the probability of success, for some $c_2 > 0$ *and* $c_3 > 0$*:*

$$\begin{cases} P^s > P^c > \overline{P} \ \textit{when}\ c < c_3 \ \&\ \pi < 6.85V \\ P^c > P^s > \overline{P} \ \textit{when}\ c_3 < c < c_2 \ \&\ \pi < 6.85V \ \textit{or when}\ c < c_2 \ \&\ 6.85V < \pi < 12.75V \\ P^c > \overline{P} > P^s \ \textit{when}\ c > c_2 \ \&\ \pi < 12.75V \ \textit{or when}\ \ \pi > 12.75V. \end{cases}$$

Under Case S, the research burden completely falls on the university's shoulders, whose payoff is then $\Pi_1^s = p^s V - \frac{c}{2}(p^s)^2$, while the firm appropriates any success and receives an expected payoff of $\Pi_2^s = p^s \pi$. Case S helps the university to eliminate the possibility of having a futile success, which the government would not reward if the firm also succeeds in research. However, the partnership part imposes high research costs on the university. The two effects go against each other, and the Proposition confirms the fact that the negative cost effect always dominates the positive effect coming from specialization, resulting in lower expected payoff to the university, even lower than its negative cartel payoff before any financial transfers are made.

Case S allows the firm to completely free ride on the university's research, making the firm's profit strictly higher compared to the game with grants. But the comparison with its cartel payoff is unclear, as its own research investment could be profit-generating when moderate enough. If the cost parameter c is sufficiently low, the firm's pure free-riding (Case S) payoff exceeds its cartel payoff, and vice versa. This can be partly explained by looking ahead at the result in (iii) on the effective probabilities of successful basic research.

The firm can realize the commercial value π as long as one of the two parties succeeds in research, i.e., $\Pi_2^j = P^j \pi - \frac{c}{2} q^2$ where $j = c, s$. In other words, the firm's payoff is directly associated with the probability of research success. In fact, when $c < c_3$ and $\pi < 6.85V$, we can show that $P^s > P^c$ and $q^s = 0 < q^c$. With higher P^s and lower research cost, it clearly follows that $\Pi_2^s > \Pi_2^c$.

These arguments show that Case S may further deepen the gap between the two parties' payoffs, featuring negative payoffs to the university. Hence, an early agreement between the two parties on how to divide the joint profits would be essential in establishing this coordination relationship.

Although Case S eliminates the duplication in research, the speed of technological progress is not necessarily the highest in this case, especially when the cost c is high or when the monopoly profit π substantially exceeds V, which is conveyed as the main message of Proposition 3 (iii).

If $c < c_3$ and $\pi < 6.85V$ (recall that $\pi > V$ is implied by Case G), then Case S does give rise to the highest propensity for innovation. But as the cost and π increase, P^s gradually declines, and when c exceeds c_2 or π exceeds $12.75V$, specialization has the lowest propensity for innovation among Cases G, C and S.

The economic intuition behind these results is simply that, when π is high relative to V, the firm is more motivated to conduct its own research (but is constrained not to do in the case of specialization), since π does not appear in the university's objective. This observation explains why Case S performs poorly in terms of propensity for innovation in such cases.

In general, Proposition 3 (iii) gives the possible rankings of the propensity for innovation for Cases G, C and S. The basic research cartel has a higher propensity for innovation than the game with grants, but Case S can lie anywhere as long as it is above the basic non-cooperative level, P^*. Particularly, when $c < c_3$ and $\pi < 6.85V$, the university alone conducts a sufficiently high level of research such that $P^s = p^s > P^c$. Therefore, using the probability of successful research alone as criterion (which may proxy for social welfare), one may say that Case S sometimes is the best from a social point of view among the three designs of R&D scheme, especially when both the research cost and the monopoly profit is comparatively low.

The last part of this section provides further comparisons between the four scenarios. Two meaningful criteria for performance evaluation are total profits of the university and the firm, for which the Pareto optimal research cartel values serve as natural upper bounds and benchmarks, and the probability of successful basic research, which we treat as a measure of social welfare.

First of all, in addition to previous results, the following ranking of total profit and success probability can be inferred quite straightforwardly:

$$\Pi_1^c + \Pi_2^c > \overline{\Pi}_1 + \overline{\Pi}_2 > \Pi_1^* + \Pi_2^*, \quad \text{and } P^c > \overline{P} > P^*.$$

The superiority of Case S over Case N is not clear in view of Proposition 3. Also, Pareto optimality of the research cartel case implies that $\Pi_1^c + \Pi_2^c > \Pi_1^s + \Pi_2^s$. However, which of Case S and Case G is more profitable does not have a clear-cut answer, as it also depends on the cost parameter c. Proposition 3 (ii) and (iii) have delivered a general view that Case S performs better than Cases G and C when c is low (with some restriction on π and V) in terms of the firm's payoff and the propensity for innovation. We present these results in the next Proposition, which provides a complete characterization of the comparisons.

Proposition 4 *The research specialization (Case S) has higher joint payoff and higher probability of success than the non-cooperative case (Case N). That is,*

$$\Pi_1^s + \Pi_2^s > \Pi_1^* + \Pi_2^* \quad \textit{and } P^s > P^*.$$

The total profit comparison between the research specialization (Case S) and the two-stage game with grants (Case G) is as follows (for some $c_4 > 0$):

$$\Pi_1^s + \Pi_2^s > \overline{\Pi}_1 + \overline{\Pi}_2 \quad \textit{if and only if} \quad c < c_4 \ \textit{and} \ \pi < 7.74V.$$

Propositions 1 (ii) and 3 (ii) imply that, while the firm actually benefits from either form of coordination (research cartel and specialization) compared to Case N, the university finds coordination detrimental in general, reflected in its negative payoffs, and more so under the specialization case. An important practical implication is that the two partners need to necessarily resort to financial transfers as part of the basic implementation of the optimal operation of either form of coordination, in order for both to benefit from such coordination. Otherwise, the university will not find it worthwhile to take part in either coordination scenario. Proposition 3 holds that Case S generates enough surplus to be mutually beneficial relative to Case N if a suitable financial transfer from the firm to the university is part of the arrangement. In contrast, recall that financial transfers are not needed in the grant game, since both entities receive higher payoffs relative to the non-cooperative scenario, i.e., $\overline{\Pi}_i > \Pi_i^*, i = 1, 2$.

The underlying intuition as to why Case S performs poorly in terms of profit when π is high relative to V is the same as above. The conclusion that both coordinated scenarios are superior to the two non-cooperative scenarios, for total profit and the pace of research as long as research costs and the monopoly profit are not too high, suggests that grants in a non-cooperative setting are not sufficient to correct the inefficiency inherent in Case N. This is another endorsement of the merits of research collaboration, even in the form of the research-specialized scenario.

In addition, the fact that Case S dominates the other three scenarios (under low R&D cost) as far as the propensity of innovation is concerned provides a strong ex post motivation for the study of this otherwise rather ad hoc scenario, which is by construction sub-optimal for joint profit. The main justification behind Case S is the avoidance of research duplication, and the high performance of this case in terms of research pace partly vindicates that goal, particularly when research is less costly. This high performance is perhaps the least easily expected result of the entire paper and reveals the importance of research duplication as a factor in the overall debate on research, at least in this particular setting.

5 Conclusion

This paper considers four different scenarios for non-cooperative and coordinated basic research between a university and a firm: a one-stage game, a two-stage game with grants, a research cartel and a cartel with research-specialized university. A university and a firm conduct basic research in order to increase their probability of success. If the university succeeds, it receives a financial reward by a policymaker and its scientific results are fully spilled over to the firm. The firm can conduct basic itself but it can also fully appropriate the university's results and profit from them. The paper is comprised of two main parts.

The first part introduces the novel scenario of a two-stage game with grants and compares its performance vis a vis the one-stage basic game. The former leads to a win-win *(resp., the same)* outcome relative to the latter *if the firm's profit exceeds (resp., is lower than) the value of the basic research:* Both the university and the firm earn higher expected profits and the effective probability of successful research (taken as a proxy for social welfare in our context without market primitives) is also higher. The university ends up with more research than in the basic static game while the firm ends up with less, but, as a result, both players emerge better off in the game with grants.

The second aim of the paper is to consider two models of research consortium, a research cartel aiming at maximizing joint profits and a scenario specializing research to the university only (called university specialization) but in the context of a partnership. The research cartel always yields a higher propensity for innovation than either of the non-cooperative scenarios. The research-specialization case always yields a higher success probability than Case N, but could be higher or lower than any of the two other scenarios, depending on the cost parameter and the relative size of the monopoly profit (w.r.t. the government grant). The research cartel (Pareto-optimal) solution yields higher profit for the firm, but not for the university. The same qualitative divergence in the profit effects also holds for the university specialization, reinforcing the potentially important idea that, in the absence of monetary transfers, coordination in basic research would only take place at the expense of the university, even though it yields substantial benefits to the firm. Naturally, our results suggest that there is scope for monetary transfers from the firm to the university to lead to mutually beneficial coordination under either of the two scenarios studied here.

In the way of policy implications for research management, we first observe that the overall analysis of this paper first sheds light on the nature of the basic research game. Indeed, if the idea of endorsing the notion of competition in basic research between a university and a firm is that it might lead to the universally desired goal of a high level of basic research, then the properties of the two models of research consortium studied in this paper run counter to that basic intuition. This paper shows the benefits of grants relative to the basic model, but highlights the substantial merits of research coordination between a university and a firm beyond that. While introducing grants into the model enhances the overall performance, the coordinated scenarios may still increase both entities' payoffs and the propensity for innovation with the help of appropriately designed financial transfers. As such, these conclusions join those from the older literature on research joint ventures in laying out the merits of research consortium.

Our model suggests new avenues for future theoretical and empirical research on the top of firm-university interaction regarding research. One promising avenue is to consider competition between firms in the commercial sector and examine how their equilibrium incentives to conduct research change in the presence of competitive pressure and opportunities for free-riding. One can also explore the effects of different schemes for coordination, involving different groups of firms and universities, in a coalition-formation approach, which might pave the way for some sort of foundations for horizontal and possibly also vertical relations in research.

6 APPENDIX

The proofs of all the results of the paper are contained in this appendix and given in the order in which they appear in the text.

6.1 Proof of Lemma 1

The reaction curves are given as $r_1(q) = \frac{(1-q)V}{c}$, $r_2(p) = \frac{(1-p)\pi}{c}$. Solving for the Nash equilibrium via $p = \frac{(1-q)V}{c}$, $q = \frac{(1-p)\pi}{c}$ yields $p^* = \frac{(c-\pi)V}{c^2-\pi V}$ and $q^* = \frac{(c-V)\pi}{c^2-\pi V}$. $\square$

6.2 Proof of Lemma 2

At the second stage, the university's problem is:

$$\max_{0\leq p\leq 1} \overline{\Pi}_1 (p, q, g) = p (1-q) (V+g) - \frac{c}{2}p^2.$$

Take the first-order condition, and we have: $\frac{d}{dp}\left(p (1-q) (V+g) - \frac{c}{2}p^2\right) = V+g-Vq-cp-gq = 0$. The firm's problem is

$$\max_{0\leq q\leq 1} \overline{\Pi}_2 (p, q, g) = q\pi + (1-q) p\pi - p (1-q) g - \frac{c}{2}q^2.$$

Take the first-order condition, and we have: $\frac{d}{dq}\left(q\pi + (1-q) p\pi - p (1-q) g - \frac{c}{2}q^2\right) = \pi - \pi p - cq + gp = 0$. Thus we need to solve

$$\begin{cases} V+g-Vq-cp-gq = 0 \\ \pi - \pi p - cq + gp = 0 \end{cases}.$$

It follows that the second-stage strategies are $\overline{p}(g) = \frac{(c-\pi)(g+V)}{c^2+(g-\pi)(g+V)}$, $\overline{q}(g) = \frac{c\pi+(g-\pi)(g+V)}{c^2+(g-\pi)(g+V)}$.

At the first stage, the firm's problem is:

$$\max_{0\leq g<\pi} \overline{\Pi}_2 (\overline{p}(g), \overline{q}(g), g) = \overline{q}(g)\pi + (1-\overline{q}(g))\, \overline{p}(g)(\pi - g) - \frac{c}{2}(\overline{q}(g))^2.$$

Plugging in $\overline{p}(g)$ and $\overline{q}(g)$, the objective function becomes $\frac{1}{2}\left(-c+2\pi+\frac{c^3(c-\pi)^2}{(c^2+(g-\pi)(g+V))^2}\right)$. Differentiating with respect to g, the first-order condition is:

$$\frac{c^3(c-\pi)^2(-2g+\pi-V)}{(c^2+(g-\pi)(g+V))^3} = 0.$$

Therefore, when $\pi > V$, $g^* = \frac{1}{2}(\pi - V)$. If $\pi < V$, $\frac{1}{2}(\pi - V)$ is not feasible, so g^* must lie on the corner. By simple reasoning, $g^* \neq \pi$, in which case the firm's gain from the one-way knowledge transfer becomes 0, thus it becomes meaningless to give high grants that encourage the university to research more. So $g^* = 0$ when $\pi < V$. It's also verifiable that $\overline{\Pi}_2(\cdot)$ is strictly higher at $g^* = 0$ than $g^* = \pi$, which we leave for the readers to verify.

Plug g^* into $\overline{p}(g)$ and $\overline{q}(g)$, we get

$$\overline{p} = \frac{2(c-\pi)(\pi+V)}{4c^2-(\pi+V)^2}, \text{ and } \overline{q} = \frac{4c\pi-(\pi+V)^2}{4c^2-(\pi+V)^2}.$$

Now we show both of them lie in $(0,1)$. Keep in mind that Assumptions 1 and 2 can be combined as $c > 2\pi > 2V > 0$. Then $c > \pi + V$. Thus $4c^2 - (\pi+V)^2 > 0$, and $\overline{p} > 0$. Is $\overline{p} < 1$? We need to show $\frac{2(c-\pi)(\pi+V)}{4c^2-(\pi+V)^2} < 1$, which is equivalent to $2(\pi - c)\frac{\pi+V}{(\pi+V-2c)(\pi+V+2c)} < 1$, further equivalent to $2(\pi-c)(\pi+V) > (\pi+V-2c)(\pi+V+2c)$ since $(\pi+V-2c)(\pi+V+2c) < 0$. Move the RHS to the LHS and check its sign, $2(\pi-c)(\pi+V) - (\pi+V-2c)(\pi+V+2c) = -2\pi c - 2Vc + \pi^2 - V^2 + 4c^2 = (\pi^2 - V^2) + 2c(-\pi - V + 2c)$, which always > 0. So $\overline{p} < 1$. For $\overline{q}$, is $\overline{q} > 0$? The denominator is positive. The numerator is also positive: $c > \pi + V$, and $4\pi > \pi + V$, so $4c\pi > (\pi+V)^2$. Thus $\overline{q} > 0$. Is $\overline{q} < 1$? It requires $4c\pi - (\pi+V)^2 < 4c^2 - (\pi+V)^2$, or $4c\pi < 4c^2$, which apparently holds. So $p, q \in (0,1)$. $\square$

6.3 Proof of Proposition 1

The need of discussing Case G here implicitly assumes that $\pi > V$. Recall in Case N, $p^* = \frac{(c-\pi)V}{c^2-\pi V}$ and $q^* = \frac{(c-V)\pi}{c^2-\pi V}$. Plug these back into the university and the firm's payoff functions, we have:

$$\Pi_1^* \equiv \Pi_1(p^*, q^*) = \frac{c(c-\pi)^2V^2}{2\left(c^2-\pi V\right)^2} > 0, \text{ and}$$

$$\Pi_2^* \equiv \Pi_2(p^*, q^*) = \frac{\pi\left(c^3\pi + 2c^2(c-2\pi)V + \pi(-c+2\pi)V^2\right)}{2\left(c^2-\pi V\right)^2}.$$

In Case G, recall when $g^* = \frac{\pi - V}{2}$, $\overline{p} = \frac{2(c-\pi)(\pi+V)}{4c^2-(\pi+V)^2}$ and $\overline{q} = \frac{4c\pi-(\pi+V)^2}{4c^2-(\pi+V)^2}$. So

$$\overline{\Pi}_1 \equiv \overline{\Pi}_1(\overline{p}, \overline{q}, \frac{\pi-V}{2}) = \frac{2c(c-\pi)^2(\pi+V)^2}{\left(-4c^2+(\pi+V)^2\right)^2}, \text{ and}$$

$$\overline{\Pi}_2 \equiv \overline{\Pi}_2(\overline{p}, \overline{q}, \frac{\pi-V}{2}) = \frac{-16c^2\pi(\pi+V)^2 - c(\pi+V)^4 + 2\pi(\pi+V)^4 + 8c^3\left(3\pi^2+2\pi V+V^2\right)}{2\left(-4c^2+(\pi+V)^2\right)^2}.$$

Assumption 1 can be written as $c > 2\pi > 2V > 0$, which leads to $c > \pi + V$.

Compare the probabilities.

$$\overline{p}-p^{*}=(\pi-c)(\pi-V)\frac{\pi V+V^{2}-2c^{2}}{(\pi+V-2c)(\pi+V+2c)(\pi V-c^{2})}.$$

We have $(\pi-c)<0$, $(\pi-V)>0$, $(\pi+V-2c)<0$, $(\pi+V+2c)>0$, and $(\pi V-c^{2})<0$. Thus the sign of the difference is the opposite sign of $\pi V+V^{2}-2c^{2}$. That is of the sign of $2c^{2}-\pi V-V^{2}$ which is >0. So $\overline{p}>p^{*}$.

$$\overline{q}-q^{*}=c(\pi-V)^{2}\frac{\pi-c}{(\pi+V-2c)(\pi+V+2c)(\pi V-c^{2})}.$$

We have $c(\pi-V)^{2}>0$, $\pi-c<0$, $(\pi+V-2c)<0$, $(\pi+V+2c)>0$, $(\pi V-c^{2})<0$. Therefore, $\overline{q}<q^{*}$.

Compare the payoffs:

$$\overline{\Pi}_{1}-\Pi_{1}^{*}=\frac{c(c-\pi)^{2}(\pi-V)\left(2c^{2}-V(\pi+V)\right)\left(-V(\pi+V)(3\pi+V)+2c^{2}(\pi+3V)\right)}{2(-2c+\pi+V)^{2}(2c+\pi+V)^{2}\left(c^{2}-\pi V\right)^{2}},$$

$$\overline{\Pi}_{2}-\Pi_{2}^{*}=\frac{c^{3}(c-\pi)^{2}(\pi-V)^{2}\left(8c^{2}-\pi^{2}-6\pi V-V^{2}\right)}{2\left(c^{2}-\pi V\right)^{2}\left(-4c^{2}+(\pi+V)^{2}\right)^{2}}.$$

It is easily verified that $\overline{\Pi}_{1}>\Pi_{1}^{*}$: $2c^{2}-V(\pi+V)>0$ since $c>V, c>(\pi+V)$. $-V(\pi+V)(3\pi+V)+2c^{2}(\pi+3V)>-V(\pi+V)(3\pi+V)+2(\pi+V)^{2}(\pi+3V)=(\pi+V)\left(2\pi^{2}+5\pi V+5V^{2}\right)>0$. And apparently $\overline{\Pi}_{2}>\Pi_{2}^{*}$: since $c^{2}>\pi^{2}, c^{2}>\pi V, c^{2}>V^{2}$, so $(8c^{2}-\pi^{2}-6\pi V-V^{2})>0$.

Compare the probabilities of innovation success. $\overline{P}-P^{*}=(\overline{p}+\overline{q}-\overline{p}*\overline{q})-(p^{*}+q^{*}-p^{*}*q^{*})$. Plug in the probabilities,

$$\overline{P}-P^{*}=\frac{c(c-\pi)(\pi-V)}{(-2c+\pi+V)^{2}(2c+\pi+V)^{2}\left(c^{2}-\pi V\right)^{2}}*A \text{ , where}$$

$$\begin{aligned}A=&8c^{5}+4c^{4}(-3\pi+V)-8c^{3}V(\pi+V)+c^{2}\left(\pi^{3}+13\pi^{2}V+3\pi V^{2}-V^{3}\right)\\&+cV(\pi+V)\left(-\pi^{2}+4\pi V+V^{2}\right)-4\pi^{2}V^{2}(\pi+V).\end{aligned}$$

$\overline{P}-P^{*}$ has the same sign as A. Keep in mind that $c>2\pi>2V, c>\pi+V, c^{2}>4\pi V$. We break the long expression of A into several parts and prove it is positive. First, $6c^{5}-12c^{4}\pi=6c^{4}(c-2\pi)>0$. $2c^{5}+4c^{4}V-8c^{3}V(V+\pi)=2c^{3}\left(c^{2}\right)+4c^{3}V*c-8c^{3}V(V+\pi)=2c^{3}\left(c^{2}-2V*(V+\pi)\right)+4c^{3}V(c-(V+\pi))>0$. So the first three terms $8c^{5}+4c^{4}(-3\pi+V)-8c^{3}V(\pi+V)=6c^{5}-12c^{4}\pi+2c^{5}+4c^{4}V-8c^{3}V(V+\pi)>0$. Second, $c^{2}\left(\pi^{2}V+\pi V^{2}\right)-4\pi^{2}V^{2}(\pi+V)>-4\pi^{2}V^{2}(\pi+V)+(4\pi V)\left(\pi^{2}V+\pi V^{2}\right)=0$. $c^{2}\left(\pi^{2}V+\pi V^{2}\right)+cV(\pi+V)\left(-\pi^{2}\right)=$

$c(\pi V)(\pi+V)(-\pi)+c^2(\pi V)(\pi+V) = c(\pi V)(\pi+V)(c-\pi) > 0$. $c^2\left(\pi^3 - V^3\right) > 0$. The remaining terms are $cV(\pi+V)\left(4\pi V + V^2\right) + c^2\left(11\pi^2 V + \pi V^2\right) > 0$. So, $A > 0$. Thus $\overline{P} > P^*$. $\square$

6.4 Proof of Proposition 2

The need of discussing Case G here implicitly assumes that $\pi > V$. The equilibrium strategies of Case C, (p^c, q^c), are solved from the first-order conditions of $\max_{0\leq p,q\leq 1} \Pi^c(p,q)$. By Assumption 3, both probabilities are guaranteed to lie in $(0,1)$. Given the strategies, the payoffs are $\Pi_1^c = p^c(1-q^c)V - \frac{c}{2}(p^c)^2$ and $\Pi_2^c = p^c(1-q^c)\pi + q^c)\pi - \frac{c}{2}(q^c)^2$.

To compare the probabilities, consider

$$p^c - \overline{p} = \frac{(c-\pi)(\pi+V)\left(2c^2 + (\pi+V)^2\right)}{\left(c^2 - (\pi+V)^2\right)\left(4c^2 - (\pi+V)^2\right)} > 0,$$

since $c > \pi + V$. Similarly,

$$q^c - \overline{q} = -\frac{3c(c-\pi)(\pi+V)^2}{\left(c^2 - (\pi+V)^2\right)\left(4c^2 - (\pi+V)^2\right)} < 0.$$

As to the payoffs,

$$\Pi_1^c = -\frac{c(c-\pi)^2(\pi-V)(\pi+V)}{2\left(c^2 - (\pi+V)^2\right)^2} < 0,$$

because $\pi > V$. We had $\overline{\Pi}_1 > \Pi_1^* = \frac{c(c-\pi)^2 V^2}{2(c^2-\pi V)^2} > 0$. So $\Pi_1^c < 0 < \overline{\Pi}_1$.

$$\Pi_2^c - \overline{\Pi}_2 = \frac{c(c-\pi)^2(\pi+V)\left(8c^4(3\pi - V) - c^2(15\pi - V)(\pi+V)^2 - 2V(\pi+V)^4\right)}{2(-2c+\pi+V)^2(-c+\pi+V)^2(c+\pi+V)^2(2c+\pi+V)^2},$$

which has the same sign as $8c^4(3\pi - V) - c^2(15\pi - V)(\pi+V)^2 - 2V(\pi+V)^4$. Break this term into two parts. First, we have $c^4(3\pi - V) - 2V(\pi+V)^4 > (\pi+V)^4(3\pi - V) - 2V(\pi+V)^4 = (3\pi - 3V)(\pi+V)^4 > 0$. For the rest, $7c^4(3\pi - V) - c^2(15\pi - V)(\pi+V)^2 = c^2\left(7c^2(3\pi - V) - (15\pi - V)(\pi+V)^2\right) > c^2\left(7(\pi+V)^2(3\pi - V) - (15\pi - V)(\pi+V)^2\right) = c^2(\pi+V)^2((21\pi - 7V) - (15\pi - V)) = c^2(\pi+V)^2(6\pi - 6V) > 0$.

To compare the probabilities of innovation success, consider $P^c - \overline{P} = (p^c + q^c - p^c * q^c) - (\overline{p} + \overline{q} - \overline{p} * \overline{q})$. Plug in the probabilities,

$$P^c - \overline{P} = \frac{c(c-\pi)(\pi+V) * B}{(-2c+\pi+V)^2(-c+\pi+V)^2(c+\pi+V)^2(2c+\pi+V)^2} \text{ , where}$$

$$\begin{aligned} B =& 8c^5 - 4c^4(5\pi + 3V) + 8c^3(\pi + V)^2 \\ &+ c^2(\pi + V)^2(7\pi + 15V) - 7c(\pi + V)^4 + (4\pi - 3V)(\pi + V)^4. \end{aligned}$$

We need to show that $B > 0$. The strategy is to break B into two parts, $B_1 + B_2$, and to show each part > 0.

The first part is

$$B_1 \equiv 8c^5 - 4c^4(5\pi + 3V) + 8c^3(\pi + V)^2 + 4c^2(\pi + V)^3.$$

The RHS equals $4c^2\left(2c^3 - c^2(5\pi + 3V) + 2c(\pi + V)^2 + (\pi + V)^3\right) \equiv 4c^2 * B_3$, where B_3 is defined to be the expression in the bracket. Differentiate B_3 w.r.t c:

$$\frac{dB_3}{dc} = 6c^2 - 2c(5\pi + 3V) + 2(\pi + V)^2.$$

Further differentiate: $\frac{dB_3^2}{d^2c} = 12c - (10\pi + 6V) > 12(\pi + V) - (10\pi + 6V) > 0$, since $c > \pi + V$. So $\frac{dB_3}{dc}$ increases in c when $c > \pi + V$. Since $c > 2\pi > \pi + V$, $\frac{dB_3}{dc} = 6c^2 - 2c(5\pi + 3V) + 2(\pi + V)^2 > 6(2\pi)^2 - 4\pi(5\pi + 3V) + 2(\pi + V)^2 = 24\pi^2 - 20\pi^2 - 12\pi V + 2\pi^2 + 4\pi V + 2V^2 = 6\pi^2 - 8\pi V + 2V^2$, a quadratic term of π which increases in π when $\pi > \frac{8}{12}V$. Since $\pi > V$, $6\pi^2 - 8\pi V + 2V^2 > 6V^2 - 8V^2 + 2V^2 = 0$. Hence, $\frac{dB_3}{dc} > 0$ when $c > 2\pi$, or B_3 increases in c when $c > 2\pi$. Replace c with 2π:

$$\begin{aligned} B_3 &= 2c^3 - c^2(5\pi + 3V) + 2c(\pi + V)^2 + (\pi + V)^3 \\ &> 16\pi^3 - 4\pi^2(5\pi + 3V) + 4\pi(\pi + V)^2 + (\pi + V)^3 \\ &= \pi^3 - \pi^2 V + 7\pi V^2 + V^3 > 0 \end{aligned}$$

So $B_1 = 4c^2 * B_3 > 0$.

The second part is

$$B_2 \equiv c^2(\pi + V)^2(3\pi + 11V) - 7c(\pi + V)^4 + (4\pi - 3V)(\pi + V)^4.$$

Differentiate B_2 w.r.t c:

$$\frac{dB_2}{dc} = -7(\pi + V)^4 + 2c(\pi + V)^2(3\pi + 11V).$$

Then $\frac{dB_2}{dc} > 0$, or B_2 increases in c when $c > \frac{7(\pi+V)^2}{6\pi+22V}$. Is $c > \frac{7(\pi+V)^2}{6\pi+22V}$? Let

$$B_4 \equiv \frac{7(\pi+V)^2}{6\pi+22V}.$$

Then

$$\frac{dB_4}{dV} = \frac{7}{22}\left(1 - \frac{64\pi^2}{(3\pi+11V)^2}\right),$$

so $\frac{dB_4}{dV} = 0$ when $V = -\pi$ or $V = \frac{5\pi}{11}$. When $V > 0$, $\frac{dB_4}{dV}$ decreases in V so that B_4 reaches its maximum at $\frac{5\pi}{11}$, that is,

$$B_4 < \frac{7\left(\pi + \frac{5\pi}{11}\right)^2}{6\pi + 22\frac{5\pi}{11}} = \frac{112\pi}{121}.$$

Hence, $\frac{7(\pi+V)^2}{6\pi+22V} < \frac{112\pi}{121} < c$. That means B_2 increases in c when $c > \frac{112\pi}{121}$. Recalling that $c > 2\pi$, we have

$$\begin{aligned} B_2 &= c^2(\pi+V)^2(3\pi+11V) - 7c(\pi+V)^4 + (4\pi-3V)(\pi+V)^4 \\ &> (2\pi)^2(\pi+V)^2(3\pi+11V) - 7(2\pi)(\pi+V)^4 + (4\pi-3V)(\pi+V)^4 \\ &= (\pi+V)^2\left(2\pi^3 + 21\pi^2V - 16\pi V^2 - 3V^3\right) \end{aligned}$$

The second bracket, $\frac{d}{d\pi}\left(2\pi^3 + 21\pi^2V - 16\pi V^2 - 3V^3\right) = 6\pi^2 + 42\pi V - 16V^2 > 0$ when $\pi > V$. Hence, $2\pi^3 + 21\pi^2V - 16\pi V^2 - 3V^3 > 23V^3 - 19V^3 > 0$ and

$$B_2 = (\pi+V)^2\left(2\pi^3 + 21\pi^2V - 16\pi V^2 - 3V^3\right) > 0.$$

Now we have proved that $B = B_1 + B_2 > 0$. Hence, $P^c > \overline{P}$. □

6.5 Proof for Proposition 3

The need of discussing Case G here implicitly assumes that $\pi > V$. It is clear that $0 < p^s < 1$ because $c > \pi + V$. First, compare the probabilities. In Case S, $p^s = \frac{\pi+V}{c}$. Then

$$p^s - p^c = -\frac{(\pi+V)\left(-c\pi + (\pi+V)^2\right)}{c(c-\pi-V)(c+\pi+V)},$$

which, since $c > \pi+V$, has the sign of $-\left(-c\pi + (\pi+V)^2\right)$. Under Assumption 3, $c\pi-(\pi+V)^2 > 0$, so $p^s - p^c > 0$. Since $q^c = \frac{c\pi-(\pi+V)^2}{c^2-(\pi+V)^2} > 0$, it is clear that $q^s = 0 < q^c$.

Now compare the payoffs. $\Pi_1^s = p^sV - \frac{c}{2}(p^s)^2$, and $\Pi_2^s = p^s\pi$. Given the previously derived expressions of Π_1^c and Π_2^c, we have

$$\Pi_1^s - \Pi_1^c = \frac{(\pi - V)(\pi + V)\left(c\pi - (\pi + V)^2\right)\left(-2c^2 + c\pi + (\pi + V)^2\right)}{2c(-c + \pi + V)^2(c + \pi + V)^2}.$$

By assumption $c\pi > (\pi + V)^2$ and $\pi > V$, so $\Pi_1^s - \Pi_1^c$ has the sign of $-2c^2 + c\pi + (\pi + V)^2 = -\left(2c^2 - c\pi - (\pi + V)^2\right)$, which is < 0 because $c^2 > c\pi$ and $c^2 > (\pi + V)^2$.

$$\Pi_2^c - \Pi_2^s = \frac{\left(c\pi - (\pi + V)^2\right)\left(c^3\pi - c^2(3\pi - V)(\pi + V) - 2c\pi V(\pi + V) + 2\pi(\pi + V)^3\right)}{2c(-c + \pi + V)^2(c + \pi + V)^2},$$

which has the sign of the polynomial $c^3\pi - c^2(3\pi - V)(\pi + V) - 2c\pi V(\pi + V) + 2\pi(\pi + V)^3$. Under our assumptions, the sign of the polynomial is indeterminate. Specifically, let c_1 be the third root[19] of $f(x) = 0$ where

$$f(x) = 2\pi^4 + 6\pi^3 V + 6\pi^2 V^2 + 2\pi V^3 + \left(-2\pi^2 V - 2\pi V^2\right)x + \left(-3\pi^2 - 2\pi V + V^2\right)x^2 + \pi x^3,$$

then combined with Assumptions 1 and 2, it can be shown (by the Reduce command in Mathematica, see the footnote on this page for detail) that $\Pi_2^s > \Pi_2^c$ if $c < c_1$, and $\Pi_2^s < \Pi_2^c$ if $c > c_1$.

Lastly, we compare the probabilities of innovation success. $P^s = p^s + q^s - p^s q^s$. The relative magnitude of P^s, P^c and $\overline{P}$ are again indeterminate, depending on the magnitude of c. Let c_2 be the fourth root of $g(x) = 0$ where

$$g(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4,$$

whose coefficients are $a_0 = -\pi^5 - 5\pi^4 V - 10\pi^3 V^2 - 10\pi^2 V^3 - 5\pi V^4 - V^5, a_1 = \pi^4 + 4\pi^3 V + 6\pi^2 V^2 + 4\pi V^3 + V^4, a_2 = 12\pi^3 + 24\pi^2 V + 20\pi V^2 + 8V^3, a_3 = -20\pi^2 - 24\pi V - 4V^2, a_4 = 8\pi - 8V$. Let c_3 be the third root of $h(x) = 0$ where

$$h(x) = \pi^3 + 3\pi^2 V + 3\pi V^2 + V^3 + \left(-\pi V - V^2\right)x + (-2\pi - 2V)x^2 + x^3.$$

Then combined with Assumptions 1 and 2, Proposition 3 (iii) can be derived by the Reduce command in Mathematica. Specifically, for the second case in (iii), since the command has returned the condition $c_3 < c < c_2$ & $\pi < 6.85V$, $c_3 < c_2$ should be implied (if $c_3 < c_2$ is not

[19]The roots are ordered from the least to the greatest. Real roots come first, followed by complex roots, which are ordered by the lexicographical order in $\mathbb{R}^2$. The root c_1 and the following c_2, c_3, c_4 (precisely, the polynomials that define these roots) are found by the "Reduce" command in the software Mathematica, which reduces complicated inequality system to the simplest possible conditions. It is then indicated that all roots involved in this paper are real numbers (since inequality conditions containing the roots are returned by Reduce).

guaranteed the command would either return more restrictions on the parameters, or return "False" implying no parameter range can make this happen).

To verify $c_3 < c_2$, we have run simulations over a wide range of values of π and V, and find that $c_3 < c_2$ is true for $\pi < 8.354V$, while $c_3 > c_2$ for $\pi > 8.354V$. In fact, we can prove that $c_3 = c_2 \approx 16.149V$ when $\pi = 8.354V$ by plugging the solution back to $g(x), h(x)$ to get 0. $\square$

6.6 Proof for Proposition 4

For Case S, the total optimal profit is

$$\Pi_1^s + \Pi_2^s = p^s (V+\pi) - \frac{c}{2}(p^s)^2 = \frac{1}{c}(\pi+V)(V+\pi) - \frac{c}{2}\left(\frac{1}{c}(\pi+V)\right)^2 = \frac{1}{2}\frac{(\pi+V)^2}{c}.$$

We compare now this sum with the sum of the Case N payoffs. Recall that the Case N equilibrium payoffs (with $p^* = V\frac{\pi-c}{\pi V-c^2}$, $q^* = \pi\frac{V-c}{\pi V-c^2}$)

$$\Pi_1^* + \Pi_2^* = p^*(1-q^*)V - \frac{c}{2}(p^*)^2 + q^*\pi + (1-q^*)p^*\pi - \frac{c}{2}(q^*)^2.$$

Since $p(1-q)V + (1-q)p\pi = p(1-q)(\pi+V)$, and $-\frac{c}{2}q^2 - \frac{c}{2}p^2 = -\frac{1}{2}c(p^2+q^2)$, we have

$$\begin{aligned}
\Pi_1^* + \Pi_2^* &= p^*(1-q^*)V - \frac{c}{2}(p^*)^2 + q^*\pi + (1-q^*)p^*\pi - \frac{c}{2}(q^*)^2 \\
&= p^*(1-q^*)(\pi+V) + q^*\pi - \frac{1}{2}c\left((p^*)^2 + (q^*)^2\right) \\
&= \frac{V(\pi-c)}{\pi V-c^2}\left(1 - \frac{\pi(V-c)}{\pi V-c^2}\right)(\pi+V) + \frac{\pi(V-c)}{\pi V-c^2}\pi \\
&\qquad - \frac{c}{2}\left(\left(\frac{V(\pi-c)}{\pi V-c^2}\right)^2 + \left(\frac{\pi(V-c)}{\pi V-c^2}\right)^2\right) \\
&= \frac{1}{2}\frac{2\pi^3V^2 + \pi^2c^3 + V^2c^3 + 2\pi Vc^3 - 2\pi V^2c^2 - 4\pi^2Vc^2}{(\pi V-c^2)^2}.
\end{aligned}$$

We want to show that $\Pi_1^s + \Pi_2^s - (\Pi_1^* + \Pi_2^*) > 0$. The LHS is

$$\begin{aligned}
&\frac{1}{2}\frac{(\pi+V)^2}{c} - \frac{1}{2}\frac{2\pi^3V^2 + \pi^2c^3 + V^2c^3 + 2\pi Vc^3 - 2\pi V^2c^2 - 4\pi^2Vc^2}{(\pi V-c^2)^2} \\
=\ &\frac{1}{2}\pi V\frac{\pi V^3 + \pi^3V + 4\pi c^3 + 2Vc^3 + 2\pi^2V^2 - 2\pi^2c^2 - 2V^2c^2 - 4\pi Vc^2 - 2\pi^2Vc}{c(\pi V-c^2)^2}.
\end{aligned}$$

It has the same sign of $B_4 \equiv \pi V^3 + \pi^3V + 4\pi c^3 + 2Vc^3 + 2\pi^2V^2 - 2\pi^2c^2 - 2V^2c^2 - 4\pi Vc^2 - 2\pi^2Vc$, which equals $\pi V^3 + \pi^3V + 2\pi^2V^2 + 2c\left(-\pi^2V + 2\pi c^2 - \pi^2c + Vc^2 - V^2c - 2\pi Vc\right)$. Check the sign of $-\pi^2 V+2\pi c^2-\pi^2 c+Vc^2-V^2 c-2\pi Vc$. We decompose: $-\pi^2 V+2\pi c^2-\pi^2 c+Vc^2-V^2 c-2\pi Vc = Vc^2 - V^2 c - \pi^2 V + 2\pi c^2 - 2\pi Vc - \pi^2 c$. $Vc^2 - V^2 c - \pi^2 V = V\left(-Vc - \pi^2 + c^2\right) > 0$, because $c > V + \pi$ and $c^2 > Vc + \pi c > Vc + \pi^2$. $2\pi c^2 - 2\pi Vc - \pi^2 c = \pi c\left(2c - \pi - 2V\right) > 0$. so $B_4 > 0$. Then, $\Pi_1^s + \Pi_2^s - (\Pi_1^* + \Pi_2^*) > 0$.

For the probability of innovation success, $P^s - P^* = (p^s + q^s - p^s * q^s) - (p^* + q^* - p^* * q^*)$. Plug in the probabilities, we have $P^s - P^* = \frac{\pi V\left(3c^3 - c\pi V - 2c^2(\pi+V) + \pi V(\pi+V)\right)}{c(c^2-\pi V)^2}$. It is true that $3c^3 - c\pi V - 2c^2(\pi + V) + \pi V(\pi + V) > 0$, because $c^3 - c\pi V > 0$ and $2c^3 - 2c^2(\pi + V) > 0$. Hence $P^s > P^*$. Note that so far the results hold without the assumption $\pi > V$ since Case G is not involved.

Lastly, again, the need of discussing Case G here implicitly assumes that $\pi > V$. Given that $c > 2\pi > 2V > 0$ and $c\pi - (\pi + V)^2 > 0$, it can be derived through the Reduce command in Mathematica that there exists some c_4, which is the fourth (real) root of $b_0 + b_1 x + b_2 x^2 + b_3 x^3 + b_4 x^4 = 0$, where $b_0 = -\pi^6 - 6\pi^5 V - 15\pi^4 V^2 - 20\pi^3 V^3 - 15\pi^2 V^4 - 6\pi V^5 - V^6$, $b_1 = 2\pi^5 + 8\pi^4 V + 12\pi^3 V^2 + 8\pi^2 V^3 + 2\pi V^4$, $b_2 = 11\pi^4 + 36\pi^3 V + 46\pi^2 V^2 + 28\pi V^3 + 7V^4$, $b_3 = -24\pi^3 - 48\pi^2 V - 24\pi V^2$, and $b_4 = 12\pi^2 - 8\pi V - 4V^2$, such that $\Pi_1^s + \Pi_2^s > \overline{\Pi}_1 + \overline{\Pi}_2$ if and only if $c < c_4$ & $\pi < 7.74V$ $\square$

References

Akcigit, U., Hanley, D., et Serrano-Vellarde, N. (2021). Back to Basics: Basic Research Spillovers, Innovation Policy, and Growth, The Review of Economic Studies, 88, 1–43.

Amir, R. (2000). Modelling imperfectly appropriable R&D via spillovers. *International Journal of Industrial Organization,* 18, 1013-1032.

Amir, R., Chalioti, E., & Halmenschlager, C. (2021). University-firm competition in basic research: Simultaneous versus sequential moves, *Journal of Public Economic Theory*, 23, 1199-1219.

Amir, R., Evstigneev, I., Wooders, J. (2003). Noncooperative versus cooperative R&D with endogenous spillover rates. *Games and Economic Behavior*, 42, 183-207.

Arora, A., Gambardella, A. (2005). The impact of NSF support for basic research in economics, *Annales d'Economie et de Statistique*, 79/80, 91-117.

Attanasi, G., My, K., B., Buso, M., Stenger, A. (2020). Private investment with social benefits under uncertainty: The dark side of public financing. *Journal of Public Economic Theory*, 22, 769– 820.

Bercovitz, J.E.L., Feldman, M.P. (2007). Fishing upstream: Firm innovation strategy and university research alliances, *Research Policy*, 36, 930-948.

Bernstein, J., Nadiri, I. (1988). Interindustry R&D spillovers, rates of return, and production in high-tech industries, *American Economic Review*, 78, 429-434.

Cabon-Dhersin, M.-L., Gibert, R. (2019). Cooperation or non-cooperation in R&D: How should research be funded? *Economics of Innovation and New Technology*, 28, 547-568.

Cabon-Dhersin, M.-L., Gibert, R. (2020). R&D cooperation, proximity and distribution of public funding between public and private research sectors, *The Manchester School*, 88, 773-800.

Cohen, W. M., Nelson, R. R., Walsh, J. P. (2002). Links and impacts: The influence of public research on industrial R&D. *Management Science*, 48, 1-23.

Correa, J., Lu, Y., Parro, F., Villena, M. (2020). Why is free education so popular? A political economy explanation, *Journal of Public Economic Theory*, 22, 973-991.

D' Aspremont, C., Jacquemin, A. (1988). Cooperative and noncooperative R&D in duopoly with spillovers, *American Economic Review*, **78**, 1133-1137.

D' Aspremont, C., Jacquemin, A. (1990). Erratum, *American Economic Review*, 80, 641-642.

David, P. A., Hall, B. H., Toole, A. A. (2000). Is public R&D a complement or substitute for private R&D? A review of the econometric evidence. *Research Policy.* 29: 497–529.

Del Rey, E. (2001). Teaching versus research: a model of state university competition, *Journal of Urban Economics*, 49, 356-373.

Friedman, M., Friedman, R. (1990). Free to Choose: A Personal Statement, Houghton Mifflin Harcourt Publishing Company, New York.

Gil-Moltó, M.J., Poyago-Theotoky, J., Zikos, V. (2011). R&D subsidies, spillovers, and privatization in mixed markets, Southern Economic Journal 78, 233-255.

Georgescu, I. (2022). Bringing back the golden days of Bell Labs, *Nature Reviews Physics*, 4, 76–78.

Griliches, Z. (1995). R&D and productivity: Econometric results and productivity issues, in P. Stoneman ed., *Handbook of the Economics of Innovation and Technological Change*, Blackwell, Oxford.

Hall, B.H., Link, A.N., Scott, J.T. (2003). Universities as research partners, *Review of Economics and Statistics*, 85, 485-491.

Hatsor, L., Zilcha, I. (2020). Subsidizing heterogeneous higher education systems. *Journal of Public Economic Theory*, 23, 318– 344.

Higón, D. A. (2016). In-house versus external basic research and first-to-market innovations, *Research Policy* 45, 816-829.

Hinloopen, J. (2000). Subsidizing cooperative and noncooperative R&D: an equivalence result? *Economics of Innovation and New Technology*, 72, 295-308.

Kamien, M., Muller, E., Zang, I. (1992). Research joint ventures and R&D cartels, *American Economic Review*, 82, 1293-1306.

Katsoulacos,Y., Ulph, D. (1998). Endogenous spillovers and the performance of research joint ventures, *Journal of Industrial Economics*, 46, 333-359.

Katz, M. (1986). An analysis of cooperative research and development, *Rand Journal of Economics*, 17, 527-543.

Kitahara, M., Matsumura, T. (2006). Realized cost-based subsidies for strategic R&D investments with ex ante and ex post asymmetries, *The Japanese Economic Review* 57, 438-448.

Lahmandi-Ayed, R., Lasram, H., Laussel, D. (2021). Is partial privatization of universities a solution for higher education?, *Journal of Public Economic Theory*, https://doi.org/10.1111/jpet.12531.

Link, A.N.,.Scott, J. (2005). Universities as partners in U.S. research joint ventures, *Research Policy*, 34, 385-393.

Lu, C.-H. (2121). Child labor and compulsory education: the effects of government education policy on economic growth and welfare, *Economic Theory*, 69, 637–666.

Murra-Anton, Z. (2021). Financial aid and early admissions at selective need-blind colleges, *Economic Theory*, https://doi.org/10.1007/s00199-021-01379-0.

Osório, A., Pinto, A. (2020). Income inequality and technological progress: The effect of R&D incentives, integration, and spillovers, *Journal of Public Economic Theory*, 22, 1943-1964.

Poyago-Theotoky, J., Beath, J., Siegel, D. (2002). Universities and fundamental research: Reflections on the growth of university-industry partnerships, *Oxford Review of Economic Policy*, 18, 10-21.

Rosenberg, N. (1990). Why do firms do basic research (with their own money)? Research Policy, 19, 165–174.

Salant, S., Shaffer, G. (1998). Optimal asymmetric strategies in research joint ventures, *International Journal of Industrial Organization* 16, 195-208.

Schumpeter, J. A. (1942). Socialism, capitalism and democracy, Harper and Brothers, New York.

Spence, M. (1984). Cost reduction, competition, and industry performance, *Econometrica,* 52, 101-121.

Stenbacka, R., Tombak, M. (2020). University-firm competition in basic research and university funding policy, *Journal of Public Economic Theory*, 22, 1017-1040.

Veugelers, R., Cassiman, B. (2005). R&D cooperation between firms and universities. Some empirical evidence from Belgian manufacturing. *International Journal of Industrial Organization*, 23, 355-379.

Wade, N. (1980). Why government should not fund science, *Science*, 210, 33.