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# Spillovers and strategic commitment in R&D

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## Abstract
This paper considers a one-stage Cournot duopoly of R&D. We characterize the Nash equilibrium of the one-stage game and provide a comparison with the two-stage version of the same Cournot model of R&D/product market competition. We look at R&D expenditures, profits, output and welfare. Under perfect symmetry, the one-stage model always leads to higher profits when the spillover parameter is not equal to 1/2. Moreover, the one-stage model implies more R&D expenditure and higher welfare if and only if the spillover parameter is greater than 1/2. The insights are robust to an $n$ -firm generalization, but the differences between the one-stage game and the two-stage game disappear as the market becomes perfectly competitive.

**Keywords** One-stage game · R&D · Spillovers

## 1 Introduction

Much of the extensive literature on innovation and market structure in industrial organization assumes research and development (R&D) take place before the associated output is produced, or in other words, that the underlying game is a two-stage game, with strategic commitment in the R&D decision.[1] In such settings, it is widely known, at least since (Brander & Spencer, 1983), that firms will use R&D for strategic purposes, in the sense that they engage in excessive R&D as a way to ensure higher market shares in the product market.

[1] The full list is too long to enumerate, but we can at least retain (Spence, 1984; d'Aspremont & Jacquemin, 1988; Kamien et al., 1992; Amir, 2000; Amir et al., 2003), among many others.

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However, in some cases, the R&D investment effort of each firm is not (perfectly) observable by its rival. This may be due to a variety of different reasons, including in particular the following factors: firms may be operating in different regions; firms may be relatively successful at not revealing the extent and/or the nature of their R&D operations (secrecy); and potential difficulties in relating partially observed R&D results or outcomes and final cost reductions. In cases where such factors play a significant role in the overall strategic interaction among the rival firms, the appropriate model for investigating the interplay between R&D and product market decisions may well be a one-stage game with both decisions taken simultaneously by each firm.

Although the literature on the topic has largely tended to follow the two-stage formulation, there are a number of important studies, where the one-stage paradigm was adopted. A partial list includes the following articles: Dasgupta and Stiglitz (1980), Vives (2008), López and Vives (2019), Amir et al. (2023) and Brander and Spencer (1983). In particular the latter authors also argue that, in the one-stage game, the firms do not engage in excessive R&D, but that in fact, their R&D levels correspond to the cost-minimizing levels of R&D for the outputs they end up producing at equilibrium.

The main goal of Brander and Spencer (1983) was to provide a comparison of the market performances of the one-stage and the two-stage models, for the case of a Cournot duopoly with general functional forms, but without spillovers in R&D. The purpose of this paper is to consider a linear version (i.e., linear demand and costs) of the duopoly model of Brander and Spencer (1983), to include R&D spillovers and an $n$-firm version of the model, and to go beyond the issues studied by Brander and Spencer (1983). This is accomplished with relative ease, since the linear structure of the duopoly allows for easily computed closed-form Nash equilibrium, both for the R&D and output levels, and also for the corresponding profits and social welfare. Exploiting this analytically convenient and widely used structure, we are able to provide a substantially more thorough comparison of the market performances implied by the one-stage and the two-stage models of R&D and product market competition. In doing so, relative to the classical paper by Brander and Spencer (1983), we obviously lose on the side of the generality of the setting and thus also of the results.

It is worth observing that, from a methodological standpoint, the present exercise is essentially a special case of the more frequently addressed issue in full-fledged economic dynamics of comparing open-loop and closed-loop Nash Equilibria in dynamic games (see, e.g., Wiszniewska-Matyszkiel 2014).

We consider a Cournot duopoly in a market for a homogeneous good with firms investing in process R&D and simultaneously deciding on output levels. Each firm benefits from its own R&D and from (involuntary and a priori unpreventable) spillovers in the rival's investment. There are two well-known models in the context of an oligopoly with cost-reducing R&D. The first, introduced by d'Aspremont and Jacquemin (1988); d'Aspremont et al. (1990), henceforth AJ, posits a framework of duopolistic R&D/Cournot competition with spillover effects that are additive in R&D outputs. The second, proposed by Spence (1984) and Kamien et al. (1992), henceforth KMZ, postulates the presence of spillover effects on R&D inputs (see also

Katz, 1986). We adopt the latter spillover model, as modified by Amir (2000) to capture in an equivalent manner the ubiquitous quadratic R&D cost function proposed by AJ and widely adopted in the follow-up literature.

In the first part of the paper, we characterize the unique Nash equilibrium of the one-stage Cournot duopoly model with R&D. The main findings may be summarized as follows. Each firm's autonomous R&D investment is decreasing in the spillover parameter, but the effective R&D expenditure (own plus spillover parts) is independent of the spillover parameter. Therefore, each firm's equilibrium output, industry price, and thus consumer surplus are also independent of spillovers. It also follows that each firm's equilibrium profit and social welfare are increasing in the spillover level. The fact that effective R&D expenditure is independent of the spillover parameter is a borderline reflection of the conventional view that technological progress is slower in economic environments, where imperfect appropriability of know-how is higher (see, e.g., Griliches, 1995; Bernstein & Nadiri, 1988; Scotchmer 2004). It is worth recalling that per-firm R&D investment is indeed decreasing in the spillover level in the present model, with this outcome being a weaker form of the same conventional view (as is easily seen by inspection).

In the second part of the paper, we provide a comparison of the one-stage game and the sequential game (two-stage game) in terms of the same dimensions of market performance. Recall that Brander and Spencer (1983) have conducted a similar exercise in the absence of spillover. They found that, relative to the one-stage model, the two-stage model will increase R&D undertaken, lower industry profits, and (under some general conditions) improve welfare. However, incorporating spillover in R&D inputs, we find that the market performance comparison depends in a critical manner on the spillover level. In short, while we confirm the Brander–Spencer results for small spillover levels (specifically, spillover parameters below 1/2), the results are reversed for high spillover levels. The one exception is equilibrium profit, which is strictly higher in the one-stage game for all spillover levels other than 1/2, while for the latter (knife-edge) case, the profit levels are exactly the same in the two games (in fact, the solutions then fully coincide).

Lastly, we extend the analysis so far conducted on duopoly to an $n$-firm oligopoly setting, and show that all insights derived earlier are robust to this market-structure generalization. In addition, we show that the firms' R&D expenditure and outputs both tend to be zero when $n$ approaches infinity or the market tends to be a perfectly competitive one, in the one-stage game as well as in the two-stage game, which renders any difference between the two game forms in regard with the market performance variables nonexistent. That is, the usual absence of market power prevents the firm from exploiting any informational advantage, thereby making the number of periods in the game irrelevant.[2]

This paper joins an extensive literature devoted to the interplay between strategic competitive forces and R&D with spillover effects. Besides the articles already cited, recent contributions include Martin (1996, 2002), Poyago-Theotoky (1999), Kline (2000), Cosandier et al. (2017), Gama et al. (2019), Cabon-Dhersin & Gibert (2020)

[2] This is a reflection of a general coincidence between one-stage and two-stage games with infinitely many players, see Wiszniewska-Matyszkiel (2014) for more on this point.

and Banerjee et al. (2023) in industrial organization;[3] as well as Lambertini et al. (2017), and Pal et al. (2023) in environmental economics.

This paper is organized as follows. Section 2 introduces the one-stage game and the symmetric equilibrium R&D/output solutions, along with the corresponding profit and social welfare. Section 3 compares the one-stage game with the standard sequential (two-stage) game in terms of overall market performance. Section 4 presents the $n$-firm case. Finally, Sect. 5 forms a brief conclusion.

## 2 The one-stage model and its market performance

This section provides a description of the basic model of R&D and Cournot competition, the only difference with the standard model in the literature being that the present version is a one-stage game. We also derive the resulting Nash equilibrium and market performance.

### 2.1 The one-stage game

We start with the description of a symmetric Cournot duopoly in a market for a homogeneous good with deterministic process R&D opportunities. The industry is a homogeneous-product Cournot duopoly with linear demand $P(q_1, q_2) = a - b(q_1 + q_2)$, where $a > 0, b > 0$ and $q_1$ and $q_2$ denote the outputs of firm 1 and 2, respectively. The firms have the same initial unit cost $c > 0$.

The process R&D opportunities are subject to (involuntary) mutual R&D spillovers. We follow the KMZ model's specification of the R&D process with spillovers taking place in R&D inputs, or investments.[4] That is, if let $y_i$ and $y_j$ be the R&D expenditures by firm $i$ and its rival firm $j$, $\{i,j\} = \{1,2\}$, firm $i$'s effective R&D inputs (own expenditure plus spillovers) amounts to $y_i + \beta y_j$, where $\beta \in [0, \overline{\beta}]$ (with $\overline{\beta} \leq 1$) is the spillover parameter capturing the proportion of the rival's R&D input that spills over to firm $i$. The spillover process will be discussed in some detail in the next subsection.

If the (effective) R&D expenditure, written as a function of the cost reduction $x$ towards the marginal production cost $c$, takes the standard quadratic form $y = \gamma x^2$, where $\gamma$ is a measurement of how costly (inefficient) the R&D process is, then the effective cost reduction for firm $i$ is $\sqrt{\frac{1}{\gamma}(y_i + \beta y_j)}$, and the final unit cost for firm $i$ is, therefore

$$c_i = c - \sqrt{\frac{1}{\gamma}(y_i + \beta y_j)}, i,j = 1,2; .i \neq j. \tag{1}$$

We depart from the standard R&D literature in industrial organization by assuming that the entire interaction is a one-shot game, i.e., firms choose both R&D levels and

[3] For a macro-economic perspective on innovation spillovers, see Akcigit et al. (2021), Hu et al. (2023a, b) and Wan & Zhang (2023).

[4] Precisely, we are following the spillover specification introduced by Spence (1984) and later adopted by many authors, including KMZ and Amir et al. (2003). The alternative specification (i.e., the AJ model), as clarified by Amir (2000), would yield a significantly different model.

outputs simultaneously (and not sequentially as in much of the literature). In other words, the model under consideration is a static game with a two-dimensional action space for each firm.

We clarify at the outset that, as is often the case in game-theoretic modeling, the simultaneity of the choices of each firm is not meant in the temporal sense, but rather in the informational sense: When a firm decides on its R&D investments (and product outputs), it has no way of knowing the other firm's decision on their R&D activities or outputs, even though the latter may have happened much earlier in time. This presumes of course that each firm does not disclose its R&D decision or progress, and that even if the actual investment becomes known, the cost reduction resulting from the firms' innovation activities remains totally unknown to its rival.

Due to the spillover effects and to assure the unit cost for both firms remains a positive level (i.e., $c_i \geq 0$), the effective action sets for R&D expenditure are inter-related via

$$\Omega_i = \{y_i : y_i \leq \gamma c^2 - \beta y_j\}, \ i,j = 1,2.$$

To guarantee that both firms would remain in the market (i.e., $q_i > 0$) even under very unequal R&D choices, we make the common assumption, which is maintained throughout the paper.

**(A1)** $a > 2c$.

In a one-shot game with simultaneous moves, given the rival's R&D expenditure and output $(y_j, q_j)$, firm $i$ solves

$$\max_{(y_i,q_i)} \pi_i(y_i, q_i, y_j, q_j) = q_i[a - b(q_i + q_j)] - q_i[c - \sqrt{\frac{1}{\gamma}(y_i + \beta y_j)}] - y_i \qquad (2)$$

subject to its effective action set $y_i \in \Omega_i$.

Prior to providing the solution of the model, we observe that the spillover structure calls for further validation, which is given next.

## 2.2 The spillover structure

This subsection addresses the tension that may be perceived between the one-stage game nature of the model and its R&D spillover structure. The objective here is to suggest a precise manner to resolve this tension, with the purported aim of making the theory at hand applicable in industries for which the features described below are realistic.

Indeed, this issue arises from the fact that, using (1), a firm can still infer, from its own observed final production cost and the spillover parameter (which is common knowledge in such games) the cost reduction achieved by the rival as a sum of its in-house R&D and the spillover coming from the rival's R&D, thus one step backward, infer the rival firm's R&D expenditure. That is to say, a firm can back out the rival's R&D expenditure if it knows its own, the spillover parameter, and its own final cost reduction. This observation underscores the tension between the one-stage feature of the game and the spillover structure of the model (Note that this issue does not arise in the literature dealing with oligopolistic R&D as a one-stage game because of the absence of spillovers, e.g., in Brander and Spencer (1983)).

We now argue that, while this tension limits the scope of the model to some extent, the issue can be resolved by invoking some realistic features that nevertheless may apply quite broadly across potential markets. In some industries, firms may need to commit to scales of operation in both R&D and output production through an upper hierarchy of the firm way ahead of the two activities actually being carried out. For R&D, this may entail fixing the overall R&D budget and a choice of R&D strategy and planning. For actual production, commitment to a level of output, which should be thought of as a capacity in the language of Kreps and Scheinkman (1983), may be sufficiently rigid not to be altered at a reasonable cost (just as conceived in the latter article and its follow-up studies). In the process of conducting R&D, spillover effects take place via professional exchanges and other human interactive activities of the scientific teams of the two firms, much in the same way these were thought of in the recent R&D literature. However, these effects remain fully non-interactive with the production commitment decision process. In other words, two-way R&D spillovers as commonly conceived in the literature are embodied in the research staff and may not reach the production management team in any direct manner. Instead, spillovers remain confined to the actors within the black box describing the R&D process. As such, the presence of these spillovers would not lead to the production managers being able to deduce the R&D expenditure of the rival firm.

Nevertheless, the tension between the (involuntary) spillover process and the one-stage game structure might admittedly suggest a restricted scope of spillover levels, with a maximum parameter being likely less than 1 (as reported above, in a deviation with respect to much of the literature, see, e.g., Amir (2000)). In summary, according to this view, even in the presence of spillover effects, the one-stage game is an economically meaningful description of the problem for industries characterized by the afore-mentioned features.

### 2.3 Nash equilibrium characterization

This subsection provides the solution of the one-stage model, along with the resulting market performance (equilibrium outputs, R&D levels and social welfare).

Given the payoff functions in (2), firm $i$ chooses both of its decision variables at the same time, treating the rival's two actions $(y_j, q_j)$ as given, in determining its best response $(y_i, q_i)$, given its own unit cost $\left(c - \sqrt{\frac{1}{\gamma}(y_i + \beta y_j)}\right)$.

It may be verified that this is a standard concave maximization problem and the first-order conditions with respect to $q_i$ and $y_i$ are given by

$$\frac{\partial \pi_i}{\partial q_i} = a - 2bq_i - bq_j - c + \sqrt{\frac{1}{\gamma}(y_i + \beta y_j)} = 0,$$
$$\frac{\partial \pi_i}{\partial y_i} = \frac{1}{2\gamma}\frac{q_i}{\sqrt{\frac{1}{\gamma}(y_i + \beta y_j)}} - 1 = 0.$$

Depending on the R&D cost (inefficiency) parameter $\gamma$, the symmetric equilibrium $(y^*, q^*)$ to this game can be easily verified to be

$$y^* = \begin{cases} \frac{\gamma(a-c)^2}{(1+\beta)(6b\gamma-1)^2} & \text{if } b\gamma > \frac{a}{6c} \\ \frac{\gamma c^2}{1+\beta} & \text{if } b\gamma \le \frac{a}{6c} \end{cases} \tag{3}$$

and

$$q^* = \begin{cases} \frac{2\gamma(a-c)}{6b\gamma-1} & \text{if } b\gamma > \frac{a}{6c} \\ \frac{a}{3b} & \text{if } b\gamma \le \frac{a}{6c} \end{cases} \tag{4}$$

If the R&D process is excessively cheap or efficient (i.e., $b\gamma \le \frac{a}{6c}$), both firms would choose to fully engage in R&D until the final unit production cost is reduced to 0 (i.e., $c - \sqrt{\frac{(1+\beta)y^*}{\gamma}} = 0$). Thus we refer below to the top line in (3) and (4) as the interior equilibrium, wherein each firm's final unit cost remains positive, and to the bottom line in (3) and (4) as a boundary equilibrium, wherein the final unit cost vanishes for both firms.

Given equilibrium R&D level $y^*$ and output level $q^*$ by each firm, the corresponding (interior and boundary) equilibrium profits are easily calculated to be

$$\pi^* = \begin{cases} \frac{\gamma(a-c)^2[4b\gamma(1+\beta)-1]}{(1+\beta)(6b\gamma-1)^2} & \text{if } b\gamma > \frac{a}{6c} \\ \frac{a^2(1+\beta) - 9b\gamma c^2}{9b(1+\beta)} & \text{if } b\gamma \le \frac{a}{6c} \end{cases}. \tag{5}$$

Lastly, social welfare, another standard indicator of market performance, can be expressed as the total consumer gain (integration along the demand curve) minus the production cost and R&D expenditures:

$$W^* = \int_0^{2q^*} (a - bt)dt - 2q^*(c - \sqrt{\frac{(1+\beta)y^*}{\gamma}}) - 2y^*, \tag{6}$$

and can be verified to have the following (interior and boundary, respectively) form:

$$W^* = \begin{cases} \dfrac{2\gamma(a-c)^2(8b\gamma(1+\beta)-1)}{(6b\gamma-1)^2(1+\beta)} & \text{if } b\gamma > \dfrac{a}{6c} \\ \dfrac{4a^2(1+\beta)-18b\gamma c^2}{9b(1+\beta)} & \text{if } b\gamma \leq \dfrac{a}{6c} \end{cases}. \tag{7}$$

Shared as a general appeal in the R&D spillover literature in industrial organization, here comes the most interesting part of our one-shot game analysis—also an important one so as to assess the model's reliability and to further provide prescriptions (if any) for policymakers—that is, to examine the comparative statics properties of this Nash equilibrium with respect to changes in the spillover parameter $\beta$.

For the equilibrium R&D expenditure $y^*$, from (3), we clearly have (both for the interior and the boundary solutions):

$$\frac{\partial y^*}{\partial \beta} < 0.$$

This is in line with standard intuition that, as $\beta$ gets higher, the public-good aspect of R&D comes to dominate due to the strong spillover effects, so free-riding increases and firms' autonomous investments decline. This effect forms one of the long-standing stylized facts about innovation in general: see, e.g., Griliches (1995) and Bernstein and Nadiri (1988) for empirical evidence and detailed overall discussion. Yet in the empirical literature, there is often some ambiguity as to whether it is per-firm R&D investment or actual technological progress that declines in industries with higher spillover effects. In the present model, the latter interpretation would amount to the effective R&D level, i.e., a firm's own R&D plus spillover R&D, being decreasing in the spillover parameter, which is clearly the stronger (more stringent) interpretation. It is seen by inspection that, with the effective R&D level $Y^* = (1+\beta)y^*$, we have

$$\frac{\partial Y^*}{\partial \beta} = 0$$

and since the firm's output decision directly relates to the effective R&D expenditure as seen in the profit function (2), we further have

$$\frac{\partial q^*}{\partial \beta} = 0.$$

In other words, effective R&D follows the aforementioned stronger interpretation of declining technological progress only in a borderline sense. Thus firms adjust their levels of R&D to a varying spillover parameter in such a way as to maintain constant technological progress, and thus also constant output, industry price and consumer surplus.

For the per-firm net profit given in (5), it is easy to verify that

$$\frac{\partial \pi^*}{\partial \beta} > 0,$$

an intuitive and quite obvious result that directly follows the preceding conclusion about the spillover-invariant property of the effective R&D level. Indeed, as $\beta$ increases, each firm's output, production costs, the industry price, thus the profit gross of its R&D expenditure remain constant, while, on the other hand, the firm is paying less in its autonomous R&D. In other words, taking into account firms' strategic adjustments of their R&D levels in equilibrium, a varying spillover parameter does not affect any of the firms' product-market decisions or performances. The only effect that higher spillovers bring about is a cost-saving effect in terms of firms' R&D activities, as firms now achieve the same level of effective R&D inputs with strictly less autonomous investments, and thus it is the spared R&D expenditure that drives up firms' net profits.

How social welfare changes with a higher technology spillover rate is often deemed as ambiguous, for that, although firms enjoy a larger portion of rivals' R&D investments and thus are able to save their own R&D expenditures, firms' strategic reactions, on the other hand, complicate the problem as they become more inclined to free ride on rivals' R&D activities, possibly resulting in both less (effective) R&D level and less output that can harm consumers' welfare. In this view, the impact of a higher spillover rate on social welfare can be broken down into two effects. One is a expenditure-saving effect (with respect to R&D activities) that enables a firm to achieve an equal amount of effective R&D inputs with less autonomous investments. And this effect clearly has a direct impact on producer surplus. The other is a cost-reduction effect (with respect to product-market performance) that is determined by the effective R&D cost-reduction amount, which can boost firms' outputs if they achieve a higher (symmetric) effective R&D cost reduction in equilibrium, an effect that affects producer and consumer surplus as a whole, but specifically on the product-market end.

With the separated R&D-end and market-end effects, the welfare analysis to our one-shot game model turns out to be quite straightforward. Recall that the model gives rise to a borderline equilibrium outcome, where firms always strategically adjust their autonomous R&D investments to a varying spillover parameter so as to maintain a constant level of effective R&D inputs, and thus the effective cost reduction, firms' outputs, industry price, all in all, firms' product-market profit (gross of R&D expenditures) and consumers' surplus, remain constant—the cost-reduction effect is neutral. Then, the overall impact of a higher spillover rate amounts solely to the expenditure-saving effect, which is always present since firms spend strictly less for their R&D activities, and is translated into a higher net profit to both firms. Therefore, social welfare increases in the spillover parameter $\beta$, with firms strictly benefiting from less R&D spendings and consumers remaining indifferent. This is easily verified by inspection of (7):

$$\frac{\partial W^*}{\partial \beta} > 0.$$

In summary, the one-shot game predicts that in an industry with technology spillovers firms tend to free-ride on the rivals' R&D investments, with its autonomous R&D investment to decrease, and the actual (or effective) R&D investment to remain constant in a higher spillover parameter. This is in line with the long-standing stylized fact that high spillover rates in an industry can slow down its technology progress due to the problem of free-riding, though our latter result supports the fact only in a borderline sense. Consumers stay unaffected by the varying level of spillovers particularly because of firms' strategic reactions (i.e., free-riding), and the social welfare improves in that firms achieve expenditure-savings in their own R&D activities as the spillover effect strengthens.

As we shall see in the next section, these results reflect significant differences with respect to the more standard two-stage game.

## 3 A comparison of the one-stage and the two-stage models

In this section, we compare our present one-stage model and the well-known model by Kamien et al. (1992) (KMZ, two-stage model) in terms of resulting propensity for R&D and overall market performance. Some of these comparisons were conducted by Brander and Spencer (1983) in a setting with general functional forms but no spillover effects. By adopting the standard specification of a duopoly with linear demand and costs, and quadratic R&D costs, we are able to expand the scope of the market performance comparison, and do so for each possible level of the spillover parameter.

To that end, we begin with a short review of the relevant results from the KMZ model, as reported in Amir (2000).

### 3.1 The KMZ model

The KMZ model is based on the standard two-stage game of R&D and product market competition in which each firm decides on its R&D expenditure and output sequentially, and as noted earlier, in an informational rather than temporal sense. Formally, in the first stage each firm simultaneously decides on its autonomous R&D investment, $y_1, y_2 \geq 0$. In the second stage, firms observe $y_1$ and $y_2$, and engage in Cournot competition in a market for a homogeneous product with linear demand $P = a - b(q_1 + q_2)$ and common marginal cost $c$, where $b > 0$ and $q_1, q_2$ are the outputs of firm 1 and 2.

The R&D process follows the version of KMZ model as adapted by Amir (2000) with spillover and quadratic R&D costs. With $\beta \in [0, 1]$ denoting the spillover parameter in R&D investment, the final (effective) cost reduction of firm $i$ is $\sqrt{\frac{1}{\gamma}(y_i + \beta y_{-i})}$ , and the final unit cost of firm $i$ is $c_i \equiv c - \sqrt{\frac{1}{\gamma}(y_i + \beta y_{-i})}$. It is well-known that a Cournot duopoly of unit costs $c_i$ and $c_j$ yields the equilibrium profit $\frac{1}{9b}[a - 2c_i + c_j]^2$ of firm $i$. Thus, accounting for firms' R&D expenditures $y_1$ and $y_2$, the payoff of firm $i$ as a function of $y_1$ and $y_2$ is

$$\pi_i(y_1, y_2) = \frac{1}{9b}\left[a - c + 2\sqrt{\frac{1}{\gamma}(y_i + \beta y_j)} - \sqrt{\frac{1}{\gamma}(y_j + \beta y_i)}\right]^2 - y_i. \tag{8}$$

Solving the first-order condition with respect to (8), $\frac{\partial \pi_i}{\partial y_i}$ for $i = 1, 2$, yields the two-stage game symmetric R&D expenditure as follows, with detailed calculations being omitted to conserve space:

$$\bar{y} = \begin{cases} \dfrac{\gamma(2-\beta)^2(a-c)^2}{(1+\beta)(9b\gamma - 2 + \beta)^2} & \text{if } b\gamma > \dfrac{(2-\beta)a}{9c} \\ \dfrac{\gamma c^2}{1+\beta} & \text{if } b\gamma \leq \dfrac{(2-\beta)a}{9c} \end{cases}. \tag{9}$$

Similar to Sect. 2, the profit maximization is subject to the constraint on firms' R&D actions, that given the R&D cost reduction, each firm's final cost $c_i$ cannot drop below zero. As a result, the equilibrium R&D expenditure $\bar{y}$ takes the interior form given in the top line of (9)[5] if R&D is reasonably costly so that $\gamma$, the parameter showing up in the quadratic R&D costs that measures the cost (inefficiency) of the R&D process is above a certain threshold. Otherwise $\bar{y}$ takes the boundary form given in the bottom line of (9) and each firm's production cost drops to zero in equilibrium.

Replacing $y_i's$ in (8) by $\bar{y}$ yields the corresponding interior and boundary equilibrium profits:

$$\bar{\pi} = \begin{cases} \dfrac{\gamma[9b\gamma(1+\beta) - (2-\beta)^2](a-c)^2}{(1+\beta)(9b\gamma - 2 + \beta)^2} & \text{if } b\gamma > \dfrac{(2-\beta)a}{9c} \\ \dfrac{a^2(1+\beta) - 9b\gamma c^2}{9b(1+\beta)} & \text{if } b\gamma \leq \dfrac{(2-\beta)a}{9c} \end{cases}. \tag{10}$$

In the remainder of this section, we restrict attention to the interior solution in comparing the one-stage and two-stage models.

### 3.2 Market performance comparison

We first assume that the interior solution conditions for both models hold, i.e., $b\gamma > \frac{(2-\beta)a}{9c}$ and $b\gamma > \frac{a}{6c}$, with the first condition relying on the spillover parameter $\beta$. We thus establish the following assumption (maintained throughout this section), such that the two conditions hold for any $\beta$:

**(A2)** $b\gamma > \frac{2a}{9c}$.

The task we undertake in this subsection is to compare our one-stage model and KMZ's two-stage model in terms of R&D expenditure, profits, output and welfare, along two potential dimensions: the equilibrium levels of these variables in one-stage

[5] As noticed here, we use $*$ (e.g., $y^*$) to denote one-stage game equilibrium variables and bar (e.g., $\bar{y}$) to denote the two-stage counterparts.

versus two-stage games, and their comparative statics with respect to the spillover parameter $\beta$.

In their setting of R&D competition absent spillover effects, Brander and Spencer (1983) find that firms engage in R&D obsessively in the two-stage game compared to the one-stage game, in an attempt to ensure itself a larger market share in the product-market competition stage. Such is referred to as the strategic use of commitment (or credible threat) regarding firms' R&D decisions. As it turns out, our model confirms and extends their findings to industries with R&D spillovers. Recall that the R&D expenditure under perfect symmetry in one-stage game and two-stage game are given in (3) and (9), and denoted by $y^*$ and $\bar{y}$, respectively. Taking their difference yields

$$\bar{y} - y^* = \left(\frac{3(a-c)^2 b\gamma^2}{(1+\beta)(9b\gamma - 2 + \beta)^2(6b\gamma - 1)^2}\right)\Big(3b\gamma(7-2\beta) - 2(2-\beta)\Big)(1-2\beta). \tag{11}$$

Given **(A1)** and **(A2)**, the second bracket can be easily verified to be positive, that is

$$3b\gamma(7-2\beta) - 2(2-\beta) > 0. \tag{12}$$

Therefore, $(\bar{y} - y^*)$ has the same sign as $(1-2\beta)$, that is, the two-stage game yields higher autonomous R&D, $\bar{y} > y^*$, if and only if $\beta < 1/2$.

In addition, it is seen by inspection of (9) that $\frac{\partial \bar{y}}{\partial \beta} < 0$, so that the two-stage game autonomous R&D decreases in the spillover parameter, as it does in the one-stage game discussed in Sect. 2.

Figure 1 depicts the above result in some detail. First note that when $\beta = 0$, firms in the two-stage framework conduct more R&D than they do in the one-stage game, which confirms (Brander & Spencer, 1983)'s findings about firms' use of R&D commitment as a strategic tool, particularly, in the absence of spillovers. Next notice that R&D decreases in the spillover parameter for both models, a conclusion we obtained earlier in the comparative statics analysis. This is in line with the common intuition that higher spillover lowers a firm's incentive for R&D, regardless of the game's information structures of interest in this paper. In other words, no matter

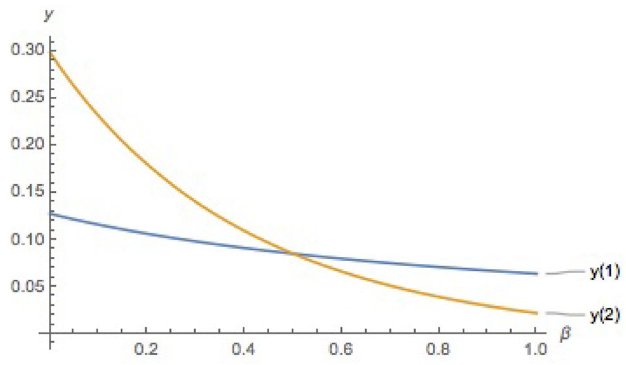


**Fig. 1** R&D expenditure: one-stage vs. two-stage

whether the rival's R&D is observable or not when firms decide on its output, in the decision of autonomous R&D, firms always become more inclined to free-ride on the rival's R&D when they see spillover strengthens. The free-rider problem is widely discussed in the field of public goods. Technology spillover connects the two streams of literature, namely R&D and public goods, in that R&D gets endowed with public-good property in the presence of spillovers, and in this regard, spillover exacerbates free-riding. Lastly, notice that the R&D expenditure decreases more rapidly in the two-stage model than in the one-stage model, implying that the two-stage information structure reinforces firms' incentive to free-ride as they see higher spillover effects. This is due to, in fact, a quite subtle observation that with a higher spillover parameter, firms' effective R&D inputs and thus their final production costs (i.e., $c_i$ and $c_j$) become more correlated with one another. Since firms' product-market performances directly depend on their relative (rather than absolute) levels of production cost, the strategic advantage of investing more in the first-stage R&D to attain a higher second-stage market share, is weakened by stronger spillover effects. In other words, spillover hampers the strategic use of R&D as a commitment tool, so that firms' autonomous R&D drops much more quickly in the two-stage game than in the one-stage game. Moreover, when the spillover parameter exceeds $\frac{1}{2}$, firms' free-riding incentive begins to override the strategic consideration, they become more passive about their autonomous investments and start to invest less in the two-stage game than in the one-stage game.

The effective R&D expenditure $Y$ equals autonomous R&D plus spillovers, thus amounts to $(1+\beta)y$ in a symmetric equilibrium, where $y$ denotes the corresponding autonomous R&D level in equilibrium. Therefore, $\bar{Y} > Y^*$ if and only if $\bar{y} > y^*$, i.e., $\beta < \frac{1}{2}$. The actual technology growth is slower in a two-stage game than in the one-stage game, because high R&D investments no longer assure a large market share in the second stage as high spillover correlates with two firms' technology progresses to a larger extent, that is, free-riding becomes so easy as to erase firms' strategic use of R&D. That said, in an industry with high spillover effects, concealing firm-wide R&D decisions from the rivals would give rise to higher technology growth.

For comparative statics, we have shown that one-stage autonomous R&D $y^*$ decreases in the spillover parameter $\beta$, while the effective R&D $(1+\beta)y^*$ remains constant. Given that the two-stage autonomous R&D $\bar{y}$ decreases faster in $\beta$ than $y^*$, the corresponding effective R&D $(1+\beta)\bar{y}$ should also decrease faster than $(1+\beta)y^*$, meaning it decreases in $\beta$. As the stylized fact suggests, high spillover slows down the actual technology growth in an industry. Either the one-stage or the two-stage model confirms the conventional wisdom, though the former model supports it in a weak sense. The intuition can be proved mathematically by noticing that $\bar{Y} = \frac{\gamma(2-\beta)^2(a-c)^2}{(9b\gamma-2+\beta)^2}$ from (9) and obviously $\frac{\partial \bar{Y}}{\partial \beta} < 0$.

The industry output and consumer surplus are closely intertwined with the effective R&D expenditure, which determines each firm's final production cost. In the two-stage game, when $\beta < \frac{1}{2}$, the two-stage effective R&D surpasses the one-stage level, leading to lower symmetric unit production cost, higher output, lower industry price, and higher consumer surplus. And vice versa. For the comparative statics, in Sect. 2 we proved that the above variables stay invariant to a varying spillover parameter in the one-stage game. But in the two-stage model, since $\frac{\partial \bar{Y}}{\partial \beta} < 0$, it follows that $\frac{\partial \bar{q}}{\partial \beta} < 0$, so higher spillover leads to lower output, higher industry price, and lower consumer surplus. Figure 2 provides an output comparison.

The fact that the one-stage game makes it possible for firms to minimize costs, as pointed out in Brander and Spencer (1983), comes in handy when comparing the one-stage profit $\pi^*$ and the two-stage profit $\bar{\pi}$. Indeed, in a symmetric equilibrium, the one-stage game always yields the highest possible profits, because firms optimize simultaneously over their R&D expenditure and output, and they do their best. The two-stage game, however, locks down firms' R&D investments when they come to decide on outputs, a structure that may possess certain strategic credit, but naturally leads to a sub-optimal decision in terms of cost minimization. Theoretically, taking difference in the profits yields

$$\bar{\pi} - \pi^* = \frac{b\gamma(a-c)^2}{(1+\beta)(9b\gamma - 2 + \beta)^2(6b\gamma - 1)^2}(2\beta - 1)^2(5 - \beta - 27b\gamma) \qquad (13)$$

Given **(A1)** and **(A2)**, we have $b\gamma > \frac{2a}{9c} > \frac{2}{9} > \frac{5}{27}$, hence $5 - \beta - 27b\gamma < 0$. It follows that $\bar{\pi} \leq \pi^*$; equality holds if and only if $\beta = 1/2$. Combined with the earlier results on the autonomous R&D levels, it tells that compared to the cost-minimizing benchmark case of the one-stage game, firms invest too much in R&D in the two-stage game for small spillover parameters (i.e., $\beta < \frac{1}{2}$), due to their strategic intention of using R&D as a way to acquire some advantage in their second-stage move, and they invest too little in R&D for large spillover parameters (i.e., $\beta > \frac{1}{2}$), as the ease of free-riding gets to dominate the strategic considerations. When $\beta = \frac{1}{2}$, two forces balance out and firms' profits in the one-stage and two-stage models coincide.

The dynamics of profit to a varying $\beta$ in the two-stage model are not that obvious. One way to examine the dynamics is to separate the impact of a rising spillover parameter into two effects. The first effect is a R&D-correction effect (when negative, it is referred to as the free-riding problem). Since in the two-stage game firms over-invest in R&D for small $\beta$, and the autonomous (effective) investment level has been

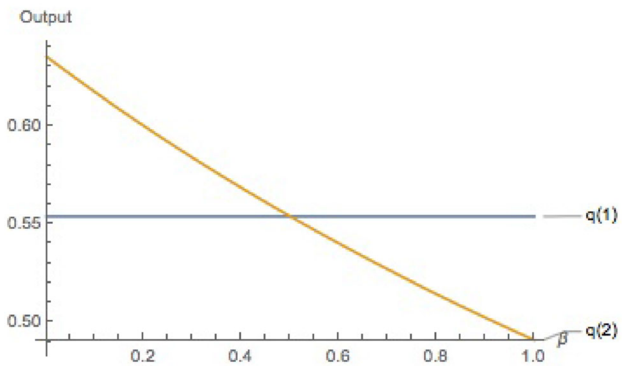


**Fig. 2** Output: one-stage vs. two-stage

shown to decrease in $\beta$, it follows that as $\beta$ increases from zero, firms' R&D decision gets corrected towards the cost-minimizing level and the two-stage profit $\bar{\pi}$ keeps approaching the one-stage profit $\pi^*$. But as $\beta$ gets higher, firms start to under-invest in R&D, while the autonomous (effective) investment level still decreases in $\beta$, thus $\bar{\pi}$ begins to depart from $\pi^*$ again. Therefore, this effect has positive influence on the two-stage profit $\bar{\pi}$ for small $\beta$'s (i.e., in the correction process) and flips to negative influence for large $\beta$'s (i.e., in the departure process). On the other hand, as discussed in Sect. 2, larger spillover always has an expenditure-saving effect in terms of R&D investments, because firms can achieve the same amount of effective R&D with less autonomous investments, for both one-stage and two-stage games. Hence, we can theoretically compute a threshold $\beta'$, such that

$$\frac{\partial \bar{\pi}}{\partial \beta} > 0 \quad \text{if and only if } \beta < \beta',$$

and we know $\beta' > \frac{1}{2}$. Precisely, when $\beta < \frac{1}{2}$, both the R&D-correction effect and expenditure-saving effect are positive, so $\bar{\pi}$ keeps growing. When $\beta > \frac{1}{2}$, the correction effect becomes negative (i.e., the free-riding problem), while the expenditure-saving effect still positive, so $\bar{\pi}$ continues to grow up to $\beta'$, since when the R&D level departs farther away from the cost-minimizing level, the free-riding problem takes over, and $\bar{\pi}$ begins to drop. Figure 3 characterizes the profit comparison.

Lastly, recall that in the one-stage model, the social welfare increases in the spillover parameter $\beta$, in that firms' output decision—thus the product-market profits, industry price and consumer surplus—stay unchanged, while firms (producer surplus) strictly benefit from a higher spillover rate due to the expenditure-saving effect. However, the dynamics in the two-stage model is not straightforward. The effective R&D level $\bar{Y}$ decreases in $\beta$, thus raising the industry price and lowering the consumer surplus. But firms' profits $\bar{\pi}$ increase in $\beta$ first and then decrease, as the incentive of free-riding gradually exacerbates and begins to dominate the expenditure-saving effect. Taking both consumer surplus and producer surplus into account, the impact of spillover on welfare is indeterminate. Specifically, from (6), the social welfare for the KMZ two-stage model can be calculated to be

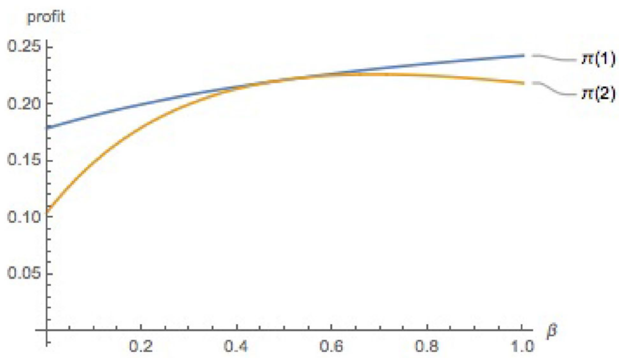


**Fig. 3** Profit: one-stage vs. two-stage

$$\bar{W} = \frac{2\gamma(a-c)^2[18b\gamma(\beta+1)-(\beta-2)^2]}{(9b\gamma+\beta-2)^2(1+\beta)}. \tag{14}$$

We manage to show that there exists a threshold of $\beta$, $\beta''$, such that when $\beta < \beta''$, the positive expenditure-saving as well as the R&D-correction effects outweigh the negative impacts on the consumer surplus, producer surplus increases faster than the speed at which consumer surplus decreases, and the social welfare increases in $\beta$. When $\beta > \beta''$, the free-riding problem takes over, accompanied by the negative impact on the consumer surplus, outweighing the expenditure-saving effect, and the social welfare decreases in $\beta$.

The comparison of one-stage and two-stage welfare can be conducted by direction calculation given the welfare expressions in (7) and (14). It can be shown that $\bar{W} > W^*$ if and only if $\beta < \frac{1}{2}$. The underlying intuition is quite clear, for $\beta > \frac{1}{2}$, that the two-stage game leads to both lower consumer surplus due to low effective R&D investments (i.e., $\bar{Y} < Y^*$) and lower producer surplus, because one-stage game with simultaneous decisions on R&D and outputs minimizes costs, so that the combined social welfare is lower, i.e., $\bar{W} < W^*$. On the other hand, when $\beta < \frac{1}{2}$, firms invest more in R&D in the two-stage game out of strategic concerns, so the consumer surplus is greater, but the producer surplus is still (and always, for $\beta \neq \frac{1}{2}$) lower than that of one-stage game. It turns out that the gain in consumer surplus outweighs the loss in producer surplus, and that the two-stage game gives rise to higher social welfare in this range of $\beta$.

Figure 4 displays the variation of welfare in response to $\beta$. Welfare in the one-stage game always increases in $\beta$, while for the two-stage game, it first increases before it diminishes. As $\beta$ increases, welfare in the two-stage model is higher than that in the one-stage model if and only if $\beta < \frac{1}{2}$.

Overall, strategic use of R&D will increase the R&D undertaken, increase output, lower industry prices and improve welfare only for small spillover levels (specifically, a spillover parameter less than $\frac{1}{2}$); the results are reversed, however, for high spillover levels, as high spillover correlates firms' production costs, the

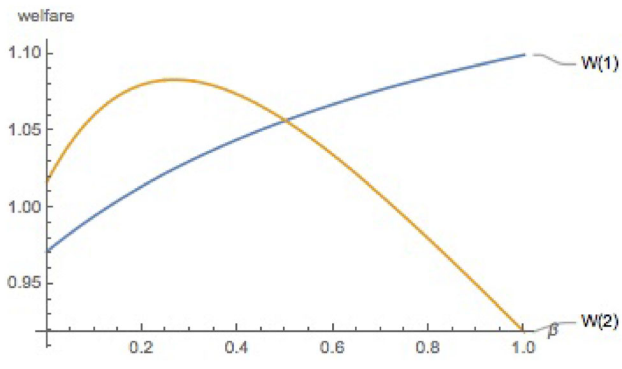


**Fig. 4** Welfare: one-stage vs. two-stage

strategic consideration diminishes, and the free-riding problem exacerbates. The one exception is profit, which is higher in the one-stage game for all spillover levels, with equality if and only if $\beta = \frac{1}{2}$, since simultaneous decision on R&D and outputs optimizes the firm's profits and minimizes costs.

Recapitulating, we have thus established the following result.

**Proposition 1** *Comparing the classical two-stage game and our one-stage game, under perfect symmetry:*

1. Each firm undertakes more autonomous (and effective) R&D in the two-stage game than in the one-stage model if and only if $\beta < 1/2$;
2. Consumer surplus follows the same comparison results as R&D;
3. The one-stage game always gives rise to strictly higher profits than the two-stage game for all $\beta \neq 1/2$ and to equal profits if $\beta = 1/2$.
4. The one-stage game leads to a superior welfare level for all $\beta > 1/2$ and to equal welfare for $\beta = 1/2$.

## 4 The *n*-firm case

The extensive analysis contrasting the one-stage and the two-stage games has been so far conducted in a duopoly scenario, for presentation simplicity. However, for standard robustness reasons, the more general $n$-firm oligopoly has a strong appeal. In this section, in a brief manner, we show that Proposition 1 does carry over to the $n$-firm scenario by a proper modification of Assumptions (A1) and (A2) to fit with the $n$-firm generalization. Furthermore, we also show that the contrast between the one-stage and two-stage games will disappear in the limit case as $n$ approaches infinity, in line with the general impression derived from one's intuition for a perfectly competitive industry.

First, let us deal with the one-stage scenario in the same spirit as in Sect. 2. To guarantee that all firms would remain in the market even under very unequal R&D choices, we need a modified **(A1)** assumption:

**(A1')** $a > nc$.[6]

The first-order conditions with respect to $q_i$ and $y_i$ are the same with $n$ firms, but then $q_j$ and $(n-1)y_j$ consist of the total output and R&D expenditure of the other $(n-1)$ firms, so $(q_n^*, y_n^*)$ for the $n$-firm symmetric equilibrium are defined by the two equations:

[6] Specifically, in a Cournot competition with $n$ firms, one can solve firm $i$ 's output to be $q_i = \frac{a - nc_i + \Sigma_{j \neq i} c_j}{b(n+1)}$. Under extremely unequal R&D choices, $c_i = c$ and $c_j = 0$ for all $j \neq i$, so $q_i = \frac{a - nc}{b(n+1)}$, which is guaranteed to be positive if $a > nc$.

$$\frac{\partial \pi_i}{\partial q_i} = 0 \Rightarrow a - b(n+1)q_n^* - c + \sqrt{\frac{(1+\beta(n-1))y_n^*}{\gamma}} = 0,$$
$$\frac{\partial \pi_i}{\partial y_i} = 0 \Rightarrow \frac{q_n^*}{2\gamma\sqrt{\frac{(1+\beta(n-1))y_n^*}{\gamma}}} - 1 = 0.$$

When $y_n^*$ hits the boundary defined by the effective action sets for R&D expenditure, it is by definition that firms' marginal cost becomes zero, so $(q_n^*, y_n^*)$ are defined instead by

$$mc = 0 \Rightarrow c - \sqrt{\frac{(1+\beta(n-1))y_n^*}{\gamma}} = 0,$$
$$\frac{\partial \pi_i}{\partial q_i} = 0 \Rightarrow a - b(n+1)q_n^* = 0.$$

Therefore, the first equation system defines the interior (in terms of R&D expenditure) solution of $(q_n^*, y_n^*)$ and the second defines the boundary solution. Solving the systems gives rise to the following symmetric equilibrium:

$$y_n^* = \begin{cases} \dfrac{\gamma(a-c)^2}{(1+\beta(n-1))(2b(n+1)\gamma-1)^2} & \text{if } b\gamma > \dfrac{a}{2(n+1)c} \\ \dfrac{\gamma c^2}{1+\beta(n-1)} & \text{if } b\gamma \leq \dfrac{a}{2(n+1)c} \end{cases}$$

and

$$q_n^* = \begin{cases} \dfrac{2\gamma(a-c)}{2(n+1)b\gamma-1} & \text{if } b\gamma > \dfrac{a}{2(n+1)c} \\ \dfrac{a}{(n+1)b} & \text{if } b\gamma \leq \dfrac{a}{2(n+1)c} \end{cases}.$$

One can verify that $(q^*, y^*)$ in equations(3) and (4) is a special case by letting $n = 2$. As later we will impose another assumption **(A2')** to ensure interior solution for both models when carrying out the comparison, now we only present the interior solutions for other market performance variables. It follows immediately that the effective R&D level, each firm's profit, and the social welfare as defined in (6) are, respectively:

$$Y_n^* = \frac{\gamma(a-c)^2}{(2b(n+1)\gamma-1)^2}, \quad \pi_n^* = \frac{\gamma(a-c)^2[4b\gamma(1+\beta(n-1))-1]}{(1+\beta(n-1))(2(n+1)b\gamma-1)^2}, \quad \text{and}$$
$$W_n^* = \frac{n\gamma(a-c)^2[2(n+2)b\gamma(1+(n-1)\beta)-1]}{(2(n+1)b\gamma-1)^2(1+(n-1)\beta)}.$$

The two-stage KMZ model can be solved routinely as we did in Sect. 3, and since the readers can conveniently refer to the original paper by Kamien et al. (1992) for calculation details, we simply present the equilibrium results for the KMZ model. One can verify that $\bar{q}_n, \bar{Y}_n, \bar{\pi}_n, \bar{W}_n$ are given by

$$\bar{q}_n = \frac{(n+1)\gamma(a-c)}{(n+1)^2 b\gamma - n + (n-1)\beta}, \quad \bar{Y}_n = \frac{\gamma(n-(n-1)\beta)^2(a-c)^2}{((n+1)^2 b\gamma - n + (n-1)\beta)^2},$$

$$\bar{\pi}_n = \frac{\gamma[(n+1)^2 b\gamma(1+(n-1)\beta) - (n-(n-1)\beta)^2](a-c)^2}{(1+(n-1)\beta)((n+1)^2 b\gamma - n + (n-1)\beta)^2}, \quad \text{and}$$

$$\bar{W}_n = \frac{n\gamma(a-c)}{2}\Big(\frac{2a(n+1)}{(n+1)^2 b\gamma - n + (n-1)\beta} + \frac{2a(\beta + n(1+n-n\beta)) - 2c(1+n)^3 b\gamma}{((n+1)^2 b\gamma - n + (n-1)\beta)^2}$$
$$- \frac{2(a-c)}{(1+(n-1)\beta)(2(n+1)b\gamma - 1)^2} - \frac{n(n+1)^2 b\gamma(a-c)}{((n+1)^2 b\gamma - n + (n-1)\beta)^2}\Big).$$

The interiority for the KMZ model requires $\sqrt{\frac{\bar{Y}_n}{\gamma}} < c$, that is, the effective cost reduction is less than the marginal cost. The condition simplifies to $b\gamma > \frac{a(n-(n-1)\beta)}{c(n+1)^2}$. Then, one can show that the following assumption, as a generalization of **(A2)**, insures that interiority is satisfied in both models.

**(A2')** $b\gamma > \frac{an}{c(n+1)^2}$.[7]

Now that we have derived the market-performance variables for the one-stage model and two-stage KMZ model, the rest is to verify that the comparison result proposed in the duopoly case still holds in the $n$-firm case. It does, and since the routine check is quite similar to that in Sect. 3, we put the proof in the Appendix.

In addition, we also show that the distinction between the two games will gradually disappear as the industry becomes more and more competitive. In fact, one obviously sees from the expressions of $q_n^*$ and $Y_n^*$ that both variables should approach zero when $n$ tends to infinity. With some simple rearrangements in the expressions of $\bar{q}_n$ and $\bar{Y}_n$, one easily arrives at

$$\bar{q}_n = \frac{\gamma(a-c)}{(n+1)b\gamma - \frac{n}{n+1} - \frac{n-1}{n+1}\beta}, \quad \bar{Y}_n = \frac{\gamma(a-c)^2}{\left(\frac{(n+1)^2}{(1-\beta)n+\beta}b\gamma - 1\right)^2};$$

From these expressions, one also confirms that $\bar{q}_n$ and $\bar{Y}_n$ should approach zero when $n$ tends to infinity. That is to say

$$q_n^* \to 0, \quad \bar{q}_n \to 0, \quad Y_n^* \to 0, \quad \text{and } \bar{Y}_n \to 0 \quad \text{as } n \to \infty.$$

It follows that, in perfect competition, firms' R&D expenditure and outputs both become zero in the one-stage game as well as in the two-stage game, which renders any difference between the two game forms in terms of the overall market perfor-

[7] It is straightforward to verify that $b\gamma > \frac{an}{c(1+n)^2} > \frac{a(n-(n-1)\beta)}{c(1+n)^2}$ and $\frac{an}{c(1+n)^2} > \frac{a}{2(n+1)c}$ for $n \geq 2$.

mance variables non-existent. Indeed, using the expressions for output and welfare, one can derive the following limit expressions for the industry output, market price, and social welfare,[8] as $n \to \infty$:[9]

$$nq_n^*, n\bar{q}_n \to \frac{a-c}{b}, \quad p_n^*, \bar{p}_n \to c, \quad \text{and} \quad W_n^*, \bar{W}_n \to \frac{(a-c)^2}{2b}.$$

This result is an unambiguous confirmation of Schumpeter (1943) classical insight that firms in perfectly competitive industries cannot be expected to engage in R&D due to their inability to finance it out of profits, thereby going against the conventional wisdom (based on static analysis) that competitive industries are best for society from a normative (efficiency) standpoint.

This result is stated in a formal sense next.

**Proposition 2** *In the n-firm oligopoly case,*

1. the four comparison results in Proposition 1 still hold;
2. the differences in R&D expenditure, consumer surplus, profit and welfare between the one-stage model and two-stage model disappear in the limit as $n \to \infty$.

The intuition behind this result lies in the fact that all firms eventually earn zero profit in perfect competition, so the information structure (or the game form in this case) has only a trivial impact on their R&D decisions. Put another way, when a firm faces against a very large number of other competitors in the industry, the usual absence of market power prevents the firm from exploiting any informational advantage, thereby making the number of periods in the game irrelevant.

## 5 Conclusion

This paper has analyzed a one-stage Cournot duopoly in a market for a homogeneous good with firms deciding simultaneously on process R&D and output levels. R&D is subject to input spillovers, i.e., investment spillovers. We argue that this setting might well be more appropriate than the more commonly used two-stage model, under some conditions on the industry environment and the R&D process.

We derive a simple characterization of the unique Nash equilibrium of the one-stage model. We show that each firm's autonomous R&D investment is decreasing in the spillover parameter, but the effective R&D expenditure (own plus spillover parts) is independent of the spillover parameter. Therefore, each firm's equilibrium output,

[8] The derivation of the limit expressions is a standard one. Since all that are involved in these formulas are fraction expressions regarding $n$, the limit of each fraction term is the quotient of the coefficients of the terms with the highest order of $n$ in the denominator and the numerator, respectively. For instance, one checks the coefficients of $n^3$ in $W_n^*$ to get $W_n^* \to \frac{\gamma(a-c)^2 2b\gamma\beta}{(2b\gamma)^2\beta} = \frac{(a-c)^2}{2b}$.

[9] It can be shown that for industry output and price, the convergence is monotone. Both the one-stage and the two-stage models satisfy the well-known property of quasi-competitiveness, a cornerstone property of classical static oligopoly models (Amir & Lambson, 2000).

industry price and consumer surplus are also independent of spillovers. Hence, each firm's equilibrium profit and social welfare are increasing in the spillover level.

In the second part of the paper, we provide a comparison of the one-stage game and the sequential game (two-stage game) in terms of the same dimensions of market performance. In the absence of spillovers, Brander and Spencer (1983) has found that, relative to the one-stage model, the two-stage model increases R&D, lowers industry profits, and improves social welfare. However, with spillover in R&D inputs, we find that the market performance comparison depends in a critical manner on the spillover level. In short, while we confirm the Brander-Spencer results for small spillover levels (specifically, a spillover parameter less than 1/2), the results are reversed for high spillover levels. The one exception is equilibrium profit, which is higher in the one-stage game for all spillover levels, with equality if and only if the spillover parameter is 1/2.

In terms of robustness to market structure, we show that all insights survive the generalization to $n$-firm oligopoly, for $n \geq 3$. In addition, all the differences between the two game forms (one-stage vs. two-stage), or by the information structure, disappear in a perfectly competitive market.

The conclusions from this comparative analysis are that the two possible models for investigating the interplay between R&D and product market competition yield significantly different results, and that the resulting market performance depends in important ways on the level of R&D spillovers.

## Appendix

**Proof for Proposition 2**

First, we need to replicate the four variable comparisons for R&D expenditure, consumer surplus, profit and welfare between the one-stage model and two-stage model.

The difference in R&D expenditure: $\bar{Y}_n - Y_n^*$ can be simplified as

$$\frac{b(a-c)^2(-1+n)(1+n)(-1+2\beta)\gamma^2(2(n+\beta-n\beta)+b(1+n)(-1-3n+2(-1+n)\beta)\gamma)}{(1-2b(1+n)\gamma)^2\left(n+\beta-n\beta-b(1+n)^2\gamma\right)^2}.$$

We claim that $(2(n+\beta-n\beta)+b(1+n)(-1-3n+2(-1+n)\beta)\gamma)<0$, so that $\bar{Y}_n - Y_n^*$ has the same sign as $(1-2\beta)$, which leads to the conclusion as in Proposition 1. The inequality can be simplified to be $\frac{2(n-(n-1)\beta)}{(1+n)(1+3n-2(n-1)\beta)}<b\gamma$. By Assumptions **(A2)** (first inequality) and **(A1)** (second inequality), $b\gamma > \frac{an}{c(1+n)^2} > \frac{n^2}{(1+n)^2} > \frac{2(n-(n-1)\beta)}{(1+n)(1+3n-2(n-1)\beta)}$, the last inequality can be verified to hold for all $n \geq 2$ and $0<\beta<1$.

Since consumer surplus $\int_0^{2q^*}(a-bt)dt$ increases in the firm's output, while the output increases in effective cost reduction, $q^* = \frac{(a-c+\text{effective cost reduction})}{b(n+1)}$, and effective cost reduction increases in R&D expenditure. So CS has the same sign comparison as the R&D expenditure. The same logic has been employed in the duopoly scenario.

Note that $\bar{Y}_n = \frac{(a-c)^2\gamma}{\left(-1+\frac{b(1+n)^2\gamma}{n(1-\beta)+\beta}\right)^2} \to 0$ when $n \to \infty$. Also $Y_n^* = \frac{\gamma(a-c)^2}{(2b(n+1)\gamma-1)^2} \to 0$ when $n \to \infty$. So $\bar{Y}_n - Y_n^* \to 0$. So the differences in R&D expenditure and CS disappear as $n \to \infty$.

The difference in profit, $\bar{\pi}_n - \pi_n^*$ can be simplified as

$$-\frac{b(a-c)^2(-1+n)^2\gamma^2(1-2\beta)^2\left(-1-2n-\beta+n\beta+3b(1+n)^2\gamma\right)}{(1+(-1+n)\beta)(1-2b(1+n)\gamma)^2\left(n+\beta-n\beta-b(1+n)^2\gamma\right)^2}.$$

Except for the negative sign in the front, all other terms are non-negative. Particularly, the term $\left(-1-2n-\beta+n\beta+3b(1+n)^2\gamma\right) > 0$ requires $3b(1+n)^2\gamma > 1+2n+\beta-n\beta$, which requires $b\gamma > \frac{1+2n+\beta-n\beta}{3(1+n)^2}$. We have shown right before, in proving for the R&D expenditure, that $b\gamma > \frac{n^2}{(1+n)^2}$, and one can verify that $\frac{n^2}{(1+n)^2} > \frac{1+2n+\beta-n\beta}{3(1+n)^2}$ always holds for all $n \geq 2$ and $0 < \beta < 1$. So $b\gamma > \frac{1+2n+\beta-n\beta}{3(1+n)^2}$ is proved. So $\bar{\pi}_n - \pi_n^* \leq 0$, while the equality holds only when $\beta = \frac{1}{2}$.

For the welfare, $\bar{W}_n - W_n^*$ can be simplified as

$$\frac{(1-2\beta)b(a-c)^2(-1+n)n(2+n)\gamma^2\left(-1-3n-2\beta+2n\beta+4b(1+n)^2\gamma\right)}{2(1-2b(1+n)\gamma)^2\left(n+\beta-n\beta-b(1+n)^2\gamma\right)^2}$$

We claim that $\bar{W}_n - W_n^*$ has the same sign as $1-2\beta$, which is stated in the Proposition. To validate that, we need $-1-3n-2\beta+2n\beta+4b(1+n)^2\gamma > 0$, or $b\gamma > \frac{1+3n+2\beta-2n\beta}{4\ (1+n)^2}$, which can be verified to be true, since $b\gamma > \frac{n^2}{(1+n)^2}$ and $\frac{n^2}{(1+n)^2} > \frac{1+3n+2\beta-2n\beta}{4(1+n)^2}$ for all $n \geq 2$ and $0 < \beta < 1$.

Now, we want to show that all these differences will disappear, moreover, in a quite monotonic manner, as $n$ approaches infinity. First, let us show this for the R&D expenditure. Differentiating $\bar{Y}_n - Y_n^*$ with respect to $n$, it equals $2b(a-c)^2\gamma^2 A$, where $A$ is the following term:

$$\frac{2\left(\frac{n+1}{2}\right)^3}{\left(\frac{n+1}{2}(-1+2b(1+n)\gamma)\right)^3} - \frac{(1+n)(1+n(-1+\beta)-3\beta)(n(-1+\beta)-\beta)}{\left(-n-\beta+n\beta+b(1+n)^2\gamma\right)^3}$$

It can be verified that for the denominators, $\frac{n+1}{2}(-1+2b(1+n)\gamma) > -n-\beta+n\beta+b(1+n)^2\gamma > 0$, and for the numerators, $\frac{(n+1)^2}{4} < (-1+n(1-\beta)+3\beta)(n(1-\beta)+\beta)$, two always hold for all $n \geq 3$ and $0 < \beta < \frac{1}{2}$. So the sign of $A$ should be negative. Thus $\bar{Y}_n - Y_n^*$ decreases in $n$ for

$0 < \beta < \frac{1}{2}$, and since $\bar{Y}_n - Y_n^* \geq 0$ for $0 < \beta < \frac{1}{2}$ and $\bar{Y}_n - Y_n^* \to 0$ when $n \to \infty$, we conclude that $\bar{Y}_n - Y_n^*$ decreases in $n$ and approaches 0 when $n \to \infty$ . Similarly, one can verify that the two inequalities for the denominators and the numerators flip signs when $1 > \beta > \frac{1}{2}$, so that $\bar{Y}_n - Y_n^*$ increases in $n$ for all $n \geq 3$ and $1 > \beta > \frac{1}{2}$. Combined with the fact that $\bar{Y}_n - Y_n^* \leq 0$ for $1 > \beta > \frac{1}{2}$ , it proves that $\bar{Y}_n - Y_n^*$ increases in $n$ to 0 as $n \to \infty$.

Because of the relation between the R&D expenditure and the consumer surplus, as argued shortly above, the consumer surplus should follow the same pattern as the R&D expenditure when $n \to \infty$.

The monotonic tendency of disappearing difference between the one-stage and two-stage models for the profit and welfare can also be proved, by playing with the differentiation terms, the inequalities and the assumptions, but they are too bulky to be presented here, so an interested reader can request from the authors a formal proof. Here, we want to give a last account on the profit and welfare as to simply prove what is stated in Proposition 2, which, the readers may notice, contains no account of monotonicity. We have shown that $\bar{Y}_n - Y_n^* \to 0$, which implies $\bar{q}_n - q_n^* \to 0$, one discussed already when we dealt with consumer surplus. Now that the profit is only affected by quantity $q$ and the effective cost reduction achieved by $Y$, one immediately gets $\bar{\pi}_n - \pi_n^* \to 0$. Lastly, as the differences of both consumer surplus and profit disappear in perfect competition, it follows that $\bar{W}_n - W_n^* \to 0$. And we have proved Proposition 2. □

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## Declarations

**Conflict of interest** All authors declare that they have no conflicts of interest.

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