Bertrand duopoly with a captive uninformed consumer segment

Rabah Amir *, Jingwen Tian †

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Abstract

This paper studies the comparative statics of increased market transparency on the pricing and profits of a Bertrand duopoly selling differentiated goods. The market consists of informed consumers who are aware of both firms' products, and uninformed consumers who are captive to one firm, and market transparency is defined as the proportion of the former. We reconsider the micro-economic foundations of the two types of consumers' demand functions and derive minimally sufficient conditions to establish clear-cut comparative statics for two separate cases of gross complements and substitutes, each case further split according to the nature of the game, strategic complements or strategic substitutes. Whether the prices increase or decrease in market transparency depends on the elasticity comparison between the two demand functions, and we reveal its close connection to the natural condition of log-supermodularity of the informed consumers' demand in the two firms' prices. The conventional wisdom that the equilibrium prices fall with more transparency is mostly confirmed, but with some exceptional cases identified.

Keywords: Bertrand duopoly, market transparency, consideration set, supermodular games, strategic complements/substitutes

*Department of Economics, the University of Iowa, Iowa City, IA, USA (e-mail: *rabah-amir@uiowa.edu*).
†Department of Economics, the University of Iowa, Iowa City, IA, USA (e-mail: *gin-tian@uiowa.edu*).

1 Introduction

In retail economics, buyers' behavior involves multiple stages of decision-making characterized by limited perception. Even if consumers are aware of the presence of all available brands, they may only consider a subset for purchase, those perceived as more reliable or likely to meet their needs more effectively.[1] This "evoked set" idea was initially put forth by Howard and Sheth (1969) in an attempt to explain the brand choice behavior of buyers, and later developed into the consideration set theory (e.g., Roberts and Lattin, 1991; Hauser and Wernerfelt, 1990; Hauser, 2014). This theory postulates that buyers experience three stages before making a purchase—awareness, consideration, choice—with the size of the brand set shrinking across the stages (Honka et al., 2017). The exclusion of less relevant brands may be rationalized by the usual cost-benefit arguments in search theory (Stigler, 1961; Stahl, 1989) or via behavioral reasons. The former include locational preference: Consumers become captive to the local store because the alternatives are *out of sight*, thus *out of mind*, or simply prohibited by transportation costs. Brand loyalty, as another explanation for the presence of a captive segment for a given brand (e.g., Chaudhuri and Holbrook, 2001), may result from consumers' satisficing behavior or from pricing strategies incentivizing repeat purchases.

In line with this literature, recent work in industrial organization posits a division of consumers according to their product brand awareness, into an informed segment that considers all brands when making purchases and an uninformed segment captive to just one firm. This gave rise to a notion of market transparency, defined as the fraction of informed consumers among all consumers (e.g., Boone and Potters, 2006; Schultz, 2004; Cosandier et al., 2018). As the market becomes more transparent (e.g., due to the ubiquitous use of the Internet), firms are faced with a larger fraction of consumers who are aware of the rival brands and their prices, which has important implications for oligopolistic price competition. This paper studies the comparative statics of increased market transparency on the pricing and profits of a Bertrand duopoly selling differentiated goods.

Market transparency may be interpreted in several ways, including buyers' limited information about the products' prices or quality (Stahl and Strausz, 2017; Schmitt and Bruckner, 2023), or buyers' limited awareness of the set of available brands that they can choose from (Varian, 1980; Saur et al., 2022). Unawareness may also be due to the dominant firm using its market power in other services to exclude or limit competitors' information.[2]

[1]Studies report for instance that consumers on average only consider 4 brands when purchasing shampoo, 5 for soft drinks, 3 for deodorant, etc. (Hauser, 2014).

[2]For instance, Microsoft was fined by the European Commission in 2013 for using its dominant position in operating systems to make Windows users unaware of competing browsers in order to lock them into the use of its own browser Internet Explorer; Google was fined in 2017 similarly for displaying its own shopping service Froogle (now Google Shopping) to users and hiding its competitors from the search results (Foucart and Friedrichsen, 2021).

Market transparency may be endogenously determined by firms' strategies and/or by consumer characteristics. For instance, transparency increases when firms advertise their (potentially) low prices to attract distant buyers who otherwise only have local price information (see, among others, Bester and Petrakis, 1995). Firms may also reach beyond their initial captive market segments by offering exceptionally high utilities to its consumers through sufficiently large investment (Foucart and Friedrichsen, 2021). On the consumer side, limited perception of horizontally differentiated goods sometimes means that consumers only perceive products sufficiently similar to their ideal version of the product. Saur et al. (2022) characterized this on the Hotelling line by a perception radius centered around the consumer's ideal product location. Limited perception thus offers a market segmentation into consumers who notice only one good and consumers who notice all goods, which is endogenously determined by the firms' location (product differentiation) choices. In this paper, however, we consider market transparency to be exogenous and study the associated comparative statics of firms' prices and profits.

One of the most important issues associated with imperfect market transparency is its impact on firms' prices and profits. The conventional wisdom is that increased transparency induces fiercer market competition, thus decreasing prices and increasing consumer surplus.[3] Empirical studies in general support the conventional wisdom. Goeree (2008) estimates a discrete-choice model in the U.S. personal computer market and finds that high markups are explained to a large extent by consumers' limited brand awareness. The prosperity of online shopping over traditional brick-and-mortar retail is believed to increase market transparency due to easier information acquisition and comparison. In line with increased transparency of Internet commerce, Brynjolfsson and Smith (2000) found that prices of homogeneous goods (books and CDs) are lower on the Internet than for physical retailers.[4]

The impact of market transparency on oligopoly pricing has also been investigated in the theoretical literature, with the conventional wisdom confirmed only with insignificant departures. The early work of Varian (1980) studied a symmetric homogeneous-good oligopoly using randomized sales to *implicitly* price discriminate[5] between the informed and uninformed consumers, and in-

[3]Increased market transparency may also affect the scope and dynamics of tacit collusion (see, e.g., Møllgaard and Overgaard, 2001; Schultz, 2005).

[4]Some studies also found that prices may be higher in electronic marketplaces than in traditional ones (see, e.g., Lee et al., 1999; Bailey, 1998). This reversion of the conventional wisdom may be explained by other factors differentiating the online and traditional marketplaces than market transparency, such as specific features of the Internet that allow online sellers to price discriminate (Bailey, 1998), or higher demand for online sellers due to better product exposure, and their ability to maintain higher reservation prices due to the absence of pressure to transport unsold products back home (Lee et al., 1999), etc.

[5]The price discrimination is implicit here: all consumers pay the same price ex post after all firms set their prices (i.e., firms cannot distinguish between the two consumer types), but pay different prices ex ante when firms randomize

creased market transparency was shown to reduce the average price paid by the uninformed consumers. Schultz (2004) studied a Hotelling duopoly choosing their locations faced with the same two types of consumers. Increasing market transparency in this setup leads to less product differentiation, and lower prices and profits. On the other hand, Boone and Potters (2006) studied a symmetric Cournot model with differentiated goods and a similar consumer base and concluded that an increase in transparency makes firms expand output due to fiercer competition, and may even lead to higer prices due to *the demand effect of transparency*, i.e., the total demand expands as more consumers become aware of all products.

Using the lattice-theoretic approach to oligopoly (see, e.g., Milgrom and Roberts, 1990; Vives, 1990, 1999; Amir and Lambson, 2000), Cosandier et al. (2018) analyzed two classes of games in the context of a differentiated Bertrand duopoly with very general demands, games of strategic complements (or supermodular games) and games of strategic substitutes (or submodular games). They found that the comparative statics of equilibrium prices in the transparency parameter depends in a crucial manner on the elasticity comparison between the demand functions of the informed and uninformed consumers. Our model follows the overall general set-up and methodology of Cosandier et al. (2018) and integrates the role of the inter-product relationship (gross substitutes or complements) in addressing the same set of questions.[6]

The theory of supermodular games has been successfully used in oligopoly theory due to its advantage in identifying a minimal set of conditions required for both (pure-strategy) equilibrium existence and general unambiguous comparative statics results (for a general survey of the methodology and applications, see Vives, 1999; Topkis, 1998).[7] The method relies on the complementarity structure between firms' strategies and dispenses with the superfluous assumptions often imposed by classical methods, such as concavity and interiority.

This paper builds upon the Bertrand duopoly model in Cosandier et al. (2018), with the same two types of informed and uninformed consumers. We reconsider the micro-economics foundations of the two demand functions, as they may be mutually related via the fact of being derived from the same representative consumer utility function. In particular, the two types of consumers maximize the same quasi-linear utility function subject to a budget constraint, but the problem of the uninformed consumers involves only a one-dimension choice as the consumption of the other good

their prices, as the model assumes that informed consumers always buy from the lowest-priced firm while uninformed consumers shop at random.

[6]The game-theoretic concept of strategic complements (substitutes) is distinct from the demand concept of gross complements (substitutes): the former describes the monotonicity of each firm's reaction curve, while the latter captures the products' inter-relationships. Detailed definitions will be given below when we introduce the model.

[7]More recent work using the same methodology and with some connection to the present paper includes Reynolds and Rietzke (2018); Gama and Rietzke (2019); Gama et al. (2020); Sabarwal (2021), among others.

is restricted to 0 due to product unawareness. This micro foundation reveals an important property of the demand elasticities: The bi-variate (informed) demand is more elastic than the single-variate (uninformed) demand when the other good's price equals the choke price of the other good's informed demand. This local elasticity comparison holds without any additional qualification.

The elasticity comparison is crucial to the comparative statics issue, and was taken in Cosandier et al. (2018) as a primary assumption. Instead, a key contribution of the present paper is to reveal its close connection to the widely used condition of log-supermodularity of the informed demand in two prices, i.e., $\partial^2 \log D^i(p_1, p_2)/\partial p_1 \partial p_2 > 0$. Log-supermodularity emerged as an important condition used in the oligopoly literature (Milgrom and Roberts, 1990). It is well-known that log-supermodular demand has a precise economic interpretation: The price elasticity of demand w.r.t one price is increasing in the other price.

Specifically, we show that the log-supermodularity of the informed demand immediately implies that it is more elastic than the uninformed demand *for all feasible prices*. Furthermore if the two goods are gross complements, it also implies that the Bertrand duopoly is a supermodular game, thus guaranteeing the existence of a pure-strategy Nash equilibrium and that both firms' equilibrium prices will decrease in market transparency. This result for gross complements thus confirms the conventional wisdom and is very general, as it requires a single broadly satisfied condition with a natural economic interpretation. However, if the informed demand is log-submodular and the elasticity comparison is reversed, the equilibrium price may also increase as the market becomes more transparent.

Second, based on the micro-economic property of the two demand functions mentioned earlier, we point out that the elasticity condition given in [Cosandier et al. (2018), Proposition 2], namely that the informed demand is less elastic than the uninformed demand, cannot hold *everywhere* if the two goods are gross substitutes. The corrected elasticity relation in this case further implies that when the two goods are gross substitutes, in line with conventional wisdom, the equilibrium price will decrease when the market becomes more transparent, although whether it holds for both firms unambiguously or just for one firm depends on the strategic nature of the game, i.e., whether the game is supermodular or submodular.

To recapitulate, the overall analysis in the present paper is divided into the two cases of gross complements and gross substitutes, each of which is further split according to the nature of the game, strategic complements or strategic substitutes. In each of the four sub-cases, we derive minimally sufficient conditions for the comparative statics of equilibrium prices and profits with respect to market transparency, emphasizing the key role of the log-supermodularity (log-submodularity) of the informed demand in implying the elasticity condition for certain cases and its interchangeability with other conditions used in Cosandier et al. (2018). The idea is that firms' equilibrium prices tend to decrease (increase) when the informed (uninformed) consumers' demand is more elastic, and supermodular (submodular) games imply both firms' prices (at least one firm's price) will change in the intuitive direction when transparency increases.

The rest of the paper is organized as follows. We introduce the duopoly problem in Section 2, and discuss the micro-economic foundations of the two types of consumers' demand functions in Section 3. Then we establish the conditions needed for the comparative statics, with gross complements discussed in Section 4, and gross substitutes in Section 5. Lastly, we conclude in 6.

2 The duopoly problem

This section presents the model Bertrand duopoly game with differentiated products and incomplete product awareness, as laid out in Cosandier et al. (2018). This is a generalization of earlier versions of the same model with particular functional forms, specifically linear and Hotelling demands by Boone and Potters (2006) and Schultz (2004), respectively.

Consider a price-setting duopoly in which each firm possesses two distinct market segments: consumers who are aware of both firms' products, called the informed consumers, and consumers who are only aware of the existence of the product of a single firm, called the uninformed consumers. The incomplete awareness may be a consequence of differing advertising intensities, limited geographical access, lack of access to information or the Internet, consumer loyalty, etc... The uninformed consumers know only one of the products and are unaware of the existence or the presence of the other. Hence, these uninformed consumers' demands depend only on the price of the one good they know about, so that their demands may be denoted $d^1(p_1)$ and $d^2(p_2)$ for two firms, respectively. We assume that half of the uninformed consumers know about each of the two goods, so we posit that these consumers end up allocating themselves equally across the two firms. The demand functions of the informed consumers for two firms are $D^1(p_1, p_2)$ and $D^2(p_1, p_2)$, respectively.

A detailed derivation of a joint theoretical foundation for these demand functions in terms of a representative consumer (who may be informed or uninformed) is provided below.

Letting $\phi \in (0, 1)$ be a key parameter that specifies the (exogenous) fraction of informed consumers; these patronize both firms according to standard (differentiated-product) Bertrand competition. It follows from the equal division rule that each firm additionally benefits from a fraction $\frac{1-\phi}{2}$ of the uninformed consumers as a captive segment.

Therefore, the total demand for, say, good 1 is

$$\phi D^1(p_1, p_2) + \frac{1-\phi}{2} d^1(p_1).$$

As for production costs, it is assumed that Firm i has a constant unit cost $c_i > 0$. Hence, Firm 1's payoff is given by

$$\Pi^1(p_1, p_2) = (p_1 - c_1)\left\{\phi D^1(p_1, p_2) + \frac{1-\phi}{2} d^1(p_1)\right\}. \tag{1}$$

That is, a fraction ϕ of the total market consists of the informed consumers, and the rest $1 - \phi$ uninformed consumers are split evenly between the two firms. Firm 2's payoff is defined analogously.

We are interested in characterizing the pure-strategy Nash equilibria of this game, with a focus on understanding the role of the transparency parameter ϕ. In particular, the comparative statics of the game with respect to ϕ, namely how the Bertrand equilibrium prices and profits will change is a central question, along with the associated minimal sufficient conditions on the overall demand system.

The methodology used in this paper is lattice theory and supermodular games. It is well-known that the lattice-theoretic methodology (Amir and Lambson, 2000; Milgrom and Roberts, 1990, 1994; Topkis, 1978) has a number of advantages over the traditional method for comparative-statics analysis. Indeed, the latter usually relies on superfluous assumptions such as the quasi-concavity of the objective function and other conditions validating the implicit function theorem, while the former yields general, unambiguous comparative-statics conclusions with a minimally sufficient set of complementarity conditions.

3 Micro-economic foundations of demand

In this section, following Cosandier et al. (2018), we amend the standard representative-consumer micro-economic foundation to integrate the informational asymmetry of the present model. The idea is to conceptualize the two demand functions $D^i(p_1, p_2)$ and $d^i(p_i)$ for Firm i as being both derived from the same representative consumer setting so as to impart some meaningful common structure to the two types of demand.

3.1 Derivation of the demand functions

Consider a representative consumer characterized by a quasi-linear utility function, $U(x_1, x_2) + y$, where y is a composite commodity for all goods other than goods 1 and 2, the price of y is normalized to 1, and income is I. In this perspective, an informed consumer performs the usual utility maximization to uncover $D^1(p_1, p_2)$ and $D^2(p_1, p_2)$, while an uninformed consumer of, say, Firm 1 (who is unaware of the existence of Firm 2), undertakes the same utility maximization to generate $d^1(p_1)$ upon the additional constraint that $x_2 = 0$ (in both the utility function and the budget constraint).

Specifically, the unrestricted problem for the informed consumer is

$$\max_{x_1,x_2,y\geq 0} \{U(x_1, x_2) + y\} \tag{2}$$
$$s.t. \;\; p_1x_1 + p_2x_2 + y \leq I$$

while the uninformed consumer of (say) Firm 1 solves (the problem for Firm 2 being symmetric)

$$\max_{x_1,y\geq 0} \{U(x_1, 0) + y\} \tag{3}$$
$$s.t. \;\; p_1x_1 + y \leq I.$$

As in Cosandier et al. (2018), a key tacit assumption throughout the paper is that the firms cannot discriminate in prices between the informed and the uninformed consumers. This may restrict the scope of the model to some extent, but is easily justified via a number of possible reasons, including (i) practical implementation hurdles, (ii) firms lacking the ability to distinguish between the two different types of consumers or (iii) price discrimination not being allowed.

The following assumptions on $U(x_1, x_2)$ are valid throughout the paper. We also assume throughout that $D^i(p_1, p_2)$ and $d^i(p_i)$, $i = 1, 2$, are twice continuously differentiable.

Assumption 1. *The utility function satisfies the following conditions,* $\forall x_1, x_2 \geq 0$*:*

(i) $U_1 \geq 0$, $U_2 \geq 0$, *and* U *is three times continuously differentiable.*

(ii) U *is differentiably strictly concave, i.e.,*

$$U_{11} < 0, U_{22} < 0, \text{ and } U_{11}U_{22} - (U_{12})^2 > 0.$$

(iii) $U_{12} \neq 0$.

(iv) $U_1(0, x_2) < \infty$, $U_2(x_1, 0) < \infty$, *and* $\lim_{a\to\infty} U_1(a, x_2) = 0$, $\lim_{b\to\infty} U_2(x_1, b) = 0$.

To provide some insight into the content of these assumptions on utility, we first note that (i) and (ii) are just standard restrictions on the utility function. Part (iii) guarantees that the two goods at hand are either gross substitutes or gross complements in a global sense, with these being the two cases under consideration in the present paper. Lastly, (iv) is needed to ensure that the demand functions derived from (2) and (3) admit a choke price, i.e., intersect the horizontal axis for a sufficiently high price, as stated later in Lemma 1. Importantly, (iv) serves to rule out some well-known utility functions such as the Cobb-Douglass function from our framework. This is w.l.o.g. as such utility functions do not fit the present model, since the maximization (3) and thus $d^i(p_i)$ are not meaningful when the consumer gets positive utility only through consuming strictly positive amounts of both goods.[8]

Substituting the constraint into the objective in the utility maximization problem (2) yields the equivalent problem

$$\max_{x_1, x_2 \geq 0} \{U(x_1, x_2) + I - p_1 x_1 - p_2 x_2\}$$

The first-order conditions are then

$$p_1 = P^1(x_1, x_2) \equiv U_1(x_1, x_2), \quad p_2 = P^2(x_1, x_2) \equiv U_2(x_1, x_2). \tag{4}$$

By the strict concavity assumption, these are necessary and sufficient for optimality, assuming I is sufficiently large. Hence, as usual, the first-order conditions define the inverse demand system (P^1, P^2), which has a negative definite Jacobian matrix as a result of the strict concavity of U. The latter property in turn implies, via the standard Inverse Function Theorem, that the system (4) is invertible and yields the direct demand system (D^1, D^2).[9] In other words,

$$\begin{cases} p_1 = P^1(x_1, x_2) \\ p_2 = P^2(x_1, x_2) \end{cases} \xrightarrow{\text{(invert)}} \begin{cases} x_1 = D^1(p_1, p_2) \\ x_2 = D^2(p_1, p_2) \end{cases}.$$

For the uninformed consumer, the analogous steps yield

$$p_1 = P^1(x_1, 0) \equiv U_1(x_1, 0), \quad p_2 = P^2(0, x_2) \equiv U_2(0, x_2). \tag{5}$$

[8] We however note that Assumption 1(iv) is only sufficient (not necessary) to our subsequent analysis. Indeed, for some utility functions violating Assumption 1(iv), the derived demand functions $D^i(p_1, p_2)$ and $d^i(p_i)$ may still be well-defined (as shown later in Example 2), and in that case our analysis carries through by allowing $f^j(p_i)$, the choke price of D^j that will be defined in the next subsection, to be infinity.

[9] Some details of this argument may be found in e.g., Vives (1999) or Amir et al. (2017).

Since $U_1(x_1, 0)$ is strictly decreasing in x_1 (as $U_{11} < 0$), and similarly for $U_2(0, x_2)$, inversion of each inverse demand function in (5) defines the demand function for the uninformed consumers of Firms 1 and 2, i.e.,

$$p_1 = P^1(x_1, 0) \xrightarrow{\text{(invert)}} x_1 = d^1(p_1),$$
$$p_2 = P^2(0, x_2) \xrightarrow{\text{(invert)}} x_2 = d^2(p_2).$$

3.2 Some useful properties of the demand functions

Since the informed and uninformed consumers are represented by the same consumer subject to incomplete product awareness, the demand functions $D^i(p_1, p_2)$ and $d^i(p_i)$ are naturally related through the utility function. Recall that, say, $d^1(p_1)$ is the consumer's demand for good 1 when x_2 is restricted to 0 in the utility maximization problem. Since $D^1(p_1, p_2)$ is derived with no restriction on x_2, an underlying relationship is that $D^1(p_1, p_2)$ equals $d^1(p_1)$ if p_2 is the choke price for D^2, i.e., the lowest price so that $D^2(p_1, p_2) = 0$ given any p_1. That is, at the choke price p_2, $x_2^* = 0$ is the solution to the unrestricted problem, hence $D^1(p_1, p_2) = d^1(p_1)$.

The following Lemma on the existence of a choke price is probably well known, but we state it (for good 1) for completeness (note that all proofs in this paper are placed in the last section). In short, the bounded derivative of U (with marginal utility going to zero for large enough quantity) and strict concavity jointly imply that for sufficiently high p_1, $x_1^* = 0$ in both (2) and (3).

Lemma 1. *Under Assumptions 1, (i) for any $p_2 < \infty$, there exists $\tilde{p}_1 < \infty$ such that $D^1(p_1, p_2) = 0$ for all $p_1 \geq \tilde{p}_1$ and (ii) there exists $\bar{p}_1 < \infty$ such that $d^1(p_1) = 0$ for all $p_1 \geq \bar{p}_1$.*

The existence of a choke price for D^2 and the Implicit Function Theorem (justified by Assumption 1(ii)) jointly justify the existence of a function $f^2(p_1)$ as follows (and the same holds analogously for good 1):

$$D^2(p_1, f^2(p_1)) = 0, \ \forall p_1 \quad (\text{and } D^1(f^1(p_2), p_2) = 0, \ \forall p_2). \tag{6}$$

Now if we let $p_2 = f^2(p_1)$, meaning $D^2 = 0$, the solutions of x_1 to (2) and (3) should coincide (and the same holds analogously for good 1), i.e.,

$$d^1(p_1) = D^1(p_1, f^2(p_1)), \ \forall p_1 \quad (\text{and } d^2(p_2) = D^2(f^1(p_2), p_2), \ \forall p_2). \tag{7}$$

Equation (7) states the important connection between d^i and D^i via the choke price p_j, which is key to our subsequent analysis. In the following discussion, we only consider prices below the choke price, i.e., before the demand curves hit the axis. The lower of the two choke prices, $f^i(\cdot)$ (where $D^i = 0$) and $\bar{p}_i$ (where $d^i = 0$), defines the upper bound of p_i. Which one of d^i and D^i hits the horizontal axis first is associated with the inter-product relationship of the two goods, i.e., whether they are gross substitutes (i.e., $D_2^1 > 0$) or gross complements (i.e., $D_2^1 < 0$).

Lemma 2. *For all non-negative prices p_1 and $p_2 < f^2(p_1)$,*

(i) if the two goods are gross substitutes, $d^1(p_1) > D^1(p_1, p_2)$, and

(ii) if the two goods are gross complements, $d^1(p_1) < D^1(p_1, p_2)$.

The proof is straightforward as one has $d^1(p_1) = D^1(p_1, f^2(p_1))$ and the sign of D_2^1 is determined by gross substitutes or gross complements. The intuition is that when the two goods are gross substitutes, the uninformed consumer's demand d^1 is greater than the informed consumer's demand D^1, as the latter has the option to buy the substitute. The relationship flips when the two goods are gross complements because the informed consumer will want to consume more of good 1 in the presence of a complementary good.

Define the feasible set of prices $(p_1, p_2) \in P^1 \times P^2 = [c_1, \bar{P}_1] \times [c_2, \bar{P}_2]$, such that $\bar{P}_2 \leq \min\{\bar{p}_2, f^2(p_1)\}$ for all $p_1 \in [c_1, \bar{P}_1]$ and $\bar{P}_1 \leq \min\{\bar{p}_1, f^1(p_2)\}$ for all $p_2 \in [c_2, \bar{P}_2]$.[10] These feasibility constraints guarantee that the prices under consideration for the overall analysis do not exceed the choke prices for both D^i and d^i, $i = 1, 2$.

Now we turn to the analysis of the connection between the derivatives of D^i and d^i, for use below. Say for good 1, differentiating (6) for good 2 with respect to p_1, we get $D_1^2(p_1, f^2(p_1)) + D_2^2(p_1, f^2(p_1))(f^2(p_1))' = 0$, or

$$(f^2(p_1))' = -\frac{D_1^2(p_1, f^2(p_1))}{D_2^2(p_1, f^2(p_1))}. \tag{8}$$

Next, differentiating (7) for good 1 with respect to p_1, we get

$$(d^1(p_1))' = D_1^1(p_1, f^2(p_1)) + D_2^1(p_1, f^2(p_1))(f^2(p_1))'. \tag{9}$$

Inspecting the second term of (9), by (8), we have

$$D_2^1(p_1, f^2(p_1))(f^2(p_1))' = -\frac{D_2^1(p_1, f^2(p_1))D_1^2(p_1, f^2(p_1))}{D_2^2(p_1, f^2(p_1))} > 0 \tag{10}$$

[10] For a more general discussion on the micro-economic foundations for the multi-variate linear demand function for differentiated products and the restrictions on the primitives of prices, see Amir et al. (2017).

because $D_2^1 = D_1^2 \neq 0$ (due to the assumption $U_{12} \neq 0$) and $D_2^2 < 0$ (due to strict concavity of U), the arguments being dropped for simplicity. Therefore by (9) and (10), we have the following inequality (and the same holds analogously for good 2):

$$(d^1(p_1))' > D_1^1(p_1, f^2(p_1)),\ \forall p_1 \quad (\text{and } (d^2(p_2))' > D_2^2(f^1(p_2), p_2),\ \forall p_2). \tag{11}$$

Equation (11) specifies the important relationship between the derivatives of, say, $d^1(p_1)$ and $D^1(p_1, p_2)$ when evaluated at the choke price of D^2, i.e., $p_2 = f^2(p_1)$. Note that $(d^1(p_1))' > D_1^1(p_1, p_2)$ is not necessarily true for all p_2, but should be true for those p_2's sufficiently close to $f^2(p_1)$ given the continuity of the demand functions.

In a nutshell, the properties of $d^i(p_i)$ and $D^i(p_1, p_2)$ for $i = 1, 2$ given in (7) and (11) are crucial to the subsequent analysis. In particular, we will show that when the game is supermodular, the elasticities of $d^i(p_i)$ and $D^i(p_1, p_2)$ are closely associated with the log-supermodularity of $D^i(p_1, p_2)$ via these properties.

Now we are ready to investigate the properties of the price game, with a first separation according to the relationship between the two goods.

4 Two goods as gross complements

In this section, we consider the case when the two goods are gross complements, i.e., the demand for either one of them is globally strictly decreasing in the other's price, or $D_2^1 < 0$. With the framework of supermodular games, it is important to distinguish the demand concept of *gross complements* from the game-theoretic notion of *strategic complements*. The latter concept is used interchangeably with the supermodularity of a game. The key property of such games is an immediate consequence of Topkis's Monotonicity Theorem: each player's reaction correspondence is increasing in rival's action.[11] The concepts of gross substitutes (i.e., $D_2^1 > 0$) and strategic substitutes (arising in submodular games) are distinguished in a similar way.

It is well-known in the recent oligopoly literature (e.g., Vives, 1999) that a Bertrand duopoly selling gross complements (substitutes) is typically of strategic substitutes (complements), although this general association may flip in both cases under general conditions, a fact that will play a major role in the analysis below as will become apparent (see e.g., Amir and Stepanova, 2006).

Let us start by providing some of the basics for our lattice-theoretic analysis. We may consider

[11] With strict supermodularity used in this paper, every selection of each player's reaction correspondence will be increasing in the rival's action.

a firm's objective to be its profit function or alternatively the log of the latter. Due to Topkis' characterization (1978), we know that a function has strictly increasing (decreasing) differences in two arguments if the corresponding cross partial derivative is strictly positive (negative). As the profit function has the functional form (1), standard differentiation yields, say, for Firm 1 for both forms of its objective:

$$\frac{\partial^2 \Pi^1}{\partial p_1 \partial p_2} = \phi\Big(D_2^1 + (p_1 - c_1)D_{12}^1\Big) \tag{12}$$

$$\begin{aligned}\frac{\partial^2 \log(\Pi^1)}{\partial p_1 \partial p_2} &= \phi\Big((\phi D^1 + \frac{1-\phi}{2}d^1)D_{12}^1 - (\phi D_1^1 + \frac{1-\phi}{2}(d^1)')D_2^1\Big) \\ &= \phi\Big(\phi(D^1 D_{12}^1 - D_1^1 D_2^1) + \frac{1-\phi}{2}(d^1 D_{12}^1 - (d^1)' D_2^1)\Big) \end{aligned} \tag{13}$$

$$\frac{\partial^2 \log(\Pi^1)}{\partial p_1 \partial \phi} = \frac{D^1 d^1}{2}\Big(\phi D^1 + \frac{1-\phi}{2}d^1\Big)^{-2}\Big(\frac{D_1^1}{D^1} - \frac{(d^1)'}{d^1}\Big). \tag{14}$$

Note that the first cross partial is taken with Π^1 while the latter two are with $\log \Pi^1$. Such a monotonic transformation of the objective function is a common technique used in oligopoly settings, because the properties of supermodularity (and increasing differences) are both of a cardinal nature and may thus be altered by monotone transformations of the objective function.

By (14), $\log \Pi^1$ has strictly increasing differences in (p_1, ϕ) if and only if $\frac{D_1^1}{D^1} - \frac{(d^1)'}{d^1} > 0$, or equivalently, $\epsilon_{D^1} > \epsilon_{d^1}$, if one multiplies both sides by p_1, that is when the informed demand is less elastic than the uninformed demand. Conversely, $\log \Pi^1$ has strictly decreasing differences in (p_1, ϕ) if and only if the informed demand is more elastic than the uninformed demand.

Another key remark is that (12) and (13) may be used interchangeably to establish the (log) supermodular nature of the game in the actions (p_1, p_2): In the perspective of Firm 1, the game is of strategic complements (resp., strategic substitutes) if $D_2^1 + (p_1 - c_1)D_{12}^1 >$ (resp., $<$)0, or if $\phi(D^1 D_{12}^1 - D_1^1 D_2^1) + \frac{1-\phi}{2}(d^1 D_{12}^1 - (d^1)' D_2^1) >$ (resp., $<$)0. The key implication of either of these two conditions, or of strict strategic complementarity (strategic substitutes), is that every selection of Firm 1's reaction correspondence is increasing (decreasing) in Firm 2's price. Similar arguments hold analogously for Firm 2.

Next, we discuss the two cases of strategic complements and strategic substitutes separately.

4.1 Strategic complements

The property of gross complements is known to be more compatible with strategic substitutes than with strategic complements in a Bertrand setting, as $D_2^1 + (p_1 - c_1)D_{12}^1 < 0$ is more likely to hold when $D_2^1 < 0$. For the association to flip, i.e., for $D_2^1 + (p_1 - c_1)D_{12}^1 > 0$ and $D_2^1 < 0$ to hold simultaneously, one needs strong supermodularity of D^1, i.e., D^1_{12} being strongly positive. The latter is the circumstance that the discussions of this subsection apply to.

In Cosandier et al. (2018), two conditions for Firm 1, $D^1_2 + (p_1 - c_1)D^1_{12} > 0$ and $\epsilon_{D^1} \leq \epsilon_{d^1}$, and the analogous ones for Firm 2 jointly imply that when the market becomes more transparent (i.e., ϕ increases), both firms' equilibrium prices will decrease regardless of the two goods being gross substitutes or complements. Our next proposition shows that for the same result to hold for gross complements, only one condition regarding each firm's demand function is needed: $D^i(p_1, p_2)$ is *(differentiably) strict log-supermodular*, $i = 1, 2$.[12]

By definition, $D^1(p_1, p_2)$ is (differentiably) strict log-supermodular (log-submodular) if the cross partial satisfies: $\partial^2 \log D^1(p_1, p_2)/\partial p_1 \partial p_2 > (<)0$. Writing out the cross partial explicitly yields the equivalent characterization:

$$D^1 D^1_{12} - D^1_1 D^1_2 > (<)0, \tag{15}$$

or equivalently,

$$\frac{D^1_1(p_1, p_2)}{D^1(p_1, p_2)} \text{ strictly increases (decreases) in } p_2. \tag{16}$$

It is well known that the log-supermodularity of demand has a precise economic interpretation: The price elasticity of demand w.r.t one price is increasing in the other price.[13] In other words, the rival's price going up works to the advantage of the firm as its demand becomes more inelastic. The log-submodularity of demand has the opposite economic interpretation. Now we are ready for the next Proposition.

Proposition 1. ***(Gross complements)*** *Assume for $i = 1, 2$, $\log D^i$ is supermodular for non-negative prices. Then we have:*

(i) The game is (log) supermodular and a Bertrand equilibrium exists for each value of ϕ.

(ii) An increase in the market transparency ϕ causes the extremal equilibrium prices of both goods to decrease.

The log-supermodularity of D^i plays a key role in proving the comparative statics stated above. The relationships between D^i and d^i presented in (7) and (11), namely $D^i = d^i$ and $D^i_i < (d^i)'$ at the choke price of D^j, immediately imply that $\epsilon_{D^i} < \epsilon_{d^i}$ at the choke price of D^j. Then, the log-supermodular D^i implies that $\epsilon_{D^i} < \epsilon_{d^i}$ holds for all p_j's less than the choke price, because in

[12] We assume *strict* log-supermodularity (log-submodularity) of the demand functions, as well as strict inequalities of the elasticity comparison in the following paper. Strict conditions are used for its simplicity while our results are robust to the weak versions of the same set of conditions.

[13] The elasticity of demand can be written as $\frac{\partial D^1(p_1,p_2)}{\partial p_1} \frac{p_1}{D^1(p_1,p_2)} = p_1 \frac{\partial \log D^1(p_1,p_2)}{\partial p_1}$, which is strictly increasing in p_2 if $\log D^1(p_1, p_2)$ is strictly supermodular in (p_1, p_2) [Topkis (1998), p.66].

this case the price elasticity of D^i w.r.t p_i is increasing in the other price. Hence, the informed demand is more elastic than the uninformed demand in a global sense (for all feasible prices).

The elasticity condition $\epsilon_{D^i} < \epsilon_{d^i}$ further implies $\partial^2 \log \Pi^i / \partial p_i \partial \phi < 0$ by (14). That is, the log of the profit function has strictly decreasing differences in the firm's own price and market transparency. Then using the elasticity condition and (15), given that $D_2^1 < 0$ for gross complements, we further prove that $\partial^2 \log \Pi^i / \partial p_1 \partial p_2 > 0$, i.e., the log profit function is strictly supermodular in (p_1, p_2). With a compact price space, the Bertrand-duopoly game satisfies the requirements for a (log) supermodular game, therefore a maximal and a minimal pure-strategy Nash equilibria exist (e.g., Topkis, 1998). Moreover, since every selection of each firm's reaction correspondence is increasing in the rival's price and will shift downward when ϕ increases, both firm's equilibrium prices decrease in ϕ.

The proposition gives a minimal set of conditions, i.e., log-supermodularity of D^i, that lead to clear-cut comparative statics when the two goods are gross complements and the game is of strategic complements. Both firms' equilibrium prices will fall as a result of increased market transparency. It is an improvement over the set of conditions used in [Cosandier et al. (2018), Proposition 1].

The micro-economics properties of D^i and d^i discussed in Subsection 3.2 are crucial for the sufficiency of log-supermodularity of D^i in deriving the comparative statics. First, log-supermodular D^i is shown to imply $\epsilon_{D^i} < \epsilon_{d^i}$ for all prices, but on the other hand log-submodular D^i, i.e., $\partial^2 \log D^i / \partial p_1 \partial p_2 < 0$, does not necessarily imply $\epsilon_{D^i} > \epsilon_{d^i}$ (because while the price elasticity of D^i w.r.t p_i will be decreasing in the other price, $\epsilon_{D^i} < \epsilon_{d^i}$ still holds at the choke of D^j). Second, the sufficiency of log-supermodular D^i in implying that the game is strictly log-supermodular, i.e., $\partial^2 \log \Pi^i / \partial p_1 \partial p_2 > 0$, is contingent on the condition $D_2^1 < 0$ or that the two goods are gross complements. For gross substitutes (discussed in the next section), additional conditions are needed for the Bertrand competition to qualify for supermodular games.

As mentioned above, gross complements are more often associated with games of strategic substitutes rather than strategic complements in the context of Bertrand competition (Vives, 1999). The latter association is established only when the demand function D^i is strongly supermodular (instead of log-supermodular), to which circumstances Proposition 1 applies. An illustrative example (Example 2) follows in the next section.

Now we look at the comparative statics for firms' equilibrium profits. Differentiating the optimal profit (1) with respect to ϕ in order to use the envelope Theorem, we have,

$$\frac{\partial \Pi^1(p_1^*, p_2^*)}{\partial \phi} = (p_1^* - c_1)\Big(D^1(p_1^*, p_2^*) - \frac{1}{2} d^1(p_1^*)\Big) + \frac{\partial \Pi^1(p_1^*, p_2^*)}{\partial p_1}\frac{\partial p_1^*}{\partial \phi} + \frac{\partial \Pi^1(p_1^*, p_2^*)}{\partial p_2}\frac{\partial p_2^*}{\partial \phi}.$$

The second term in the right-hand side is zero by the first-order condition. With some simple algebra, we get (where the arguments of the profit and demand functions are the equilibrium prices, suppressed for clarity)

$$\frac{\partial \Pi^1}{\partial \phi} = (p_1^* - c_1)\Big(D^1 - \frac{1}{2}d^1 + \phi D_2^1 \frac{\partial p_2^*}{\partial \phi}\Big). \tag{17}$$

By Lemma 2, when the two goods are gross complements, $D^1(p_1, p_2) > d^1(p_2)$ for all feasible prices, so $D^1 > \frac{1}{2}d^1$. In addition, by Proposition 1 we have $\partial p_i^*/\partial\phi < 0$ for $i = 1, 2$, so $\phi D_2^1(\partial p_2^*/\partial\phi) > 0$. Therefore, the equilibrium profit of Firm 1 will increase when the market becomes more transparent (and analogously for Firm 2). We have just proved the following result.

Corollary 2. ***(Gross complements)*** *Under the assumptions in Proposition 1, an increase in the market transparency ϕ causes the (extremal) equilibrium profits of both firms to increase.*

Intuitively, there are two profit effects associated with increased market transparency. The direct effect, measured by $D^i - \frac{1}{2}d^i$, is the change in a firm's total market demand when more consumers become informed. This effect is positive for gross complements because the informed consumers' demand is greater than the uninformed consumers' demand by Lemma 2. The indirect effect, measured by $\phi D_j^i(\partial p_j^*/\partial\phi)$, is the change in the demand of Firm i's informed consumers when the rival responds to increased market transparency by adjusting its price, in this case downward according to Proposition 1. This effect is also positive for gross complements. Therefore, the two effects work in tandem to raise both firms' equilibrium profits with increased market transparency.

4.2 Strategic substitutes

So far we have limited our discussion of gross complements to games of strategic complements (i.e., supermodular games), where both firms' reaction correspondences increase in rival's price. Now we turn to the case of strategic substitutes or submodular games, where both firms' reaction correspondences decrease in the rival's price. The latter property is in fact more compatible with the two goods being gross complements in standard (differentiated-good Bertrand competition (Vives, 1999). This case requires D^i to be log-submodular. As mentioned earlier, log-submodular D^i does not imply $\epsilon_{D^i} > \epsilon_{d^i}$ in a global sense. Hence, the elasticity condition is also needed to establish clear-cut comparative statics. Moreover, it is well-known that in submodular games, unambiguous comparative statics can only be reached for one firm, but not both, when the two players' reaction correspondences shift in the same direction.

Proposition 3. ***(Gross complements)*** *Assume that for all feasible prices, $i = 1, 2$,*

(i) $\log D^i$ *is submodular, and*

(ii) $\epsilon_{D^i} > \epsilon_{d^i}$.

Then we have:

(i) The game is (log) submodular and a Bertrand equilibrium exists for each value of ϕ.

(ii) An increase in the market transparency ϕ causes the extremal equilibrium price of at least one good to increase.

The idea is that condition (ii) implies that $\log \Pi^i$ has strictly increasing differences in $(p_i; \phi)$ via (14). Conditions (i) and (ii) together imply that $\log \Pi^1(p_1, p_2)$ is strictly submodular in (p_1, p_2). Hence, every selection of each firm's reaction correspondence is decreasing in the rival's price and will shift upward when ϕ increases. Therefore, at least one firm's price increases in ϕ (but not necessarily both prices). Here, pure-strategy equilibrium existence follows via the usual arguments from the fact that this is a two-player submodular game (see Vives, 1999).

Note that condition (i) can be replaced by the condition $D^i_j + (p_i - c_i)D^i_{ij} < 0$ as is used in [Cosandier et al. (2018), Proposition 2]. The two conditions are known to be close cousins in such oligopoly settings. Both conditions are crucial to show the submodular nature of the game: $D^i_j + (p_i - c_i)D^i_{ij} < 0$ implies that $\Pi^i(p_1, p_2)$ is submodular in (p_1, p_2), while log-submodularity of D^i and the elasticity condition jointly imply that $\log \Pi^i(p_1, p_2)$ is submodular in (p_1, p_2).

The comparative statics result for the submodular game is less sharp than that derived in the supermodular game as is given in Proposition 1, since only one firm's equilibrium price will increase with certainty. The ambiguity is the consequence of the downward sloping of both firms' reaction curves, as there are two opposing forces at work when the market transparency parameter increases. As the market becomes more transparent, the reaction curve of, say, Firm 1, shifts up for each value of p_2. Meanwhile, p_2 also increases (i.e., shifts up for each value of p_1 in the same manner), which causes a downward adjustment of Firm 1's price along its reaction curve. The aggregate effect is thus indeterminate which depends on the relative magnitude of the two forces.

In terms of the comparative statics for the equilibrium profit, we need to examine the sign of (17). We know that $D^i > \frac{d^i}{2}$ for gross complements by Lemma 2, i.e., the direct effect results in both firms' total market demand to increase as more consumers become informed, thus raising profits. The indirect effect, $\phi D^i_j(\partial p^*_j/\partial\phi)$, measures the change in the demand of a firm's informed consumers when the rival's equilibrium price is adjusted as a result of increased transparency. Proposition 3 implies two possible scenarios: both firms' equilibrium prices increase, or one firm's price increases while the other firm's price decreases. Therefore for Firm i, the rival's equilibrium price can either go up or go down when market transparency increases. If Firm j's price goes up, i.e., $\partial p_j^*/\partial\phi > 0$, then $\phi D_j^i(\partial p_j^*/\partial\phi) < 0$, thus the two effects work in opposite directions for Firm i and the aggregate effect is indeterminate. If Firm j's price goes down, then $\phi D_j^i(\partial p_j^*/\partial\phi) > 0$. Here, the two effects work in tandem, and Firm i's equilibrium profit increases with certainty. We have just proved the following Corollary.

Corollary 4. ***(Gross complements)*** *Under the assumptions in Proposition 3, for $i = 1, 2$, $i \neq j$, when the market becomes more transparent:*

(i) if $\partial p_j^*/\partial\phi > 0$, then the change of Firm i's (extremal) equilibrium profit is indeterminate;

(ii) if $\partial p_j^*/\partial\phi < 0$, then Firm i's (extremal) equilibrium profit increases.

5 Two goods as gross substitutes

In this section, we examine the comparative statics of the equilibrium prices and profits when the two goods are gross substitutes (i.e., $D_2^1 > 0$). There are two subcases, namely when the Bertrand pricing game is of strategic complements (supermodular games) and when it is of strategic substitutes (submodular games).

As is well-known for Bertrand pricing games (e.g., Vives, 1999), the case with the two goods being gross substitutes is more often associated with games of strategic complements than with games of strategic substitutes. Mathematically, $D_j^i > 0$ is more compatible with the condition $D_j^i + (p_i - c_i)D_{ij}^i > 0$, the latter implying the game to be supermodular via (12), i.e., $\partial^2\Pi^i/\partial p_1\partial p_2 > 0$. However, the association can flip if the demand is strongly submodular, i.e., D_{ij}^i is strongly negative. Before formal discussions of the two subcases, we want to give an important remark on the elasticity condition for gross substitutes.

It has been revealed in Cosandier et al. (2018) and our preceding analysis that the elasticity comparison between D^i and d^i plays a crucial role in determining the comparative statics for the equilibrium prices and profits. In particular, it determines the sign of $\partial^2 \log \Pi^i/\partial p_i\partial\phi$, which affects whether the reaction correspondence of each firm shifts up or down with increased market transparency. Further inspection of the elasticity comparison by invoking the useful properties of the demand functions discussed in subsection 3.2 leads to a critique of the elasticity condition used in [Cosandier et al. (2018), Proposition 2]: $\epsilon_{D^i} \geq \epsilon_{d^i}$ for all prices. In fact, $\epsilon_{D^i} \geq \epsilon_{d^i}$ cannot hold *for all feasible prices* if the two goods are gross substitutes, given that D^i and d^i are derived from the same utility function. In particular, we have, say, for Firm 1, the opposite $\epsilon_{D^1} < \epsilon_{d^1}$ holds at the choke price of D^2, i.e., $p_2 = f^2(p_1)$.

Formally, we first note that by Lemma 2, $d^2(p_2) > D^2(p_1, p_2)$ for gross substitutes, so the choke price of D^2 is feasible as $d^2(f^2(p_1)) > D^2(p_1, f^2(p_1)) = 0$. But then (7) and (11) imply that

$$\frac{D_1^1(p_1, f^2(p_1))}{D^1(p_1, f^2(p_1))} < \frac{(d^1)'(p_1)}{d^1(p_1)}, \quad \text{or } \epsilon_{D^1} < \epsilon_{d^1} \text{ when } p_2 = f^2(p_1). \tag{18}$$

That is, for Firm 1, the informed consumers' demand is always more elastic than the uninformed consumers' demand at the rival's choke price (regardless of gross complements or substitutes). Moreover, (18) should hold for any p_2 sufficiently close to $f^2(p_1)$ due to the continuity of the demand functions. Here, (18) combined with the fact that $p_2 = f^2(p_1)$ is feasible for gross substitutes implies the invalidity of the elasticity condition given in [Cosandier et al. (2018), Proposition 2], unless a sufficiently large neighborhood around the choke $f^2(p_1)$ is excluded from the feasible set of prices. We have just proved the following Lemma for Firm 1 (and analogous for Firm 2).

Lemma 3. *When the two goods are gross substitutes, there exists some $\epsilon > 0$ such that $\epsilon_{D^1} < \epsilon_{d^1}$ for those feasible prices such that $p_2 \geq f^2(p_1) - \epsilon$.*

Note that the same arguments do not necessarily extend to the case of gross complements. The key difference is the feasibility of the choke price $p_2 = f^2(p_1)$. For gross complements, Lemma 2 states that $D^2(p_1, p_2) > d^2(p_2)$, thus the choke price of D^2 is no longer feasible as $d^2(f^2(p_1)) < 0 = D^2(p_1, f^2(p_1))$, i.e., in this case d^2 binds first. In a nutshell, (18) does not rule out the possibility of $\epsilon_{D^i} \geq \epsilon_{d^i}$ being a valid condition *for all feasible prices* provided that the two goods are gross complements, simply because $f^j(p_i)$ is not feasible. The linear demand system given in the next example corresponds to the latter case.

Example 1. *The following linear demand system derived from a quadratic quasi-linear utility function*[14] *was given in [Cosandier et al. (2018), Section 4]. The two demand functions for Firm 1 (and analogously for Firm 2) corresponding to the informed and the uninformed consumers are*

$$\begin{aligned} D^1(p_1, p_2) &= \frac{2a_1b_2 - \gamma a_2}{4b_1b_2 - \gamma^2} - \frac{2b_2}{4b_1b_2 - \gamma^2}p_1 + \frac{\gamma}{4b_1b_2 - \gamma^2}p_2 \\ d^1(p_1) &= \frac{a_1}{2b_1} - \frac{p_1}{2b_1}. \end{aligned}$$

Here, we have $4b_1b_2 > \gamma^2$ by the concavity assumption of U. By the third term of D^1, the two goods are gross complements if $\gamma < 0$ and are gross substitutes if $\gamma > 0$. Write out the elasticity formula

[14] This is the widely used linear demand system coming from a representative consumer maximizing a quasi-linear quadratic utility function. This was originally due to Bowley and ubiquitous in industrial organization (Singh and Vives, 1984; Amir et al., 2017).

for Firm 1, and rearrange the terms, then we have

$$\epsilon_{D^1} - \epsilon_{d^1} = \frac{(-\gamma)(a_2 - p_2)}{(a_1 - p_1)(2a_1 b_2 - \gamma a_2 - 2b_2 p_1 + \gamma p_2)}. \tag{19}$$

Note that $p_1 < a_1$ *by* $d^1 > 0$, $p_2 < a_2$ *by* $d^2 > 0$, *and* $2a_1 b_2 - \gamma a_2 - 2b_2 p_1 + \gamma p_2 > 0$ *by* $D^1 > 0$. *The intersection of these constraints (and the marginal cost constraint) for Firm 1 and Firm 2 forms the feasible set of prices. Therefore,* $(\epsilon_{D^1} - \epsilon_{d^1})$ *has the same sign as* $(-\gamma)$.

Here, for the special linear demand system, the elasticity comparison is determined by whether the two goods are gross substitutes or gross complements. It follows that for all feasible prices, $\epsilon_{D^1} > \epsilon_{d^1}$ *if* $\gamma < 0$ *(gross complements), and* $\epsilon_{D^1} < \epsilon_{d^1}$ *if* $\gamma > 0$ *(gross substitutes). The same arguments hold for Firm 2 analogously.*

First note that Lemma 3 holds trivially for gross substitutes, as in the case at hand $\epsilon_{D^1} < \epsilon_{d^1}$ *holds true for all feasible prices. However, note that* (18) *is a general argument without distinguishing gross complements or substitutes, so the fact that* $\epsilon_{D^1} > \epsilon_{d^1}$ *holds for all feasible prices here for gross complements appears to contradict* (18)*, which is not true. By definition, the choke of* D^2, $f^2(p_1)$ *has the following expression:*

$$D^2(p_1, f^2(p_1)) = 0 \Rightarrow f^2(p_1) = a_2 - \frac{\gamma(a_1 - p_1)}{2b_1}. \tag{20}$$

Replacing p_2 *in* (19) *by* $f^2(p_1)$*, one gets*

$$\epsilon_{D^1} - \epsilon_{d^1} = \frac{-\gamma^2}{(a_1 - p_1)(4b_1 b_2 - \gamma^2)} < 0,$$

which verifies that (18) *is true for gross complements. However, note that the choke price of* D^2, $p_2 = f^2(p_1)$ *is not feasible here, because* $\gamma < 0$ *implies* $f^2(p_1) > a_2$ *via* (20)*, with* a_2 *being the choke price of* d^2.

This example shows that the elasticity comparison between D^i and d^i demands extra caution paid to the underlying properties of the demand functions and the feasible set of prices. In the following section, we then restrict attention to the case when $\epsilon_{D^i} < \epsilon_{d^i}$ for all feasible prices, $i = 1, 2$.

5.1 Strategic complements

As is analyzed in the case of gross complements, the log-supermodularity of D^i means that the informed consumers' demand becomes more elastic as the rival's price decreases, which further implies that $\epsilon_{D^i} < \epsilon_{d^i}$ for all p_j below the choke price via (18). Then by (14), the elasticity condition implies that $\log \Pi^i$ has strictly decreasing differences in $(p_i; \phi)$, so that an increase in market transparency causes every selection of each firm's reaction correspondence to shift down. But here, log-supermodular D^i combined with the elasticity condition no longer imply $\partial^2 \log(\Pi^i)/\partial p_1 \partial p_2 > 0$ (due to the flipped sign of D_2^1). For the game to be supermodular, we need the additional condition $D_j^i + (p_i - c_i) D_{ij}^i > 0$, $i = 1, 2$, $i \neq j$.

Proposition 5. ***(Gross substitutes)*** *Assume that for non-negative prices, $i = 1, 2$, $i \neq j$,*

(i) $\log D^i$ is supermodular, and

(ii) $D_j^i + (p_i - c_i) D_{ij}^i > 0$.

Then we have:

(i) The game is supermodular and a Bertrand equilibrium exists for each value of ϕ.

(ii) An increase in the market transparency ϕ causes the extremal equilibrium prices of both goods to decrease.

The two conditions guarantee that the game is strictly supermodular and that each firm's log-profit function has strictly decreasing differences in one firm's own price and ϕ. Then, every selection of each firm's reaction correspondence is increasing in the rival's price and will shift down when the market becomes more transparent. Hence both firms' (extremal) equilibrium prices decrease. In comparison with [Cosandier et al. (2018), Proposition 1], the main message here is that the elasticity condition $\epsilon_{D^i} < \epsilon_{d^i}$ is interchangeable with the log-supermodularity of D^i. Indeed, conditions (i) and (ii) often go in tandem with each other, as both are in favor of the supermodularity of D^i (i.e., $D_{ij}^i > 0$) and gross substitutes (i.e., $D_j^i > 0$).

The next Example 2 is given to illuminate a couple of important points. First, the example illustrates a circumstance where the Bertrand competition exhibits strategic complements when the two goods are gross complements (instead of its usual association with strategic substitutes in Bertrand settings, see, e.g., Vives, 1999). Thus it is a direct application of Proposition 1, and we demonstrate that the compatibility is due to the strong supermodularity of D^i (rather than $\log D^i$). Second, we show that the utility function used in the example always gives rise to log-supermodular demand D^i regardless of the two goods being gross substitutes or complements, contrasting the linear example given earlier. The example verifies the comparative statics results of Proposition 1, namely log-supermodular D^i is the only condition needed for the case of gross complements. Meanwhile, it gives a circumstance when the results in Proposition 5 (for gross substitutes) fail to hold because one of the conditions is violated.

Example 2. *Consider the utility function $U(x_1, x_2) = \sqrt{x_1} + 2\sqrt{x_2} + 2\gamma\sqrt{x_1 x_2}$. Following the* *utility maximization in (4) and (5), we derive the following inverse demand functions and the corresponding demand functions:*

$$\begin{cases} P^1(x_1,x_2) = \frac{1+2\gamma\sqrt{x_2}}{2\sqrt{x_1}} \\ P^2(x_1,x_2) = \frac{1+\gamma\sqrt{x_2}}{\sqrt{x_1}} \end{cases} \xrightarrow{\textit{(invert)}} \begin{cases} D^1(p_1,p_2) = (\frac{2\gamma+p_2}{2p_1p_2-2\gamma^2})^2 \\ D^2(p_1,p_2) = (\frac{\gamma+2p_1}{2p_1p_2-2\gamma^2})^2 \end{cases}$$

and

$$P^1(x_1,0) = \frac{1}{2\sqrt{x_1}} \xrightarrow{\textit{(invert)}} d^1(p_1) = (\frac{1}{2p_1})^2,$$
$$P^2(0,x_2) = \frac{1}{\sqrt{x_2}} \xrightarrow{\textit{(invert)}} d^2(p_2) = (\frac{1}{p_2})^2.$$

The concavity of $U(x_1,x_2)$ requires $\gamma + 2p_1 > 0$, $2\gamma + p_2 > 0$ and $\gamma^2 < p_1p_2$. With simple algebra, we get

$$\frac{\partial^2 \log D^1}{\partial p_1 \partial p_2} = \frac{\partial^2 \log D^2}{\partial p_1 \partial p_2} = \frac{2\gamma^2}{(\gamma^2 - p_1p_2)^2} > 0, \quad \textit{and} \quad \frac{\partial D^1}{\partial p_2} = \frac{\partial D^2}{\partial p_1} = \frac{-\gamma(\gamma+2p_1)(2\gamma+p_2)}{2(p_1p_2-\gamma^2)^3}.$$

The two equations reveal two important features of the game:

(i) The first equation verifies that D^1 and D^2 are always strictly log-supermodular.

(ii) The second equation implies that the two goods are gross complements if $\gamma > 0$, gross substitutes if $\gamma < 0$.

Recall that by Proposition 1, the log-supermodularity of D^1 and D^2 implies that the Bertrand competition is of strategic complements when the two goods are gross complements. Here, it corresponds to the case $\gamma > 0$. The compatibility between gross complements and strategic complements is due to the strong supermodularity of D^1 and D^2: When $\gamma > 0$, we have

$$D^1_{12} = \frac{\gamma(2\gamma+p_2)(2\gamma^2+(3\gamma+4p_1)p_2)}{2(p_1p_2-\gamma^2)^4} > 0, \quad D^2_{12} = \frac{\gamma(\gamma+2p_1)(\gamma^2+2(3\gamma+p_2)p_1)}{2(p_1p_2-\gamma^2)^4} > 0.$$

To solve the game, one can write down the first-order condition $\partial\Pi^i/\partial p_i = 0$ for $i = 1,2$ and solve for (p_1,p_2) for any given value of ϕ. However, the FOC turns out to involve high-order terms of $p_1^4p_2^3$, thus numerical simulation is used instead to illuminate the comparative statics for the game at hand. Fix $c_1 = 0.3$, $c_2 = 0.5$.

Let $\gamma = 0.3$, so the two goods are gross complements. The (unique) equilibrium price (p_1,p_2) monotonically decreases when ϕ increases from 0.1 to 0.9: p_1 decreases from 0.5504 to 0.4911, p_2 from 0.9361 to 0.8194, which is consistent with Proposition 1.

Let $\gamma = -0.4$, so the two goods are gross substitutes. When ϕ increases in the range $[0.2, 0.5]$, the two firms' prices move in opposite directions: p_1 increases from 0.5971 to 0.6, while p_2 decreases from 0.8951 to 0.8, as shown in Figure 2. This conflicts with the comparative statics result of Proposition 5. The reason is that condition (ii) $D^i_j + (p_i - c_i)D^i_{ij} > 0$, required for the game to be supermodular, does not always hold for both firms, which is left for the readers to verify. This is illustrated in Figure 2, which shows that while the reaction curve of Firm 2 slopes upward, the reaction curve of Firm 1 slopes downward, in which case the game is neither supermodular nor submodular.

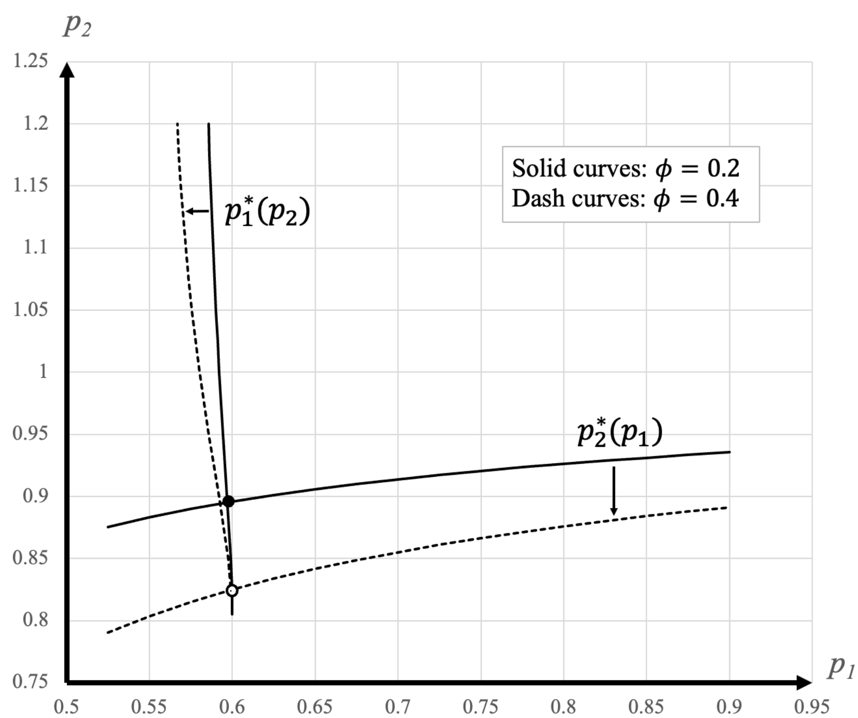

Figure 1: The reaction curves shift down when ϕ increases from 0.2 to 0.4

Note: $c_1 = 0.3, c_2 = 0.5, \gamma = -0.4$. With increased ϕ, p_2^* decreases while p_1^* increases.

Next, we turn to the comparative statics for the equilibrium profits, which requires checking, say, for Firm 1, the additive sign of $\left(D^1 - \frac{1}{2}d^1\right)$ and $\phi D^1_2(\partial p_2^*/\partial\phi)$. The latter term can be signed via Proposition 5: $\phi D^1_2(\partial p_2^*/\partial\phi) < 0$, because $\partial p_2^*/\partial\phi < 0$ and $D^1_2 > 0$. That is, the indirect effect on Firm 1's informed demand resulting from Firm 2's downward price adjustment is negative, as the two firms sell gross substitutes. However, the sign of the first term $\left(D^1 - \frac{1}{2}d^1\right)$ (evaluated at equilibrium prices) is indeterminate because Lemma 2 says that $D^1(p_1, p_2) < d^1(p_1)$ for gross substitutes. If at the equilibrium prices, $D^1 < \frac{1}{2}d^1$, then the two effects go in tandem and $\partial\Pi^1/\partial\phi < 0$. Otherwise, the two effects go in opposite directions and the overall result is ambiguous. We have just proved the following result for Firm 1 (and analogously for Firm 2).

Corollary 6. ***(Gross substitutes)*** *Under the assumptions in Proposition 5, for $i = 1, 2$, $i \neq j$, when the market becomes more transparent,*

(i) if $D^i > \frac{1}{2}d^i$, then the change of Firm i's (extremal) equilibrium profit is indeterminate;

(ii) if $D^i < \frac{1}{2}d^i$, then Firm i's (extremal) equilibrium profit decreases.

5.2 Strategic substitutes

Lastly, we turn to the case of strategic substitutes when the two goods are gross substitutes. With similar arguments for Proposition 1, the log-submodularity of D^i and the elasticity condition $\epsilon_{D^i} < \epsilon_{d^i}$ jointly imply that $\partial^2 \log \Pi^i/\partial p_1 \partial p_2 < 0$, thus the game is strictly log-submodular and every selection of each firm's reaction correspondence has negative slopes. Meanwhile, $\epsilon_{D^i} < \epsilon_{d^i}$ implies that $\partial^2 \log \Pi^i/\partial p_i \partial\phi < 0$, hence every selection of each firm's reaction correspondence shifts down when the market becomes more transparent. Therefore, at least one firm's (extremal) equilibrium price decreases. The result is formalized in the following Proposition.

Proposition 7. ***(Gross substitutes)*** *Assume that for all feasible prices, $i = 1, 2$,*

(i) $\log D^i$ is submodular, and

(ii) $\epsilon_{D^i} < \epsilon_{d^i}$.

Then we have:

(i) The game is (log) submodular and a Bertrand equilibrium exists for each value of ϕ.

(ii) An increase in the market transparency ϕ causes the extremal equilibrium price of at least one good to decrease.

Note that condition (i) can be replaced by the condition $D^i_j + (p_i - c_i)D^i_{ij} < 0$ as used in [Cosandier et al. (2018), Proposition 2]. The latter implies that $\partial^2\Pi^i/\partial p_1\partial p_2 < 0$, thus ensuring that the game is strictly submodular.

Remark. Condition (ii) requires $\epsilon_{D^i} < \epsilon_{d^i}$ for all feasible prices, which can be relaxed by condition (i): the elasticity comparison only needs to hold at the lowest rival's price, $p_j = c_j$, and the inequality holds for other prices by the log-submodularity of D^i. That is, say, for Firm 1, if we have $\epsilon_{D^1} < \epsilon_{d^1}$ at $p_2 = c_2$ (i.e., the first inequality below), then it follows by log-submodular D^1 that

$$\frac{(d^1)'(p_1)}{d^1(p_1)} > \frac{D^1_1(p_1, c_2)}{D^1(p_1, c_2)} \geq \frac{D^1_1(p_1, p_2)}{D^1(p_1, p_2)}, \quad \text{for all } p_2 \geq c_2.$$

As for the comparative statics for equilibrium profits, the arguments preceding Corollary 6 apply here similarly with strategic substitutes, except that only one firm's equilibrium price decreases with certainty (instead of both prices). The next result comes immediately from (17).

Corollary 8. *Under the assumptions in Proposition 7, if (i)* $D^i < \frac{1}{2}d^i$ *and (ii)* $\partial p_j^*/\partial\phi < 0$, $i \neq j$, *then Firm i's equilibrium profit decreases when the market becomes more transparent.*

In general, whenever the informed consumers' demand is more elastic than the uninformed consumers' demand, $\epsilon_{D^i} < \epsilon_{d^i}$, (every selection of) each firm's reaction correspondence shifts down when the market becomes more transparent, leading to downward adjustments of equilibrium prices. In supermodular games, both firms' prices go down with certainty (due to the positive reciprocal feedback), while in submodular games, only one firm's price goes down with certainty (due to the negative reciprocal feedback). If $\epsilon_{D^i} > \epsilon_{d^i}$, (every selection of) each firm's reaction correspondence shifts up when the market becomes more transparent and similar arguments apply for supermodular games and submodular games, respectively.

The log-supermodularity (log-submodularity) of D^i plays a key role in determining the comparative statics. First, $\partial^2 \log D^i/\partial p_1 \partial p_2 > 0$ implies $\epsilon_{D^i} < \epsilon_{d^i}$ by invoking the underlying properties of D^i and d^i, the elasticity condition further implying the downward shifting of firms' reaction correspondences. Moreover, $\partial^2 \log D^i/\partial p_1 \partial p_2 > 0$ often goes in tandem with the condition $D_j^i + (p_i - c_i)D_{ij}^i > 0$, the latter guaranteeing the game to be supermodular. Meanwhile, as the log-supermodularity (log-submodularity) of D^i partially determines the sign of $\partial^2 \log \Pi^i/\partial p_1 \partial p_2$ as reflected in (13), it is also closely associated with the supermodular (submodular) nature of the game.

6 Conclusion

This paper has considered a differentiated Bertrand duopoly in a market consisting of two types of consumers, one fully informed of the existence of both firms/goods and the other aware of just one good and split evenly between the two firms. Using the methodology of supermodular games (e.g., Vives, 1999; Milgrom and Roberts, 1990; Amir and Lambson, 2000), we derive a parsimonious set of conditions, based on the micro-economic foundation of the two types of consumers' demand functions, that establish unambiguous comparative statics results on the firms' equilibrium prices and profits as market transparency is (exogenously) increased.

The log-supermodularity (log-submodularity) of the informed consumers' (bivariate) demand plays a key role in two ways. First, it partly determines the supermodular (submodular) nature of the game, thus ensuring equilibrium existence and facilitating comparative statics analysis. More importantly, it is shown to be closely related to the elasticity comparison between the two demand functions, which directly determines the direction of comparative statics, i.e., whether the equilibrium prices increase or decrease when there is higher market transparency. We show that when the two goods are gross complements, log-supermodularity of the informed demand is the only condition required for the equilibrium price of both firms to increase with increased market transparency. More broadly, we find that the conventional wisdom that prices fall with more transparency is confirmed for most cases, with an exceptional case being when the two goods are gross complements and the informed demand is less elastic than the uninformed demand.

7 Proofs

Proof of Lemma 1. First, we prove (i). Let us fix some $p_2 < \infty$. There are two cases to consider.

First, assume that $p_2 \leq U_2(0,0) < \infty$. Because the marginal utility goes to zero for large x_i's given by Assumption 1(iv), there exists $a \geq 0$ such that $p_2 = U_2(0,a)$ as $U_{22} < 0$ everywhere. Now let $\tilde{p}_1 = U_1(0,a) < \infty$. Because $U_{11} < 0$ everywhere, it holds that $U_1(x_1,a) - \tilde{p}_1 < 0$ for all $x_1 > 0$. Therefore, for p_2 and prices larger than $\tilde{p}_1$, utility maximization yields $(x_1^*, x_2^*) = (0,a)$.

Second, fix some p_2 such that $U_2(0,0) < p_2 < \infty$. Then $U_2(0,x_2) < p_2$ for any $x_2 \geq 0$. For any $x_2 \geq 0$, there exists some $\tilde{p}_1$ (dependent on x_2) such that $\tilde{p}_1(x_2) = U_1(0,x_2) \in [0,\infty)$, and $U_1(x_1,x_2) - p_1 < 0$ for all $x_1 > 0$, $p_1 \geq \tilde{p}_1(x_2)$, due to $U_{11} < 0$. Let $\tilde{p}_1 = \inf\{\tilde{p}_1(x_2) : x_2 \geq 0\}$. Hence $U_1(x_1,x_2) - p_1 < 0$ for all $x_2 \geq 0$, $x_1 > 0$, $p_1 \geq \tilde{p}_1$. Thus for such prices, utility maximization yields $(x_1^*, x_2^*) = (0,0)$. Now the proof for part (i) is complete.

For (ii), simply let $\bar{p}_1 = U_1(0,0) < \infty$. Then $U_1(x_1,0) - \bar{p}_1 < 0$ for all $x_1 > 0$, due to $U_{11} < 0$. Thus $x_1^* = 0$ for the utility maximization (3) for all prices larger than $\bar{p}_1$. Q.E.D.

Proof of Lemma 2 In the case of gross substitutes, $d^1(p_1) = D^1(p_1, f^2(p_1)) > D^1(p_1,p_2)$ for all p_1 and $p_2 < f^2(p_1)$. The first equality is by (7) and the second inequality is due to the property $D_2^1 > 0$ for gross substitutes. In the case of gross complements, $d^1(p_1) = D^1(p_1, f^2(p_1)) < D^1(p_1,p_2)$, as in this case $D_2^1 < 0$. Q.E.D.

Proof of Proposition 1. First, we prove (i). By (7) and (11), we have the first inequality in the following

$$\frac{(d^1)'(p_1)}{d^1(p_1)} > \frac{D_1^1(p_1, f^2(p_1))}{D^1(p_1, f^2(p_1))} \geq \frac{D_1^1(p_1,p_2)}{D^1(p_1,p_2)}, \tag{21}$$

and the second inequality follows from the log-supermodularity of D^1, as defined by (16), for

$p_2 \leq f^2(p_1)$. With gross complements, $D_2^1 < 0$, dividing both sides of (15) by $D_2^1 D^1$ yields

$$\frac{D_{12}^1(p_1, p_2)}{D_2^1(p_1, p_2)} < \frac{D_1^1(p_1, p_2)}{D^1(p_1, p_2)}. \tag{22}$$

Combining (21) and (22), we have

$$\frac{(d^1)'(p_1)}{d^1(p_1)} > \frac{D_1^1(p_1, p_2)}{D^1(p_1, p_2)} > \frac{D_{12}^1(p_1, p_2)}{D_2^1(p_1, p_2)}. \tag{23}$$

That is, we have both $D_{12}^1 D^1 - D_1^1 D_2^1 > 0$ and $D_{12}^1 d^1 - (d^1)' D_2^1 > 0$.

Therefore, by (13), the log-supermodularity of D^1 implies that $\log \Pi^1$ is strictly supermodular in (p_1, p_2). (Similar arguments hold for $\log \Pi^2$.) Consequently, every selection of each firm's reaction correspondence is increasing in the rival firm's price. With a compact price space, the Bertrand-duopoly game thus satisfies the requirements for a log-supermodular game. Therefore, a maximal and a minimal pure-strategy Nash equilibria exist (e.g., Topkis, 1998).

Next we prove (ii). As shown above, the log-supermodularity of D^1 implies (21), which is equivalent to $\epsilon_{d^1} > \epsilon_{D^1}$ upon multiplying both sides by p_1. It then follows that $\log \Pi^1$ has strictly decreasing differences in $(p_1; \phi)$ via (14). (Similar arguments hold for $\log \Pi^2$.) Hence, an increase in ϕ will cause every selection of each firm's reaction correspondence to shift downward and by Theorem 6 in Milgrom and Roberts (1990), both firms' prices in the extremal equilibria will decrease in response to an increase in ϕ. Q.E.D.

Proof of Proposition 3. First, we prove (i). Note $\epsilon_{D^i} > \epsilon_{d^i}$ can be written as $\frac{D_i^i}{D^i} > \frac{(d^i)'}{d^i}$. Furthermore, by log-submodularity of $D^i(p_1, p_2)$, we have, say, for Firm 1

$$\frac{(d^1)'(p_1)}{d^1(p_1)} < \frac{D_1^1(p_1, p_2)}{D^1(p_1, p_2)} < \frac{D_{12}^1(p_1, p_2)}{D_2^1(p_1, p_2)}.$$

The former inequality is the elasticity comparison, while the latter inequality results from log-submodular D^1 by dividing both sides of (15) by $D^1 D_2^1$. Then we have $D_{12}^1 D^1 - D_1^1 D_2^1 < 0$ and $D_{12}^1 d^1 - (d^1)' D_2^1 < 0$. By (13), $\log \Pi^1(p_1, p_2)$ is strictly submodular in (p_1, p_2). (Similar arguments hold for $\log \Pi^2$.)

That is, the log-submodularity of demand and the elasticity condition jointly imply that the Bertrand game is strictly log-submodular, hence every selection of each firm's reaction correspondence is decreasing in the rival firm's price. As this is a two-player game, by taking the reverse order in one firm's action space (e.g., Topkis, 1998), it follows that (at least two extremal) pure-strategy Nash equilibria exist (each with the strategy of one player being minimal and that of the other player maximal).

Next we prove (ii). Note that the elasticity condition, or $\frac{D^i_i}{D^i} > \frac{(d^i)'}{d^i}$ implies that $\log \Pi^i$ has strictly increasing differences in $(p_i; \phi)$ via (14). Hence an increase of ϕ will cause every selection of each firm's reaction correspondence to shift upward, and consequently at least one firm's (extremal) equilibrium price will increase. Q.E.D.

Proof of Proposition 5. First, we prove (i). Condition (ii) $D^i_j + (p_i - c_i)D^i_{ij} > 0$ implies that $\Pi^i(p_1, p_2)$ is strictly supermodular in (p_1, p_2) via (12). Hence every selection of each firm's reaction correspondence is increasing in rival's price. So the Bertrand game is supermodular, and it follows that a maximal and a minimal pure-strategy Nash equilibria exist (e.g., Topkis, 1998).

Next we prove (ii). Following the same arguments given in the proof of Proposition 1, one can show that the log-supermodularity of D^i implies that $\log \Pi^i$ has strictly decreasing differences in $(p_i; \phi)$. Hence an increase of ϕ will cause every selection of each firm's reaction correspondence to shift downward. By Theorem 6 in Milgrom and Roberts (1990), both firms' prices in the extremal equilibria will decrease. Q.E.D.

Proof of Proposition 7. First, we prove (i). Given log-submodular D^1, divide both sides of (15) by $D^1 D^1_2$ and noting $D^1_2 > 0$, we have

$$\frac{(d^1)'(p_1)}{d^1(p_1)} > \frac{D^1_1(p_1, p_2)}{D^1(p_1, p_2)} > \frac{D^1_{12}(p_1, p_2)}{D^1_2(p_1, p_2)},$$

while the former inequality is implied by condition (ii). Then it follows that both terms in (13) are negative, and thus $\log \Pi^1(p_1, p_2)$ is strictly submodular in (p_1, p_2). (Similar arguments hold for Firm 2.) Consequently, every selection of each firm's reaction correspondence is decreasing in rival's price. Here, the log-submodularity of D^i implies that the game is strictly log-submodular. As this is a two-player game, by taking the reverse order in one firm's action space (e.g., Topkis, 1998), it follows that (at least two extremal) pure-strategy Nash equilibria exist (each with the strategy of one player being minimal and that of the other player maximal).

Next we prove (ii). The elasticity condition (ii) implies that $\log \Pi^i$ has strictly decreasing differences in $(p_i; \phi)$ by (14). Therefore, an increase of ϕ will cause every selection of each firm's reaction correspondence to shift downward. It follows that at least one firm's extremal equilibrium prices will fall. Q.E.D.

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