Critical Mass and Campaign Success:
A Behavioral Model of Reward-Based Crowdfunding

Jingwen Tian

Department of Economics, The University of Iowa. gin-tian@uiowa.edu.

Abstract

We set out to solve the bimodal puzzle regarding campaigns' outcomes in reward-based crowdfunding: Most campaigns either fail with little fund raised or succeed by a small margin. The proposed analytical model consists of a sophisticated entrepreneur and boundedly rational buyers whose decisions are based on a meaningful behavior rule associated with network externalities and time effects. Buyers are assumed to bear hassle costs for pledging and thus only pledge when the campaign has progressed well by their arrival time. The model suggests that the campaign's success hinges on reaching a critical mass at an early stage, with the bimodal funding outcomes neatly predicted by the closed-form solution. The model also yields herding dynamics which is argued to be (bounded) rational and efficient. Rich economic and managerial implications about the best timing to utilize the social network, the caveats in designing discriminatory prices, the (stricter) quality requirement due to buyers' pledging failure risk for campaigns to succeed in crowdfunding, and on targeted marketing, campaign monitoring, re-campaigning decision... are drawn from the model, giving sights for the entrepreneur to better steer strategy and achieve success in crowdfunding.

Keywords: Behavioral crowdfunding, critical mass, bounded rationality, network effects, rational herding

1 Introduction

With the progressing of the Information Age and the development of Internet Technology, crowdfunding emerges as a new way in which firms interact with outside stakeholders (Allon and Babich, 2020). Utilized mostly by startups as a new capital source (Agrawal et al., 2014; Belleflamme et al., 2015), a powerful marketing device (Burtch et al., 2013; Mollick, 2014), and sometimes a demand revelation tool (Chemla and Tinn, 2019), crowdfunding provides rich research opportunities in the fields of marketing and operations management (Allon and Babich, 2020). Despite a rapidly growing empirical literature on this new topic, the theory counterpart is relatively limited, with a handful of models approaching different aspects of crowdfunding: comparison between different fundraising forms (Belleflamme et al., 2014), screening and moral hazard (Strausz, 2017), funding dynamics and the role of donors (Deb et al., 2019), etc.

Among other crowdfunding forms, reward-based crowdfunding features an All-or-Nothing provision point policy. A funding goal needs to be pre-specified and satisfied before the pre-determined deadline, in which case the entrepreneur receives all funds (less a platform fee) and starts production. Failing to reach the goal results in full refunds to buyers, and thus nothing to the entrepreneur. Although some crowdfunding platforms (e.g., Indiegogo) provide a Keep-it-All option which allows the entrepreneur to get all collected funds without achieving the goal, All-or-Nothing is hitherto the dominant form imposed by the biggest such platform, Kickstarter.

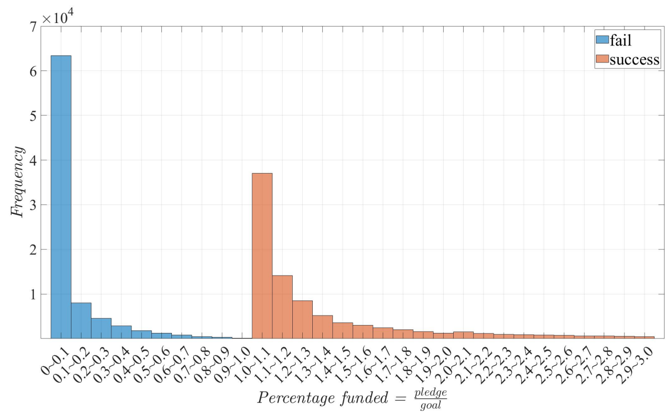


Figure 1: Most campaigns either fail with little funds collected or succeed by a small margin

Note: The data contains 184,828 Kickstarter projects from April 2009 to June 2020. A thin tail above 3.0 is not displayed. Data source: Web Robots.

In a macro view, the fundraising dynamics is known to exhibit several noteworthy features. First, despite some well-known remarkable successes (e.g., Pebble Watch), shown in Figure 1, most campaigns fall in two areas regarding their funding outcomes, ending up either barely funded (0-10% funded) or just adequately funded (100%-110%) (Alaei et al., 2016; Mollick, 2014). Second, the entrepreneur's own social network seems to play an important role in contributing (Agrawal et al., 2014; Mollick, 2014) especially in the early funding stage (Agrawal et al., 2011). Third, early-stage contributions are positively related to the campaign's success probability (Gao et al., 2019). Specifically, the funding outcome seems to hinge on reaching a critical mass early on after the campaign starts, while failing to do so may make buyers' funding propensity drop dramatically and may lead to a potential failure (Li et al., 2020). Fourth, evidence of rational herding is found on many crowdfunding platforms (Astebro et al., 2019; Wei et al., 2021; Zhang and Liu, 2012). Understanding the cause and mechanism underlying these phenomena is crucial for any entrepreneur who wants to pursue success on these new and influential platforms.

This paper sets out to solve the *two-spike puzzle*, or the bimodal pattern shown in Figure 1: Why do most campaigns either fail with little funds collected or succeed by a small margin? Following the literature of behavioral industrial organization wherein rational firms often optimize against a group of myopic and sometimes naive buyers (e.g., Gabaix and Laibson, 2006), we propose an analytical model with a sophisticated entrepreneur and a crowd of boundedly rational buyers whose decisions are based on a meaningful behavioral rule associated with network externalities and time effects (Li et al., 2020). We show that buyers' hassle costs coupled with the behavioral pledging rule are sufficient to explain the two spikes and other afore-mentioned crowdfunding features.

To model buyer's pledging decision, the major buyer-side risk considered is *pledging failure risk*: If the campaign fails, buyers get refunds but lose all hassle costs. Hassle costs capture the fact that: After pledging, some buyers may actively engage in monitoring the campaign, thus may suffer from feelings of loss when the campaign fails. Buyers also bear the opportunity cost of committing the money and forgoing the outside options such as bank deposit (Chakraborty et al., 2021). These hassle costs hinder buyers from pledging to badly performing campaigns that seem to fail. While the theory literature abounds in discussions of other buyer-side risks related to asymmetric information or moral hazard, this elementary pledging failure risk, which is also the key risk associated with crowdfunding's special All-or-Nothing policy, remains less than fully addressed.

Taking a step back, among those theory studies that did bring up the pledging failure risk (e.g., Chakraborty et al., 2021; Deb et al., 2019), buyers are often equipped with sophisticated knowledge of the market demand, the arriving pattern and behavior of other players, and update beliefs by Bayes' rule. We point out that the plausibility of this perfect rationality and the notion of game theoretic equilibrium are in doubt in the crowdfunding context, as buyers hardly know all these market characteristics or can achieve perfect coordination. Instead, simple rules of updating information may be more plausible than full Bayesian updating (Loch and Wu, 2007) in the setting. In fact, perfect rationality does not always prevail due to unavailability of information (as in our model), tendency to follow routinized behavior (Rosenthal, 1993), costly gathering and manipulation of information (Radner, 1996), to list a few. In contrast, we let buyers be boundedly rational and make decisions based on publicly observable information via a behavioral rule associated with network externalities and time effects, two factors found to be relevant in Li et al. (2020).

This paper contributes to the crowdfunding literature in two aspects, namely, to explain and to enlighten. First, the proposed behavior model explains the afore-mentioned crowdfunding features theoretically. It predicts two high-probability events, namely barely-funded failures and small-margin successes, and demonstrates that sufficient demand plus reaching a critical mass early on are two major forces for campaign success. Though these two forces have been investigated by existing literature empirically(e.g., Li et al., 2020), the underlying mechanism, in particular, how different types of buyers account for campaign's success or failure, is yet to be clarified. This paper adds to the literature by identifying that early support (when) from dedicated buyers (who) is one crucial factor driving good campaigns to success, with sufficient common buyers (or, total demand) being another prerequisite. So failures may result from either bad luck or bad quality. The model also underpins herding in crowdfunding, another empirically confirmed phenomenon, and holds that such herding is both rational and (with high probability to be) efficient.[1]

Second, based on the behavioral rule, we derive rich economic and managerial implications. In the extension, we show that early support from social network is more effective than last-minute support, and alarm that discriminatory prices need to be designed with caveats. Economically, we argue that qualitative projects eligible to initiate production in traditional markets may still be doomed to failure in crowdfunding due to buyer's pledging failure risk, thus crowdfunding's fruits may only be in the reach of sufficiently high-quality projects. Managerially, the model suggests for efficient marketing, the entrepreneur should target the right crowd (dedicated buyers) at the right time (early stage); it predicts funding dynamics that can be monitored, and potentially corrected by the entrepreneur when the sign of failure shows up; it also provides clues for the entrepreneur to distinguish back-quality project from back luck, thus advising in whether to re-campaign or not.

The rest of the paper is arranged as follows. Section 2 provides a crowdfunding theory literature review. We present the baseline model in Section 3, extensive discussions in Section 4. Economic and managerial implications are summarized in Section 5. Section 6 concludes the paper.

2 Literature Review

There is a rapidly growing empirical literature for crowdfunding, but rather limited on the theory side. For the latter, several dynamics models are proposed to address some, but different from this paper, *bimodal* patterns of crowdfunding. Alaei et al. (2016) utilized a novel stochastic process, namely Anticipating Random Walk, to show that crowdfunding projects either succeed with high probability or fail with certainty. Deb et al. (2019) studied a dynamic game between buyers and a donor who wants to nudge the campaign towards success with a minimal budget, and proved donations can have spikes only at the start or the deadline, a prediction consistent with their high-frequency data collected from Kickstarter. Ellman and Fabi (2019) explained the U-shape bidding

[1]All insights are illustrated in a simple baseline setting, while the supplement material includes robustness checks with regard to demand uncertainty, strategic waiting, random arriving time, and greater buyer heterogeneity. Indeed, the two-spike feature emerges as such a strong, easily derived outcome following the behavioral rule that it naturally extends to small, realistic perturbations of the basic assumptions.

profile in a dynamic game with endogenous inspection costs. In addition, static models extend the scope to other crowdfunding issues, including asymmetric information (about project quality) and signaling (Chakraborty and Swinney, 2021), pre-ordering (reward-based) versus profit-sharing (equity) crowdfunding (Belleflamme et al., 2014), crowdfunding as a demand learning tool (Chemla and Tinn, 2019), optimal product line design (Hu et al., 2015), optimal campaign design for the price and duration (Zhang et al., 2017) and for the funding form (Chang, 2020), crowdfunding interacting with traditional financing methods (Belleflamme et al., 2010; Babich et al., 2020), etc.

Though moral hazard is a potential risk in crowdfunding, statistics suggest that blatant fraud is actually rare except for frequent delivery delays (Mollick, 2014). Strausz (2017) showed that an information-restricted and payout-deferred mechanism can help prevent moral hazard, as buyers can defer their payout to the after-market once the campaign succeeds. Chang (2020) echoed the role of retail market in deterring moral hazard.

Herding in crowdfunding is studied by several empirical works. Astebro et al. (2019) and Zhang and Liu (2012) found evidence of herding in equity and debt crowdfunding markets respectively, both agreeing with the rational herding hypothesis, i.e., buyers do not naively mimic their peer backers' behavior but actively engage in information aggregation and observational learning. Wei et al. (2021) documented herding in reward-based crowdfunding with a pre-funding stage, as a way for uninformed buyers to avoid costly information acquisition. Our model tries to underpin the (bounded) rationality and efficiency of herding for reward-based crowdfunding.

The most relevant theory study to ours is Chakraborty et al. (2021) (CS). Both studies distinguish buyers by their non-valuation types, address the risk of campaign failure, and extend the model to menu designs. However, CS focused on strategic waiting buyers who may forget to return. The provision of early-bird discounts is thus explained as to induce no-delay equilibrium that entails higher revenue. In contrast, strategic waiting is not the focus of this paper (though its impact will be analyzed in the supplement material), where the behavioral rule associated with pledging failure risk weigh in instead. Another difference lies in the assumption of buyers' knowledge. CS took a conventional path and assumed buyers have a common knowledge of the underlying (uniform) demand and update beliefs by Bayes rule. In contrast, we impose no publicly inaccessible knowledge on buyers and resort to behavioral rule to explain the stylized facts.

The behavior rule we specify is intimately related to the empirical study of Li et al. (2020). They found buyers' perceived campaign success probability is positively related to network externalities (percentage funded prior to the buyer's arrival), negatively related to the time effects (elapsed time). While they employed a hazards model to fit with data, as a theory counterpart, we adopt a simple linear relationship for tractability. Our model echos some of their empirical findings, such as high-value projects may fail if not achieving a critical mass early on due to unfavorable random arrivals (bad luck), and that early contribution (promotion) is more efficient than last-minute support.

3 The Model

In this section, we introduce the baseline behavioral model, predict that a campaign either fails with little funds raised or succeeds by a small margin, and relate these two funding outcomes to two high-probability events: the pledging cascade (or herding) starts early, or never starts.[2] To conserve space, all proofs are moved to the supplement material (hereafter referred to as the **Sup**).

3.1 Model setup and assumptions

3.1.1 Assumptions about the entrepreneur

An entrepreneur wants to finance the production of an innovative consumption good via crowdfunding. The production involves a fixed cost of K and a unit marginal cost $c \in (0, 1)$. To initiate a crowdfunding campaign, among other preparations, the entrepreneur needs to design three economically meaningful factors: the monetary goal, the campaign's duration, and the product price.[3] First, following Belleflamme et al. (2014) and Zhang et al. (2017), we assume that the entrepreneur wishes to finance exactly the amount of the production cost, thus the monetary goal is $K + cG$, where G is the number of backers the entrepreneur wants to capture (thereafter referred to as *the backers' goal*).[4] Second, crowdfunding platforms usually restrict the campaign's duration within a certain length (maximally 60 days for Kickstarter). For simplification, we assume the entrepreneur adopts an exogenously determined duration, either the maximal length for the longest public exposure, or a specific deadline constrained by some production timeliness requirement (e.g., the mascot design for a festival).[5] Lastly, notice the entrepreneur's pricing decision is equivalent to deciding the backers' goal G, since with a single price p, we have $p = \frac{K+cG}{G}$. Hence, the backers' goal (or price) is the focal decision variable of the model.[6]

[2]This simple, tractable model delivers the basic intuitions and insights in a succinct manner, despite its highly-stylized nature, since a lot of realistic factors such as demand uncertainty, buyers' heterogeneity (at a higher degree) and arrival patterns are oversimplified to this end. These factors are addressed in the supplement material using a combination of theoretic reasoning and numerical examples.

[3]For now we restrict attention to a single-pricing scenario, while price discrimination is discussed in Section 4.

[4]Note each backer is assumed to need at most one unit of the good.

[5]In fact, strategic duration design is meaningful in this context only when used as a device to signal the campaign's quality (shorter duration may reflect the confidence of capturing all needed backers in a timely manner), or when the temporal discounting of payoffs is taken into account (as in Zhang et al., 2017), both not being the case in our model.

[6]Hu et al. (2015) made an extensive discussion on the crowdfunding menu design. Combining heterogeneous product valuation and random arriving, they derived the optimal pricing strategy under different scenarios. However, they adopted a two-period model wherein buyers have no transaction cost associated with their pledging decisions, and focused on the coordination between different types of buyers. In contrast, this paper explores how hassle costs

The entrepreneur's objective is to maximize the campaign's ex-ante success probability (following Alaei et al., 2016). Indeed, being a success-probability maximizer instead of a profit maximizer is not at odds in the crowdfunding context. Besides financing, entrepreneurs frequently use crowdfunding as a "proof-of-concept" vehicle (Alaei et al., 2016) for various purposes, such as revealing the aggregate demand (Chemla and Tinn, 2019), marketing a new product (Burtch et al., 2013; Mollick, 2014), attracting further funding from venture capital or business angles (a famous example is Pebble Watch in Mollick, 2014), etc. As success in crowdfunding means more than fulfilling the funding needs, the objective is indeed not restricted to revenue (Allon and Babich, 2020).

To fold up, the entrepreneur's problem is to choose an optimal backers' goal G (or price p) to maximize the probability that the campaign achieves its monetary goal of $K + cG$ before the specified deadline. As will become clear later, the backers' goal instead of price as the decision variable better captures the underlying trade-off, thus is mostly invoked in the following analysis.

3.1.2 Assumptions about the buyers

After the campaign is launched, buyers arrive sequentially and make a pledging decision (*buy* or *leave*) once they observe all public information of the campaign. Some crowdfunding platforms facilitate the option of *strategic waiting* for hesitating backers, allowing them to return to the campaign later, mostly until the end of the campaign, as a way to reduce the pledging risk in case the campaign fails (Chakraborty et al., 2021). Therefore, if the buyer is not impressed by the current campaign progress (i.e., she decides to *leave* at the moment), she might come back at the end of the campaign to make the pledging decision once again. On the other hand, we posit that if the buyer is already willing to pledge given the current progress, she would refrain from waiting and *buy* right away.[7] Under these assumptions, the pledging dynamic should remain the same as one without any wait-and-see option, except for the possibility that some of the leaving buyers may return at the end to decide again. Therefore, we start the model without considering the wait-and-see option and leave such discussion to the Sup.[8]

Buyers are, to this end, homogeneous in their product valuation v, normalized to 1, but heterogeneous in their *hassle cost* $\in \{0, h\}$. Indeed, hassle cost should be the key ingredient in such context where full refund is guaranteed, and where asymmetric information, moral hazard are abstracted out. It includes the efforts of monitoring the campaign, the opportunity cost of com-

enter buyers' decision-making procedure via a behavioral channel and thereby affect the campaign's outcome.

[7] A possible justification is that the option of strategic waiting may be associated with additional costs (time, sign-up, etc.), thus discouraging such buyers to delay their pledge.

[8] In addition, strategic waiting involves high degree of unpredictableness, such as how many leaving buyers would choose to come back, and how they might behave/coordinate when they return at the deadline. We thus posit that the entrepreneur does not further take buyers' option of strategic waiting into account for campaign design.

mitting the money and forgoing the outside options (Chakraborty et al., 2021; Li et al., 2020). In other words, hassle costs lie behind backers' pledging risk in fear of campaign failure. For now, we adopt a dichotomy of buyer's type regarding their heterogeneous hassle costs.[9] **Common buyers** represent a conservative type who has a positive hassle cost h. They are concerned whether the campaign can end up successfully and only pledge when such odds are high. **Dedicated buyers** have no hassle cost, willing to pledge as long as they like the project and find the price reasonable.

The dedicated buyers' zero hassle cost may be explained from at least three different angles. First, such buyers may have pledged out of impulse (as documented in Hausman, 2000) and have no plan to engage in tracking the campaign afterwards. Second, they may gain utilities from the supporting action itself.[10] Third, these backers may receive "community benefits" (Belleflamme et al., 2014) as they participate in developing and customizing the product, and as they establish positive relationships with the entrepreneur via customer engagement (Pansari and Kumar, 2017). These additional utilities cancel out the hassle costs. Along the logic, dedicated buyers could be identified as the impulse consumers for a certain product category (e.g., electronics fanatics for a crafted mechanical keyboard), engaging customers who value the interaction in the campaign community, unconditional supporters from the entrepreneur's external social network (friends and family), and the internal social network (Buttice et al., 2017) developed inside the platform.

The dichotomy proposed here—buyers with zero, and positive (which is allowed to exhibit higher heterogeneity), hassle costs—directly relates to buyers' different pledging behaviors in crowdfunding and turns out to be the key in generating the two probability spikes observed in reality.

In the baseline model, buyers' arrivals are highly stylized. T buyers sequentially visit the campaign and make their pledging decisions, among which there are T_d dedicated buyers and $(T - T_d)$ common buyers. So T is the total demand. A crucial assumption underlying our model (also tacit in most of other theoretic models) is that T and T_d must be independent of buyers' pledging decisions.[11] Buyers' arriving order $\tilde{O}$ is a random variable with a typical element $o \in \mathbb{O} \subset \{CB, DB\}^T$ where $\mathbb{O}$ contains $C_T^{T_d}$ elements (cases).[12] Each case $o \in \mathbb{O}$ is assumed to happen with the same probability. For tractability, we also assume buyers arrive at the campaign in equal time intervals, that is, the t^{th} buyer arrives when $\frac{t}{T}$ time has elapsed. This simplified arrival process can

[9] It will become clear that the key momentum driver is the buyers with zero hassle cost (see Subsection 3.2), while the main results are robust to higher degrees of heterogeneity for positive hassle costs (see Sup Section S1.4).

[10] In fact, crowdfunding entrepreneurs are frequently found to engage in supporting peers' projects as a way to reciprocally accumulate social capital (Buttice et al., 2017).

[11] Our model cannot address issues caused by the possibility that better funding progress improves the campaign's public exposure, by the platform's algorithm or by social media reference, thus bringing in more interested buyers.

[12] $C_T^{T_d}$ denotes the combinatorial number regarding T_d-combinations of a set containing T elements. We define $C_T^0 \equiv 1$. CB and DB are short for common and dedicated buyers. o is one realization of their arriving order. An example of o is $(DB, ..., DB, CB, ...CB)$ where the initial T_d arrivals are DBs and the last $(T - T_d)$ arrivals are CBs.

be viewed as a mean approximation of a Poisson process where buyers' arrival rate is fixed.[13]

The utilities conditional on pledging for dedicated and common buyers are, respectively,

$$u_d = (1-p)\phi \text{ and } u_c = (1-p)\phi - hp, \tag{1}$$

where $\phi \in [0,1]$ is the buyer's anticipation of the campaign's success probability. With full-refund policy, the only loss is buyers' hassle costs when campaign fails. As the hassle cost is irrecoverable, buyers only pledge when ϕ is sufficiently high. This pledging failure risk is the key risk associated with crowdfunding's All-or-Nothing policy. Several theory models (e.g., Chakraborty et al., 2021; Deb et al., 2019) have modeled buyers' anticipation of the campaign's success probability, but by equipping buyers with sophisticated market knowledge or publicly unavailable information (e.g., the donor's behavior, the demand's distribution). In contrast, we address the pledging failure risk through a behavioral channel wherein buyers use a meaningful rule of thumb to form such anticipations and base their decisions on publicly available information. An elaboration of the behavioral rule follows shortly. Also, we assume the total hassle cost hp is linear in price, as higher price usually induces greater monitoring efforts or incurs higher opportunity costs.

Normalize the outside option to be of zero utility. Then, dedicated buyers pledge when $(1-p)\phi \geq 0$, or $p \leq 1$. Common buyers pledge when $(1-p)\phi - hp \geq 0$, or

$$\phi \geq \frac{hp}{1-p}. \tag{2}$$

To this end, it is worth asking whether the model's setup is applicable to equity or debt crowdfunding. First, notice that buyers have a known valuation for the reward (product), $v = 1$, which is not the case in equity crowdfunding, where backers get a claim to the business thus the value of pledge depends on the *uncertain* profitability of the business in question. Debt crowdfunding does provide a certain payoff if there is a pre-specified interest rate and no default risk. The difference is that lenders can choose any investment amount at their will (usually above a threshold), which is another variable yet to be modeled, and the number of potential lenders alone cannot pin down the potential funds available. But in the reward-based crowdfunding modeled by this paper, each backer is assumed to want just one unit of the product and make one decision of *buy* or *leave*. Therefore, the model at hand fits the context of reward-based crowdfunding best.

[13]Its robustness to stochastic arrival is checked in the Sup Section S1.3.

3.1.3 The behavioral rule

In this section, we elaborate on the behavioral rule by which buyers form their anticipations of the campaign's success probability ϕ. On this issue, the conventional theory tactics usually involve imposing (different levels of) sophistication on buyers, while the bimodal puzzle shown in Figure 1 seems to keep eluding such analysis. Therefore, our behavioral model may be seen to fill this gap exactly. But it does not mean that this behavioral rule stand opposite of sophistication (or rationality); rather, we argue that buyers have bounded rationality, less than fully due to the costly, if not completely inaccessible, observation and manipulation of information (Radner, 1996).

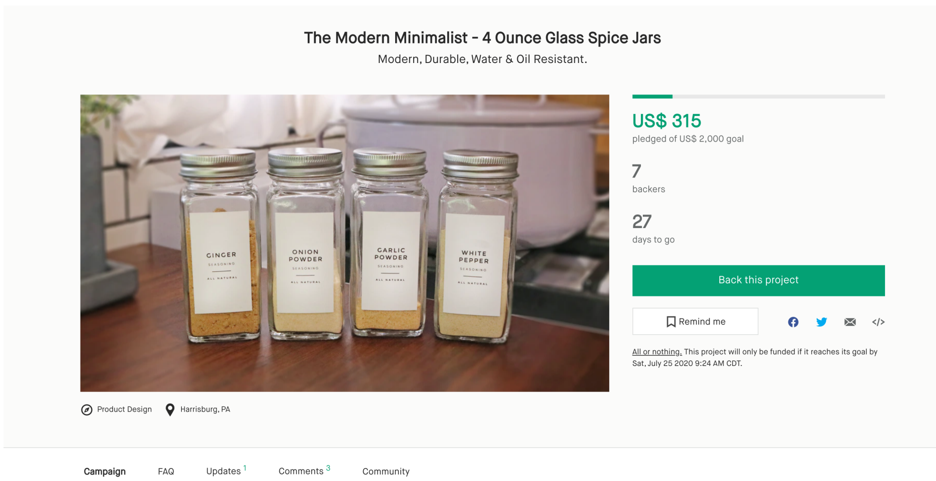


Figure 2: A typical campaign reports its progress on the top of its website.

We start with the publicly available information that can be used by buyers to form their anticipations. Figure 2 displays part of a typical campaign's website: the monetary percentage funded (fund/goal) illustrated by the green-filled bar, then current number of backers (denoted by n), remaining time, inception date and more information on the lower page. We assume buyers rely on two key observable elements to form their ϕ: the percentage funded (fund/goal) and the fraction elapsed time (time elapsed/total duration). Li et al. (2020) empirically proved that backers' perceived success probability increases in the former, identified as positive network externalities, and decrease in the latter, identified as negative time effects.[14]

In our setting, when the t^{th} buyer arrives, $t \in \{1, 2, .., T\}$, the fraction elapsed time is $\frac{t}{T}$ since buyers arrive in equal time intervals. For single-pricing without donation consideration, the percentage funded is $\frac{n}{G}$, where n is the number of pledged backers, $n \in \{0, 1, ..., t-1\}$, and G is the number of backers required for success (the backers' goal). That is, we assume buyers' ϕ increases in $\frac{n}{G}$ and decreases in $\frac{t}{T}$. For tractability, we further assume ϕ is linear in the campaign's *funding*

[14] Li et al. (2020) incorporated a third factor of the campaign's social media exposure, which falls outside our scope.

progress, which is defined to be $\frac{n/G}{t/T}$. Note that we have restricted that no buyer arrives exactly at the inception of the campaign ($t \neq 0$) for $\frac{n/G}{t/T}$ to be well defined.

Definition 1. *The campaign's* ***funding progress*** *is defined to be the ratio of the percentage funded and the fraction elapsed time. That is,* $funding\ progress = \frac{n/G}{t/T}$.

Assumption 1. *The anticipated success probability is a linear function of the funding progress:*

$$\phi(t, n, T, G) = \begin{cases} \rho \frac{n/G}{t/T} & , \text{ if } n + 1 < G \\ 1 & , \text{ if } n + 1 \geq G \end{cases} , \text{ where } \rho \in \left(0, \frac{G}{T}\right]. \tag{3}$$

Here are some clarifications for the functional form of ϕ. First note that $n < t$, since there are at most $(t-1)$ pledges when the t^{th} buyer arrives. When $\frac{n}{t} \to 1$, $\frac{n/G}{t/T} \to \frac{T}{G}$, so for $\phi \leq 1$, the slope ρ, representing how optimistic the buyer is, must satisfy $\rho \leq \frac{G}{T}$. This is a natural requirement for the buyers to perceive a probability lying in $(0, 1)$. To simplify notation, let $\rho = \frac{G}{rT}$ where $r \geq 1$, then $\rho \frac{n/G}{t/T} = \frac{n}{rt}$. Also, if the buyer can complete the campaign by herself, or $n + 1 \geq G$, then $\phi = 1$. The *funding progress* reflects how well the campaign has progressed by the buyer's arrival. For instance, in Figure 2, the campaign has received 315 USD out of a goal of 2,000 USD, whereas 3 out of 30 days (the duration information is displayed on the lower page) have passed. Therefore, $\frac{t}{T} = \frac{3}{30}$ and $\frac{n}{G} = \frac{315}{2000}$, and the *funding progress* $= \frac{15.75\%}{10\%} = 1.575$. Funding is currently ahead of time in the sense that if funds continue to grow at the same speed, the campaign will get 157.5% funded.[15] Without sophisticated knowledge of the market, buyers are posited to use this simple, intuitive linear function as a rule of thumb[16] to estimate the campaign's success probability.

No doubt, without sophisticated understanding of the stochastic fundraising process, buyers can hardly achieve perfect rationality, which, indeed, is absent in many real world situations due to the unavailability of information (as it is here), the tendency to follow routinized behavior (Rosenthal, 1993), the costly gathering and manipulation of information (Radner, 1996), etc. Rather than perfect, this behavioral rule may be seen to reflect bounded rationality of crowdfunding buyers.[17]

Lastly, we claim that the choice of this linear form is mainly motivated by tractability. What lie at the core of the behavior rule, as suggested by Li et al. (2020), are the positive network externalities and negative time effects.[18] Despite its simplicity, this rule will be shown to accord very well with the stylized facts in crowdfunding.

[15] But buyers do not assume that the speed would continue, otherwise it should be $\phi = 1$ if $\frac{n/G}{t/T} \geq 1$.

[16] Rosenthal (1993) gave examples in varying contexts where people seem to follow certain rules of thumb instead of doing case-by-case optimizations to make their decisions.

[17] We show in Sup S2.4 that such a linear form will emerge if buyers make a simple yet rational update about the campaign's fundraising speed.

[18] The simulation result in Li et al. (2020) also seems to suggest the plausibility of such a linear relation in some contexts (cf. Footnote 20).

3.2 The buyer's decision

As discussed above, a dedicated buyer makes her decision independently of ϕ due to her zero hassle cost, pledging as long as $p < 1$. Note that $p < 1$ is always required since v is normalized to 1 for all buyers.[19] Henceforth, we assume $p < 1$, thus dedicated buyers always pledge upon arrival. Since $p = \frac{K+cG}{G}$, it follows that a natural restriction for G is $G > \frac{K}{1-c}$.

With a positive hassle cost $h > 0$, common buyers only pledge when the campaign's current progress seems promising. Combining (2) and (3), we know that the t^{th} buyer, if she turns out to be a common buyer, pledges if and only if $n + 1 \geq G$ (succeeding immediately), or

$$\rho \frac{n/G}{t/T} \geq \frac{hp}{1-p}, \tag{4}$$

where all variables are *perfectly observable* to the buyer as she knows her own ρ and h. Replacing $\rho\frac{n/G}{t/T}$ by $\frac{n}{rt}$ and p by $p = \frac{K+cG}{G}$, (4) is equivalent to

$$\frac{n}{t} \geq \frac{rhp}{1-p} = \frac{rh}{(\frac{G}{K+cG} - 1)} \equiv k \equiv k(G), \tag{5}$$

So common buyers pledge (if the campaign does not succeed immediately) if $\frac{n}{t} \geq k$, in other words, if sufficient backers (n) have revealed themselves before this buyer's arrival (t).

Definition 2. *Let $k \equiv k(G) \equiv \frac{rh}{(\frac{G}{K+cG}-1)}$ be the* ***critical mass*** *that determines whether or not the current progress can convince a common buyer to pledge.*[20]

Define the right hand side of (5) as the common buyers' *critical mass* k. $k > 0$ since $p < 1$. Recall that $\frac{n}{t} < 1$, so if the entrepreneur wants to capture any common buyer, the critical mass k must be set (via the choice of G) below 1. We thus restrict attention to $k \in (0, 1)$.

Lemma 1. *For any common buyer to pledge, the critical mass k must be set below 1.*

Buyers make their decisions according to (4), while the equivalent criterion (5) is the key to our analysis. An immediate application is: $\frac{0}{1} < k \in (0, 1)$. If the first arrival is a common buyer,

[19] In fact, $p = 1$ is also feasible, in which case common buyers would never pledge because the right hand side of (4) becomes infinity. We thus ignore this case in our context.

[20] Our definition of the critical mass is closely related to, but different from the time-varying critical threshold defined in Li et al. (2020). To see the connection, rewrite (4) as $\frac{n}{G} \geq \frac{t}{T}\frac{hp}{\rho(1-p)}$, the right hand side is analogous to the time-varying critical threshold defined in Li et al. (2020), which depends linearly on the fraction elapsed time $\frac{t}{T}$ in our model. The linear relation is consistent with the empirical simulation for average-valuation projects in Figure 7(a) of Li et al. (2020), except for an intercept that is missing here. The intercept and curvature of the time-varying threshold in their setting seem to vary with different product valuations, which is beyond the scope of our model as we focus on one typical campaign and normalize its product valuation to 1.

overwhelmed by uncertainty, the buyer would not pledge (nor can she perform more strategically because she does not know she is the first). In fact, no initial *common* buyer will pledge until sufficient *dedicated* buyers have revealed themselves since $\frac{0}{t} < k$ for all t. This highly abstracted, baseline model thus predicts that the early-stage capital comes solely from dedicated buyers. Adding random arrival or other realistic perturbation allows early funds to come from common buyers as well, but only in small fractions. Somewhat agreeing, Agrawal et al. (2011) found that friends and family as an important source of dedicated buyers are disproportionately active in the beginning, while other buyers gradually increase their investing propensity as the capital accumulates.

Once sufficient dedicated buyers have arrived and pledged, the first common buyer observing $\frac{n}{t} \geq k$ (the critical mass being reached) will be convinced to pledge. So does the next buyer and buyers thereafter, since $\frac{n+1}{t+1} > \frac{n}{t} \geq k$. Thus, a *pledging cascade*, or herding,[21] happens after $\frac{n}{t}$ first crosses the threshold k. This is summarized in Lemma 2.

Definition 3. *The* ***pledging cascade****, or* ***herding****, refers to the situation that once the first common buyer decides to pledge, all subsequent buyers will pledge regardless of their types.*

Lemma 2. *Only dedicated buyers would pledge before the campaign reaches the critical mass k, after which a pledging cascade (herding) starts.*

Given some $k < 1$, the campaign's outcome hinges on how fast it collects funds from dedicated buyers to reach the critical mass. In the baseline setting, all common buyers arriving before reaching the critical mass will be lost by the campaign, while those arriving afterwards can be captured. If the critical mass is never reached, the funds only accrue at the occasional arrivals of dedicated buyers, leading to a potential failure if dedicated buyers alone cannot cover the target amount. Thus the model predicts a pattern of steady growth in funds after a campaign reaches its critical mass, though undoubtedly, the real funding dynamics are affected by other relevant factors and exhibit higher complexity, particularly exhibiting two funding spikes (U-shape) at the beginning and the end of the campaign (e.g., Figure 3 in Li et al., 2020). We address these factors and the formation of the U-shape funding dynamics in the Sup Section S1.3.

Folding back to the model, if the campaign has T_d dedicated buyers out of T buyers, its outcome is fully pinned down by the critical mass k and the buyers' arriving order (recall it is the random variable $\tilde{O}$ with a typical element o). Let the random variable $\tilde{N}$ denote the number of *captured buyers*, with a typical element $N \in \{T_d, T_d + 1, ..., T\}$: A campaign receives at least T_d pledges from the dedicated buyers (since $p < 1$) and at most T pledges. For the convenience of analysis,

[21] Pledging cascade and herding are used interchangeably in this paper. Though they refer to the same phenomenon, the former stresses its relation to the bounded rational pledging rule proposed in the paper, while the latter stresses the behavioral aspect of the phenomenon.

alternatively let random variable $\tilde{L} = T - \tilde{N}$ denote the number of *lost buyers*, with a typical element $l \in \{0, 1, ..., T - T_d\}$: A campaign loses at most all the common buyers. There are in total $C_T^{T_d}$ arriving orders, each happening with the same probability and corresponding to a unique *lost buyer* number l. For instance, if $o = (CB, ..., CB, DB, ...DB)$ where all common buyers arrive in the front, then the campaign loses all of them and $\tilde{L} = T - T_d$ regardless of k.

The entrepreneur affects $k = \frac{rh}{(\frac{G}{K+cG} - 1)}$ via setting the backers' goal G, where the former is clearly a decreasing function of the latter, thus sometimes denoted by $k(G)$. While a lower G is easier to achieve, it raises k and makes it harder to capture common buyers. Eventually, the campaign succeeds if $T - \tilde{L} \geq G$ and fails otherwise. Given some k, the derivation of $\tilde{L}$'s probability mass function (p.m.f) is purely technical and can be achieved using numerical methods,[22] while due to the structural nature of (5), namely n and t are both integers, an analytical expression is only attainable for two regular forms of k, $k \in \mathbb{K} = \{\frac{1}{x}, \frac{y-1}{y} : x, y \in \{2, 3, 4, ...\}\}$. In the following, we first present the closed-form result for such k, discuss its implications, and then formalize the impact of k on $\tilde{L}$ with a Proposition about the first-order stochastic dominance relationship among arbitrary $k \in (0, 1)$, which also indicates that these special-form k's serve as good approximations for arbitrary $k \in (0, 1)$ in estimating $\tilde{L}$'s distribution.

3.2.1 Campaign's outcome with two regular forms of k

Given T, T_d and $k(G)$, the key to characterize the lost buyers $\tilde{L}$'s p.m.f is to figure out a way to place two types of buyers properly such that the pledging cascade starts right at the $(l + 1)^{th}$ common buyer, thus the campaign loses exactly l buyers. For example, if $k = \frac{1}{2}$, the campaign does not lose any buyer (i.e., $l = 0$) if and only if the first arrival is a dedicated buyer, so the second buyer facing $\frac{n}{t} = \frac{1}{2} \geq k$ always pledges following which the cascade starts. The complexity of the problem grows quickly in l, and the methodology is fully presented in Sup Section S2.

Also note that the interesting scenario is $T_d < G$: Since the campaign always captures all dedicated buyers, this means the campaign's success probability is less than 1.[23] Because the lowest feasible G the entrepreneur can choose is $\frac{K}{1-c}$ where p reaches its upper bound 1, so throughout the paper we only consider the scenario $T_d < \frac{K}{1-c}$, that is, there is no way the entrepreneur can assure 100% success, and she has to capture common buyers to reach the goal.

Assumption 2. $T_d < \frac{K}{1-c}$, *thus the entrepreneur has to capture common buyers to reach the goal.*

[22] Explanations for the theoretic/numerical methods to calculate $\tilde{L}$'s p.m.f for $k \in (0, 1)$ are included in the Sup.

[23] When $T_d \geq G$, the campaign succeeds for sure. $\tilde{L}$'s p.m.f may be of value in its informativeness of *how many* buyers the succeeding campaign is able to capture. Such discussion falls outside the scope of the paper. Nevertheless, we provide $\tilde{L}$'s p.m.f for the case $T_d \geq G$ in the Sup (S2.1, S2.2, and Figure S6) for completeness.

For notation convenience, when $k = \frac{1}{x}$, divide l by $(x-1)$, and let $(n-1)$, $(m-1)$ denote the quotient and remainder.[24] The next Propositions fully characterize the lost buyers' p.m.f for $k = \frac{1}{2}, \frac{1}{3}, ...$ or $k = \frac{2}{3}, \frac{3}{4}, ...$

Proposition 1. *Suppose $k = \frac{1}{x}$ for some $x \in \{2, 3, 4, ...\}$. Then,*

(1) For all $0 \le l \le min(T_d(x-1)-1, T-T_d-1)$, $\text{Prob}(\tilde{L} = l) = \frac{mC^{n-1}_{x(n-1)+(m-1)} C^{T_d-n}_{T-(x(n-1)+m)}}{((x-1)(n-1)+m)C^{T_d}_T}$.

(2) For all $T_d(x-1)-1 < l \le T-T_d-1$ (if well defined), $\text{Prob}(\tilde{L} = l) = 0$.

(3) $\text{Prob}(\tilde{L} = T-T_d) = 1 - \sum_{l=0}^{T-T_d-1} \text{Prob}(\tilde{L} = l)$.

Proposition 2. *Suppose $k = \frac{y-1}{y}$ for some $y \in \{2, 3, 4, ...\}$, and assume $y-1 \le T_d$.*[25] *Then,*

(1) For all $0 \le l \le min(\frac{T_d}{y-1}-1, T-T_d-1)$,$\text{Prob}(\tilde{L} = l) = \frac{C^{l}_{(l+1)y-2} C^{T_d-(l+1)(y-1)}_{T-((l+1)y-1)}}{(l+1)C^{T_d}_T}$.

(2) For all $\frac{T}{y-1}-1 < l \le T-T_d-1$ (if well defined), $\text{Prob}(\tilde{L} = l) = 0$.

(3) $\text{Prob}(\tilde{L} = T-T_d) = 1 - \sum_{l=0}^{T-T_d-1} \text{Prob}(\tilde{L} = l)$.

The somewhat bulky expressions reflect the technical complexity of the problem. While we encourage readers to focus on the implications behind the mathematical expressions, several clarifications might be valuable to help digesting the math expressions.

First, the two propositions are equivalent when $x = 2$ or $y = 2$ as both represent $k = \frac{1}{2}$. Second, if the critical mass k is too high, that is, x too small (or y too big) to the extent that the interval in (2) of each proposition is well defined, then $\text{Prob}(\tilde{L} = l) = 0$ for these big l's. Intuitively, if the critical mass k is too high relative to $\frac{T_d}{T}$, falling short of dedicated buyers, the campaign cannot reach the critical mass after having lost l common buyers, thus resulting in losing all common buyers.[26] Last, in (3) the probability of losing all CBs (critical mass never reached and cascade never happens) is 1 minus the sum of other probabilities (cascade happens at some point).

3.2.2 Implications of the Probability Mass Function

Propositions 1 and 2 have three major implications: There tend to be two probability spikes in the p.m.f.; Herding almost always (probabilistically) leads to campaign's success; $\tilde{L}$'s distribution is heavily affected by $\frac{T_d}{T}$ while almost invariant to the scale to T_d and T.

A. Two probability spikes

[24] For example, if $l = 5$ and $x = 4$, then $n = 2$ and $m = 3$.

[25] If $y - 1 > T_d$, $\text{Prob}(\tilde{L} = T - T_d) = 1$.

[26] For example, if the campaign has 11 buyers, 8 CBs and 3 DBs, $k = \frac{1}{3}$, then it is not possible to lose exactly 6 buyers. To lose 6 buyers, 6 CBs need to be placed in the front, accompanied by n DBs in a certain way, for the $(6+n+1)^{th}$ buyer to observe $\frac{n}{7+n} \ge \frac{1}{3}$ and start the cascade. The smallest such n is 4 but there are only 3 DBs. $\tilde{L} = 7$ is also not possible for the same reason. But it is possible to lose all CBs, simply by not letting the cascade start (e.g., placing 8 CBs in the front).

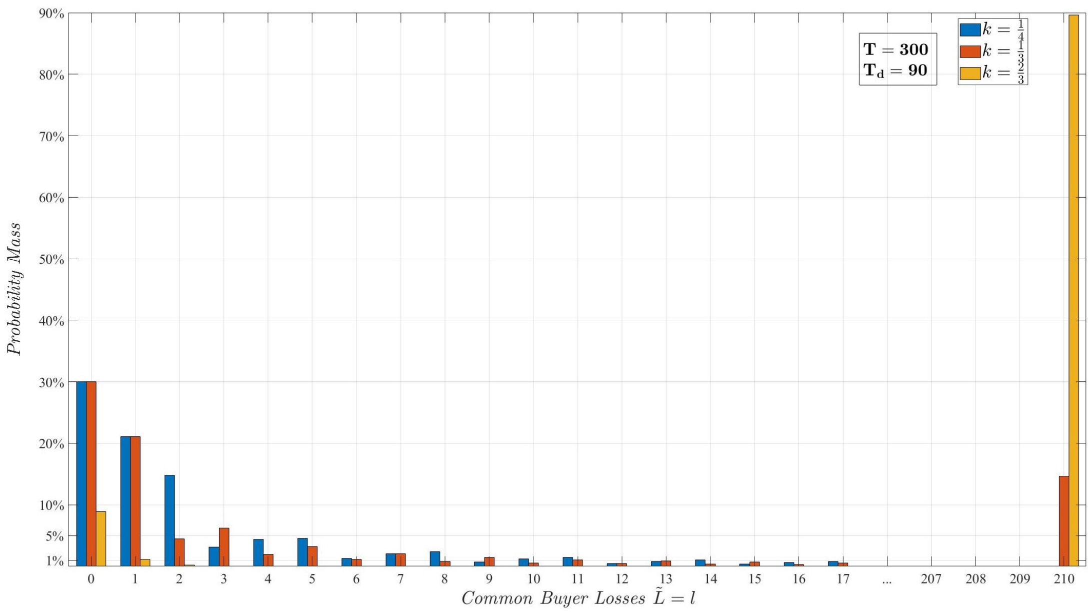


Figure 3: $\tilde{L}$'s p.m.f for varying critical masses

Note: This figure displays $\tilde{L}$'s p.m.f for $k = \frac{1}{4}, \frac{1}{3}$ and $\frac{2}{3}$. All omitted probabilities are less than 0.01. Because the p.m.f decreases quickly to a negligible level as l increases, there tend to be two spikes around $\tilde{L} = 0$ and $T - T_d$. Also, as the critical mass k increases, $\tilde{L}$'s p.m.f becomes worse in the sense that the campaign tends to lose more buyers. Consequently, when k is small (e.g., $k = \frac{1}{4}$), the mass at $\tilde{L} = T - T_d$ may not exist—campaigns with a very low critical mass are likely to capture almost all buyers.

As shown in Figure 3, there tend to be *two probability spikes* in $\tilde{L}$'s p.m.f: The campaign either captures almost all buyers ($\tilde{L} \approx 0$, referred to as the *all-captured spike*) or captures only the dedicated buyers ($\tilde{L} = T - T_d$, referred to as the *dedicated-only spike*). Mathematically proved in the Sup, $\tilde{L}$'s p.m.f will quickly (may not monotonically) decrease to a negligible level as l increases, giving rise to the all-captured spike, while such rapid decrease naturally leads to the dedicated-only spike according to (3) of each proposition. The two spikes correspond to two high-probability events: the cascade starts very early (so $\tilde{L}$ is low) or never starts (i.e., $\tilde{L} = T - T_d$). If the campaign quickly reaches the critical mass in the beginning, it will capture all subsequent buyers no matter how they arrive in order (which results in high probability). Otherwise, if the campaign fails to attain the critical mass for a long period of time, that is when many common buyers have arrived too early to see an encouraging progress (which also happens with high probability if the critical mass is hard to achieve), the momentum ebbs, dedicated buyers hardly able to compensate for the negative time effects, and the campaign will lose all subsequent common buyers.

This pattern is reminiscent of the crowdfunding's two-spike feature in Figure 1—most campaigns either fail with little funds collected or succeed by a small margin—but three steps away. First, T_d must to be very low that the failing campaign collects barely nothing. Due to the exogenuity of T_d

in the model, the barely-funded spike found in data may reflect the fact that most failing campaigns lack in dedicated buyers, which conversely, is responsible for the campaign's failure in the first place (more on this point in Implication (C)). Discussions about the importance in harnessing dedicated buyers will be picked up in Section 5. Apart from the lack of dedicated buyers, another crowdfunding mechanism also facilitates the formation of the barely-funded spike: Backers are allowed to cancel their pledge while the project is still live (e.g., on Kickstarter). Therefore, pledged dedicated buyers may retrieve their pledge when it becomes apparent that the campaign will fail, not waiting for refund until the advent of actual failure. The lack of dedicated buyers and the cancel policy may be said to work in tandem in forming the barely-funded spike observed in data.

Second, the entrepreneur must set the backers' goal G slightly below the total number of backers T for successful campaigns to exceed the target only by small margins. Interestingly, the two-spike feature itself makes it optimal for the entrepreneur to do so. As will be elaborated in Subsection 3.3, since the p.m.f decreases rapidly in l, the pro of lowering G (easilier achievable goal) is soon dominated by the con (harder reachable critical mass k), thus the optimal G^* is always set below the all-captured spike before the probability diminishes to a negligible level. This logic applies as long as $\tilde{L}$'s p.m.f decreases rapidly around the all-captured spike, which renders any further lowering of G unprofitable. So the result should be robustness to demand uncertainty (see Sup S1.1) or random arrival (see Sup S1.3), each adding some variance to the baseline model along either the demand, or the time dimension, naturally resulting in higher variance around the two spikes, but retaining the rapidly diminishing trend of $\tilde{L}$, though how fast it diminishes (or how close G^* is to T) depends on the variance of the underlying uncertainty for the demand or the time.

Lastly, the sawtooth pattern in Figure 3 is different from that in Figure 1, in that the former predicts the highest probability to show up at $\tilde{N} = T$ (capturing all buyers) but the later shows it actually happens at $\tilde{N} = G$ (capturing exactly the backers' goal). Keeping only a parsimonious amount of basic yet key ingredients, the simple setup at hand can only deliver an explanation for the two statistical spikes in the campaign's funding outcomes, but for not their sawtooth pattern. An attempt to address the issue inevitably involves modeling the backers' incentives and strategies in a more complex way. We provide a possible explanation for the sawtooth pattern in Sup S1.1 by considering the interplay of donors (Deb et al., 2019) and strategic-waiting buyers.

B. Rational and efficient herding

Herding is a common phenomenon in traditional financial markets, recently also documented for different crowdfunding markets: reward-based (Xiao et al., 2021), equity (Astebro et al., 2019), microloan (Zhang and Liu, 2012), prefunding (Wei et al., 2021). While the rationality of crowdfunding herding is overall approved, in that backers do not passively mimic others' choices but engage in observational learning during the process (Zhang and Liu, 2012), such learning is mostly postulated to mitigate buyers' uncertainty for campaign's quality. Thus one novel aspect of this paper is that, absent any *quality-related risk*, our model implies that herding can also emerge from the relief of buyers' *pledging failure risk* once the critical mass is reached. Differing from past empirical works, this model addresses crowdfunding herding from a theoretic angle and by a different risk measure, and below we reason that such herding is both rational in terms of why it arises, and (with high probability to be) efficient in terms of where it leads to.

Lemma 2 formalized herding to be a natural pledging cascade resulting from the pledging rule (5), a behavioral rule-of-thumb that reflects bounded rationality. So the underlying assumption is that buyers do not passively mimic existing backers, but actively examine, with the information at hand, whether the success odds are high enough to cover the hassle costs. Such herding, originating from the bounded-rational behavioral rule, may be said to be (boundedly) rational as well.

Herding would be ex post efficient if it de facto resolves buyers' pledging failure risk, i.e., if herding does lead to success for the campaign. Recall that the two spikes represent two high-probability events: herding happens very early (leading to outcomes near the all-captured spike) or herding never happens (leading to the dedicated-only spike). And success relies on capturing more than G^* buyers, while we will proceed to show G^* is set close to T, the all-captured spike. It means that the absence of herding foretells the campaign's failure, while herding almost always leads to success. Is it possible that herding may sometimes lead to failure? The answer is yes, because it is still possible that herding happens rather late when the campaign has lost quite a few common buyers (the intermediate values of l), but intermediate l is low-probability event as seen in Figure 3. In these cases, herding is still rational, because the mismatch of herding and failure is due to buyers' inability to *ascertain* the success. That is, buyers' *perceived success probability* is high enough to cover hassle costs (so they herd), though the outcome turns out to be a failure.[27]

To summarize, herding originates from the specified behavioral rule. High, timely funded percentage is an indicator of high network externality, thus high success probability, in which case buyers are less likely to engage in rewardless pledging at a cost. That is, the funding progress is informative of the campaign's outcome, hence the buyers who utilize these information to relieve their risks, and the herding phenomenon emerging during the process, can be deemed as (boundedly) rational. The herding is also ex post efficient in that it almost always leads to the campaign's success, which may be seen as a full resolution of buyers' pledging risk.

[27]Recall that ϕ is always less than 1 unless $n + 1 \geq G$, i.e., the campaign is already successful or the buyer can complete the campaign by herself. That is the only way buyers can ascertain success.

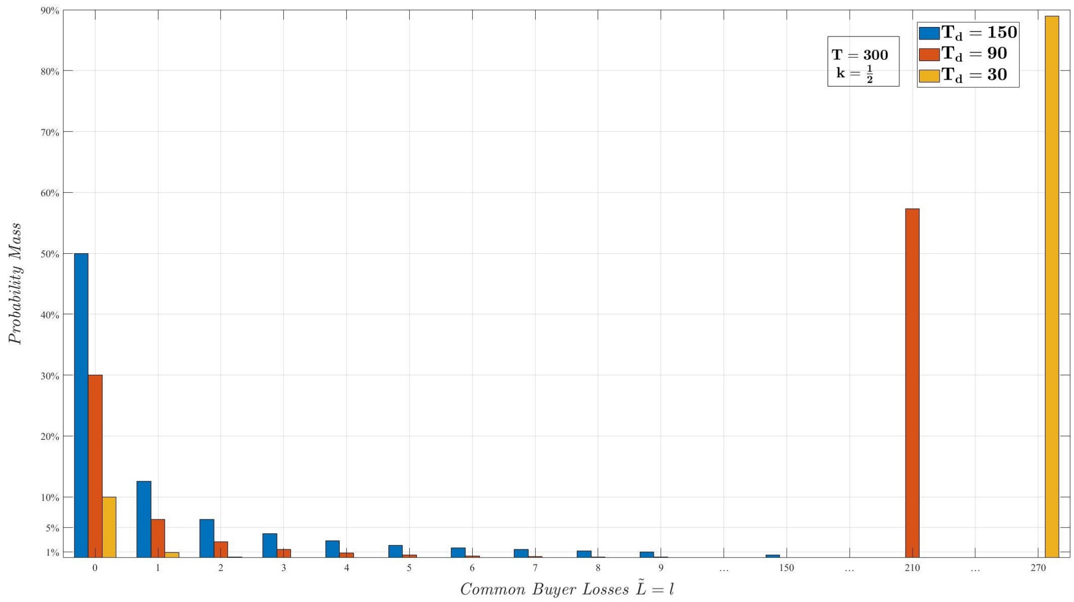


Figure 4: $\tilde{L}$'s p.m.f for different $\frac{T_d}{T}$ ratios

Note: This figure displays $\tilde{L}$'s p.m.f for $T_d = 30, 90, 150$, given $T = 300$ and $k = \frac{1}{2}$. Higher proportion of dedicated buyers improves $\tilde{L}$'s distribution, in that the campaign tends to lose less buyers.

C. The demand, T, T_d, and the dedicated-to-all ratio $\frac{T_d}{T}$

The conventional economics tactic to model the market's demand system is to fully characterize it by a demand curve, where buyers' type (heterogeneity) is distinguished by their value, or willingness to pay. Upon a further inspection of the behavioral rule described in Subsection 3.1 and encapsulated in (5), one might notice that this model distinguishes buyers by their hassle costs, or more specifically, their perceived critical mass $k = \frac{rh}{(\frac{G}{K+cG}-1)}$, which completely determines the buyers' pledging decision and behaviors.[28] The Propositions imply that fixing k, the campaign's outcome (its $\tilde{N}$'s distribution) relies on three dimensions of the demand: how many buyers in total (T), how many dedicated buyers (T_d), and the dedicated-to-all ratio ($\frac{T_d}{T}$).

By the expressions given in Propositions 1 and 2, $\tilde{L}$'s distribution is almost invariant to the scale of T and T_d, but heavily affected by the dedicated-to-all ratio.[29] A quick example is, when $k = \frac{1}{x}$, Prob($\tilde{L} = 0$) $= C_{T-1}^{T_d-1}/C_T^{T_d} = \frac{T_d}{T}$, while a graph illustration is provided in Sup Figure S4. The portion of dedicated buyers determines how fast the critical mass can be reached, thus directly affecting the lost buyers' distribution. Since the campaign's outcome is fully characterized

[28]Common buyers have positive k while dedicated buyers have $k = 0$. So if robustness can be proved for higher heterogeneity in k (argued in Sup S1.4), the model actually represents a large relaxation of traditional demand system regarding where the buyer-side heterogeneity may come from, not only from v, but also from r and h.

[29]Changing the scale of T and T_d would stretch the support of $\tilde{L}$. What stays invariant is $\tilde{L}$'s p.m.f at the polar (around the two spikes), while the middle stretch (of different lengths) share very low probabilities.

by $\tilde{N} = T - \tilde{L}$ where $\tilde{N} \in [T_d, T]$, T and T_d have different impacts on $\tilde{N}$ as follows.

T has three impacts on $\tilde{N}$. First and foremost, $k = \frac{rh}{(\frac{G}{K+cG}-1)}$ where the maximal feasible G is T (the backers' goal less than total number of backers), so higher T raises the highest G or lowers the lowest k. Intuitively, higher total demand means buyers will arrive with higher frequency, so the t^{th} buyer will arrive when a smaller fraction of time has elapsed, bringing her perceived ϕ up, and her k down. A lower k improves $\tilde{L}$'s distribution (Figure 3) by tilting the probability bars to the all-captured spike, thus tilting $\tilde{N}$'s distribution to the T-polar. The second impact of T is to increase the upper bound of $\tilde{N}$, allowing for more capturable buyers. So far, both impacts are positive. Third, higher T results in a lower dedicated-to-all ratio, which negatively impacts $\tilde{L}$'s ($\tilde{N}$'s) distribution by tilting the probability bars to the dedicated-only spike (the T_d-polar), as shown in Figure 4. Similarly, (higher) T_d has a twofold impact on $\tilde{N}$: it raises both the $\tilde{N}$'s lower bound and the $\frac{T_d}{T}$ ratio, thus improving $\tilde{L}$'s ($\tilde{N}$'s) distribution by tilting the probability bars to the all-captured spike (the T-polar). Conversely, as illustrated by the yellow bars in Figure 4, low T_d results in high probability of capturing only the dedicated buyers, thus high probability of failure.

In summary, a higher portion of dedicated buyers is always beneficial, in securing a larger number of pledges at the minimum and helping the campaign quickly accumulating funds and reaching the critical mass. Having more common buyers (fixing T_d while increasing T), on the other hand, benefits the campaign by increasing the capturable buyers at the maximum and by lowering the critical mass for common buyers, but may harm the distribution as it becomes more difficult to reach the critical mass with a lower portion of dedicated buyers. A direct take-away is that without a solid foundation of dedicated buyers, the attempts to bring in more common buyers might be of no avail if the critical mass is yet to be reached. The overall impact of having more common buyers does not have a clear-cut answer, and needs case-by-case examination.

3.2.3 General k and first-order stochastic dominance

While Propositions 1 and 2 deliver the three implications neatly, it is naturally to ask whether these desired properties, especially the two-spike feature that serves as a corner-stone for all subsequent analysis, would survive for arbitrary $k \in (0, 1)$. Also, the foregoing analysis calls for a formalization of the impact of k on $\tilde{L}$'s distribution, i.e., what it means by saying lower k tilts $\tilde{L}$'s distribution to the all-captured spike. This section settles the two issues in one by establishing a first-order stochastic dominance (FOSD) relationship regarding $\tilde{L}$'s distribution for different $k \in (0, 1)$.

Consider two identical campaigns except for their backers' goal, resulting in $k_A \leq k_B$, $k_A, k_B \in (0, 1)$. Recall that cascade happens when $\frac{n}{t} \geq k$. If cascade has started for Campaign B, it should have started for Campaign A since $\frac{n}{t} \geq k_B \geq k_A$. In other words, the cascade always starts (weakly) earlier for Campaign A, meaning for any random arriving order, Campaign A should lose (weakly) less buyers than Campaign B. This logic gives rise to the FOSD relationship in the next Lemma.

Lemma 3. *Assume Campaigns A and B have the same number of common and dedicated buyers. If their critical masses are such that* $k_A \leq k_B$*, then* $\tilde{L}_B$ *weakly FOSD* $\tilde{L}_A$*, that is,* $\text{Prob}(\tilde{L}_B \leq l) \leq \text{Prob}(\tilde{L}_A \leq l)$ *for all* $l \in \{0, 1, ..., T - T_d\}$*.*

The FOSD relationship immediately implies that $\tilde{L}$'s cumulative mass function (c.m.f) of any k lies between the c.m.f's of its two nearest regular-form k's. If a function is bounded above and below by two functions exhibiting the two-spike feature (in its corresponding p.m.f), the function should exhibit the feature as well. In fact, if two k's are very close to each other, their $\tilde{L}$'s p.m.f's may share the same value for many low l's (e.g., $k = \frac{1}{4}, \frac{1}{3}$ in Figure 3) due to the integer constrains on n and t. Moreover, as $\tilde{L}$'s p.m.f decreases quickly to a negligible level, sharing the same values at low l's indicates their p.m.f's are almost the same. Figure S7 in Sup provides a graph illustration.

3.3 The entrepreneur's decision

In Section 3.2, we analyzed buyers' behavior and derived $\tilde{L}$'s p.m.f given T, T_d and k (k is set by the entrepreneur via $k(G)$). Now, we are interested in the decision of backers' goal G by a sophisticated, success-probability maximizing entrepreneur. The setup of a sophisticated seller optimizing against boundedly rational buyers follows the literature of behavioral industrial organization wherein firms are treated as fully rational agents who sometimes exploit the myopia or naivete of buyers (see e.g., Gabaix and Laibson, 2006). The goal of this section is to show that the entrepreneur would set the optimal backers' goal G^* close to T due to $\tilde{L}$'s two-spike feature.

First, the entrepreneur needs to have *some extent* of market knowledge in order to maximize success probability. In the baseline, we assume somewhat strongly that the entrepreneur knows the exact market demand T and T_d, and show the optimal G^* is close to T. In Sup S1.1, a more general setting is considered where the entrepreneur knows only the distributions of T and T_d, rather than precise point estimations. Though, how demand uncertainty impacts $\tilde{L}$'s p.m.f essentially, is to take a weighted sum of $\tilde{L}$'s p.m.f for each point estimation and end up with higher variance around (instead of obliterating) the all-captured and dedicated-only spikes. And if $\tilde{L}$'s p.m.f under demand uncertainty retains the two-spike feature (it does), the main message of this section should carry over. In addition, the entrepreneur needs to know other key parameters in the maximization: here, it is buyers' product valuation v, common buyers' hassle cost h, and the anticipation parameter ρ (or r) which reflects how optimistic the buyer feels about the campaign's progress.

It may appear unreasonable to ask the entrepreneur to know so much about the market; it is, indeed. But recall that only three factors matter in determining $\tilde{L}$'s distribution, T, T_d, and k, or put in another way, how many each type of buyers there are, plus what each type's critical mass is—This is actually a simpler, reduced version of any demand curve which has infinitely many types of buyers distinguished by their willingness-to-pay. Essentially, the entrepreneur needs to estimate how k is affected by her goal-setting G, and this is where v, h, r weigh in. Moreover, it will become clear that the main result (G^* close to T) only requires the entrepreneur to accurately estimate T (or T's distribution, as discussed in Sup S1.1). Failing to do so for all other parameters will result in a suboptimal decision, but not affect the main result.

Now let $F(\cdot \mid k)$ denote $\tilde{L}$'s cumulative function given some k, i.e., $F(L \mid k) = \sum_{l=0}^{L} \text{Prob}(\tilde{L} = l \mid k)$. The entrepreneur needs to solve

$$\max_{G \in \left[\frac{K}{1-c}, T\right]} F(T - G \mid k) \qquad s.t. \;\; k = k(G) = \frac{rh}{\left(\frac{G}{K+cG} - 1\right)}.^{30} \tag{6}$$

The optimal G^* maximizes the probability that the campaign reaches the backers' goal (i.e. $\tilde{L} \leq T - G$), while $F(\cdot \mid k)$'s distribution is jointly determined by T, T_d and k. The feasible set of G is $\left[\frac{K}{1-c}, T\right]$, for $p = \frac{K+cG}{G} \leq 1$ and for the backers' goal not exceeding the total number of buyers.

A trade-off associated with the choice of G makes the maximization problem meaningful. Since $k(G)$ decreases in G, a higher goal gives rise to an easilier reachable critical mass, and as $k(G)$ decreases, $\tilde{L}$ is stochastically lower (c.f., Lemma 3) thus $F(\cdot \mid k(G))$ shifts up. However, higher backers' goal is harder to achieve, reflected in $F(T - G \mid k(G))$ deceasing in the first G.

The first economic implication is that, by Assumption 2 and Lemma 1, $k < 1$ is a necessary condition for the campaign to *be possible to succeed* since the entrepreneur cannot solely rely on dedicated buyers' funds. The lowest k is $k(T)$, and $k(T) < 1$ requires $T > \frac{K}{\frac{1}{rh+1} - c}$.

Proposition 3. *If* $T < \frac{K}{\frac{1}{rh+1} - c}$*, the campaign's success probability is 0.*

Notice that $\frac{K}{\frac{1}{rh+1} - c} > \frac{K}{1-c}$. It yields a strong implication that, even if the total demand ($T * v = T$) is sufficient to cover the production cost ($K + c * T$), i.e., $T \geq \frac{K}{1-c}$, the campaign still fails for certain if $T < \frac{K}{\frac{1}{rh+1} - c}$. This stands in contrast to models which tacitly assume perfect coordination among buyers, where the campaign is deemed successful as long as total demand meets the goal (e.g., Chakraborty and Swinney, 2021; Strausz, 2017) and buyers' hassle costs are absent. In other words, higher demand (than what is needed to cover production costs) is necessary for the campaign's success as a way to compensate for buyer's pledging failure risk.

An analytical solution for (6) requires $F(L \mid k)$'s expression. Sup Figure S8 shows that $F(L \mid k)$ is not everywhere continuous in k (whereas L, taking integer values, is naturally discrete) and does

[30] Though G is allowed to take non-integer values, it is proved in Sup S2.6 that G^* must be an integer. It follows that there exists a solution to the problem.

not have a closed-form expression. Therefore, our main goal is not to give the analytical solution but to characterize it as much as possible. To get some intuition, first ignore the discontinuity in $F(L \mid k)$ and take the first-order condition (FOC) of (6):

$$-f(T-G \mid k(G)) + (-\frac{dF}{dk})(-\frac{dk}{dG}) = 0, \tag{7}$$

where $f(\cdot \mid k(G))$, formerly denoted as $\text{Prob}(\tilde{L} = \cdot \mid k(G))$ for the discrete version, is the density function corresponding to $F(\cdot \mid k(G))$, with its two-spike feature illustrated in Subsection 3.2.[31] We know k decreases in G, specifically, $-\frac{dk}{dG} = \frac{rhK}{((1-c)G-K)^2} > 0$.

To characterize $\frac{dF}{dk}$, we use simulation[32] to get $F(L \mid k)$'s expression for $\frac{T_d}{T} = 0.1, 0.2, ..., 0.9$ and $k = 0.10, 0.11, ..., 0.90$, and summarize $F(L \mid k)$'s properties regarding k with assistance of its graphs (see Figure S8 in Sup) in the following Lemma.

Lemma 4. *By simulation, $F(L \mid k)$ as a function of k exhibits the following properties:*

(1) $F(L \mid k)$ is close to 1 (0) for sufficiently large (small) $\frac{T_d}{T}$ or sufficiently small (large) k.

(2) $F(L \mid k)$ decreases monotonically in k, may flattening when $F(L \mid k)$ is close to 1 or 0, but decreasing steadily in between, with occasional downward jumping.

(3) When $F(L \mid k)$ decreases steadily in k, the decreasing rate is slower for smaller L.

The first property is straightforward. $F(L \mid k)$ decreasing in k reflects Lemma 3 (FOSD), while the flattening when $F(L \mid k)$ is close to 1 or 0 is due to the behavioral algorithm, that for sufficiently small or large k's $F(L \mid k)$ does not change much. The last property comes from the fact that when L is large, $F(L \mid k)$ stays close to 1 for a wider range of small k's, thus it must decrease faster to reach 0 when k approaches 1. Given these properties of $F(L \mid k)$, we claim that the maximization is approximately a convex problem in the continuous space (and the convexity is quite obvious in simulations, see the following Figure 5, and Figure S9 in Sup). This is because when G increases (i) $T - G$ is smaller and $k(G)$ smaller, so $f(T - G \mid k(G))$ increases,[33] (ii) $-\frac{dF}{dk}$ decreases because $L = T - G$ is smaller by Lemma 4 (3), (iii) $-\frac{dk}{dG}$ decreases by its expression. Therefore, the LHS of (7) decreases in G, thus the maximization is convex. Combining the convexity of the maximization and the fact that the optimal G^* must be an integer, we can characterize G^* by (7).

Proposition 4. *Given the convexity of (6), start from $G = T$ and decrease G by 1 each time, then*

[31] By ignoring the discontinuity, we mean that for the integer L argument, one can think of $F(L \mid k)$ as a continuous approximation, and for the k argument, one can focus on the continuous parts of the function.

[32] Recall $F(L \mid k)$ depends on T, T_d and k, while fixing k, it is insensitive to the scale of T and T_d but sensitive to $\frac{T_d}{T}$. In the simulation, we take $T = 500$.

[33] Since $f(L \mid k(G))$ does not decrease monotonically in L or k but has the general tendency (see Figure 3), we call the original problem *approximately* convex. This means that the problem is convex if the G grid is not too fine.

the optimal G^* *is the first* G *such that* $F\big(T-(G^*-1)\mid k(G^*-1)\big) < F\big(T-G^*\mid k(G^*)\big)$*, or*

$$\Big(-f(T-G^*+1\mid k(G^*-1))\Big)+\Big(F\big(T-G^*\mid k(G^*)\big)-F\big(T-G^*\mid k(G^*-1)\big)\Big)>0. \quad (8)$$

When G decreases, the LHS of (7) increases, thus the optimal G^* occurs when the LHS of (7) first becomes >0. That is when the benefit of including $f(T-G^*+1\mid k(G^*-1))$ is too small to offset the general shifting down of $F\big(T-G^*\mid\cdot\big)$ due to higher k, i.e., $k(G^*-1)>k(G^*)$, as illustrated by (8). The way we characterize the solution ignores the jumps of $F(L\mid k)$ in k. However, (downward) jumps in $F(L\mid k)$ should be avoided by the entrepreneur because it represents a sudden worsening of the success probability. Mathematically, a downward jump when moving from G to $G-1$ means the second bracket in (8) suddenly increases by much, and (8) may be immediately satisfied.

By the two-spike feature of $f(\cdot|k)$, i.e., $f(L|k)$ soon approaches 0 when L increases, and by Lemma 4 (2) that $F(L\mid k)$ decreases steadily in k when the success probability is not too close to 1 or 0 so the second bracket in (8) is always sufficiently positive, the next corollary follows.

Corollary 5. *The optimal* G^* *is close to* T*, where* $\tilde{L}$*'s p.m.f has not decreased to a negligible level.*

How close is G^* to T? Unfortunately, this question cannot be answered analytically. Using simulation methods, for all $\frac{T_d}{T}=0.1,0.2,...,0.9$ and $k=0.10,0.11,...,0.90$, we find that $F(L+1\mid k)-F(L\mid k+0.01)$, which is the benefit of lowering G by 1 if the induced k increases by 0.01, often becomes negative within $L\leq 10$, $L=24$ for the most. Simulation sets $T=500$, but the thresholds should remain with similar values for other T's since $\tilde{L}$'s distribution is invariant to its scale. If the induced k increases more than 0.01, this threshold of L should be even smaller, and vice versa.

Notice that the foregoing analysis still holds if the entrepreneur wrongly estimates v, h and r. Incorrect estimation of those parameters will distort $k=rh/(\frac{G}{K+cG}-1)$ from its real value, which may make the entrepreneur's choice of G suboptimal. However, reducing G (thus increasing k) at a wrong level still gives rise to Corollary 5 since $\tilde{L}$'s p.m.f decreases quickly for *any* k. One caveat is that, since the analysis relies on Lemma 4 (2), for those campaigns whose ex ante success probability is either close to 1 or 0, this Corollary may not hold, meaning G^* may not be that close to T, giving rise to potential blockbuster successes (i.e., funds far exceeds the target, for an example, see the first subfigure in Sup Figure S9). The comparative statics analysis, though, cannot be performed satisfactorily because it requires precise characterization for the second-order properties of $F(L\mid k)$, i.e., $\frac{d^2F}{dk^2}$, since both k and $\frac{dk}{dG}$ are affected by the parameters h,r,K,c, but the irregularity of $F(L\mid k)$ function (as shown in Sup Figure S8) forbids such exploitation.

Figure 5 displays a group of $\tilde{L}$'s c.m.f's. Given these parameters, the optimal G^* falls somewhere around 285 with a success probability around 79%. Not shown in the figure, if ρ takes a more

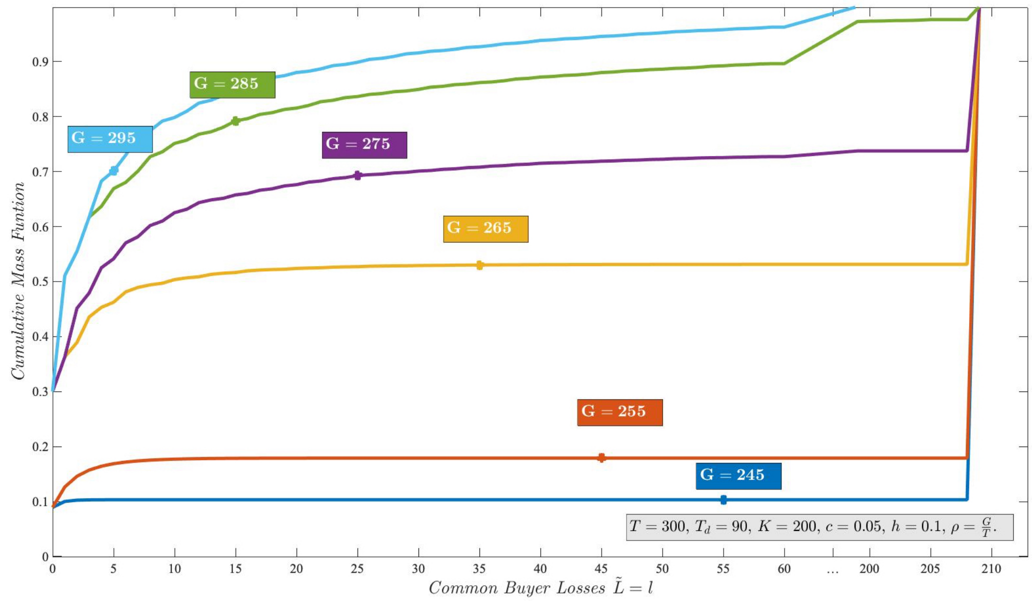


Figure 5: $\tilde{L}$'s c.m.f for varying G, where $k = \frac{rh}{(\frac{G}{K+cG}-1)}$

Note: This figure shows $\tilde{L}$'s c.m.f $F(l \mid k(G))$ for varying backers' goals. The parameters are specified in the text box. Recall $\rho = \frac{G}{rT}$ where $r \geq 1$, represents how optimistic buyers feel about the campaign. Here it takes the most optimistic value, i.e., $r = 1$. Comparing different curves, as G decreases, $\tilde{L}$'s c.m.f flattens more quickly due to a higher $k(G)$. The height of each dot marker represents the campaign's **success probability** for each G, i.e., $F(T - G \mid k(G))$. Connecting these dots shows the convexity of the maximization problem. Clearly, the benefit of reducing G (to include more l) diminishes quickly, because $\tilde{L}$'s distribution worsens quickly as $k(G)$ increases. The optimal G^* falls somewhere around 285 with a success probability around 79%.

pessimistic value at $\frac{G}{2T}$ (or $r = 2$), the optimal G^* is around 295 with a success probability around 17%. Notice even if the entrepreneur estimates r differently, G^* is still close to T.

Though comparative statics (for G^*) are not clear, how parameter change impacts the success probability is straightforward. From (6), higher r, h, K, c would raise k, while $F(T - G \mid k)$ decreases in k for *any* level of G, so the (optimal) success probability decreases in these parameters. In addition, higher c and K raise up G's feasible lower bound, $\frac{K}{1-c}$, meaning the goal-setting (price-setting) is limited to a narrower range. In a nutshell, more pessimistic buyers, higher hassle costs, and higher production costs reduce the campaign's (optimal) success probability.

To fold up, we characterize the optimal G^* for the (discontinuous) maximization problem by analyzing its continuous counterpart and argue that G^* should be set close to T. The results apply to all campaigns that is not ex ante close to a certain success/failure. The behavioral rule gives to two polar funding outcomes (all-captured, and dedicated-only), further leads the success-probability maximizing entrepreneur to set backer's goal close to the total demand[34] and thus pins down the barely-funded failures and just-adequately-funded successes revealed in data. Conversely, the data

[34] With demand uncertainty, it is the expected value of total demand as shown in Sup S1.1.

also vindicates the plausibility of the behavioral rule, and we proceed to discuss more implications.

4 Extensions

In this section, we extend the baseline model to answer two applied questions: (i) What is the best timing to utilize the entrepreneur's own social network (as expected, easily answered)? (ii) How to design an effective early-bird offer (this one is more complex)? Due to space constraints, the robustness check is moved to Sup S1, including an extensive discussion for demand uncertainty, strategic waiting, random (and denser early) arrival, and higher level of buyer-side heterogeneity.

4.1 Friends and family: when to utilize social network

Personal network plays an instrumental role in the success of small startup businesses, especially when the latter seeks funding opportunities in crowdfunding (Agrawal et al., 2014; Mollick, 2014). More evidence indicates that, the entrepreneur's friends and family (Agrawal et al., 2011) and her internal social capital established within the platform (Colombo et al., 2015) contribute mainly in the early funding stage, breeding a self-reinforcing mechanism to accelerate the campaign's success—The effectiveness of such early-stage support is immediately justified by our model, one that stresses reaching a funding critical mass early for the campaign to succeed.

In the same spirit as Li et al. (2020) who empirically tested the effectiveness of early, versus late promotions, we consider two polar cases: early-stage and last-minute support from social network. Results show that, first, it is true that having social network's support is better than not having; early or late, unconditional pledges from friends and family help reaching the target. However, early support is more effective, because it builds the momentum and helps the campaign to reach the critical mass early on, thus allowing subsequent common buyers to be captured. Last-minute supports only help increasing funds, but not with the preceding funding dynamics.

Proposition 6. *Consider a campaign originally having T buyers, T_d dedicated buyers and a backers' goal G. Now assume the entrepreneur has $n_0 \geq 1$ many friends and family who would pledge unconditionally. Let P_s, P_s' and P_s'' denote the campaign's ex-ante success probability without support, with early-stage support (pledging before any other buyer arrives) and last-minute support (pledging after the last buyer has arrived), respectively. Then $P_s' > P_s'' > P_s$.*[35]

[35]Strictly, the latter inequality should be $\geq$. $P_s'' > P_s$ holds if $\text{Prob}(\tilde{L} = T - G + 1) \neq 0$, which is often the case as the optimal goal G^* is close to T, meaning $\tilde{L}$ has not decreased to a negligible level at $l = T - G^* + 1$.

4.2 Price discrimination: the design of early-bird offers

Price discrimination is a common and easily implementable scheme in crowdfunding. Campaigns are often seen to include several differently priced rewards of essentially the same product, granting the early buyers (e.g., the initial 20 backers) with some discounts, known as the Early-Bird offer. This scheme encourages buyers to pledge early and thus accelerates the funding progress, but on the flip side, the entrepreneur has to raise the normal (or late-bird) price to fulfill the capital requirement. Therefore, the lower the early-bird price is, the easier to capture early buyers; but the higher the late-bird price must be set, and the more difficult to capture late birds.[36] With that being said, early-bird offers must be carefully designed in order to exert positive impacts.

For presentation simplicity, we restrict our attention to a dual-pricing scheme, while similar arguments can be applied to higher levels of discriminatory prices. Consider a campaign with T buyers, T_d dedicated buyers, production cost K, marginal cost simplified to $c = 0$, and a backers' goal G. Under single-pricing, the price is $p = \frac{K+cG}{G} = \frac{K}{G}$ with critical mass $k = \frac{rhp}{1-p}$ by (5). Now, consider a dual-pricing scheme with the same backer' goal G.[37] Suppose the entrepreneur provides an early-bird offer at price $p_1 < p$ to the initial G_1 backers. Let $K_1 = p_1 G_1$ be the total funds expected from early birds. Then, $K - K_1$ funds need to be collected from $G - G_1$ many late birds, thus the late-bird price is $p_2 = \frac{K-K_1}{G-G_1}$. Note that $p_1 < p$ (or $\frac{K_1}{G_1} < \frac{K}{G}$) implies $p_2 > p$ (or $\frac{K-K_1}{G-G_1} > \frac{K}{G}$): The provision of early-bird discount is at the cost of raising the late-bird price.

The new critical masses become $k_1 = \frac{rhp_1}{1-p_1}$ for early birds and $k_2 = \frac{rhp_2}{1-p_2}$ for late birds. Note $p_1 < p < p_2$ implies $k_1 < k < k_2$: While it is easier to start the pledging cascade in the early-bird stage, the cascade may stop when the first late bird arrives. A good design must balance two prices well to avoid such stagnation. The next proposition provides a sufficient condition to strictly increase the campaign's ex-ante success probability, and the following example illustrates the importance of balancing the early- and late-bird prices.

Proposition 7. *The dual-pricing profile* ***strictly increases*** *the campaign's ex-ante success probability over a single-pricing profile if:*

(1) (non-stopping pledging cascade) $\frac{G_1}{G_1+T-G+1} \geq k_2$*, and*

(2) (strict improvement) There exist $n \leq \min(G-1, T_d)$ *and* $l \leq T-G$ *such that* $k > \frac{n}{n+l+1} \geq k_1$.

Example 1. *This example shows that good and bad designs may have opposite impacts on the campaign's success probability. Suppose* $K = 70$*,* $c = 0$*,* $T = 100$*,* $T_d = 30$*,* $h = 0.2$ *and* $r = 1$*.*

[36] Early and late birds are distinguished by the prices they face.

[37] We assume the dual-pricing scheme maintains the same backers' goal. Otherwise there is too much flexibility, and the scheme design needs resorting to numerical methods, and is not theoretically tractable. For the same reason, we do not discuss the optimal design.

Under single-pricing, the optimal backers' goal is found via a numerical method to be $G^* = 98$, *with the price* $p = \frac{K}{G^*} = 0.71$ *and critical mass* $k = \frac{rhp}{1-p} = 0.5$. *The campaign's ex-ante success probability is* $\text{Prob}(\tilde{L} \leq 2 \mid k = 0.5) = 38.9\%$.

(1) (A Good Design) The entrepreneur has two decision variables regarding the dual-pricing profile: the early-bird price p_1 *and the offer amount* G_1. *Two degrees of freedom allow for various combinations of the design. Baselines are:* $p_1 < p$, *but the offer cannot be over-provided. Consider* $p_1 = 0.5$ *being offered to the initial* $G_1 = 10$ *buyers. Then* $k_1 = \frac{rhp_1}{1-p_1} = 0.2$, *a substantial decrease from* $k = 0.5$, *and condition (2) is obviously satisfied. Now check the late-bird price.* $p_2 = \frac{K - p_1 G_1}{G^* - G_1} = 0.74$ *and* $k_2 = \frac{rhp_2}{1-p_2} = 0.57$. *As* $\frac{G_1}{G_1 + T - G^* + 1} = \frac{10}{13} > k_2$, *condition (1) is also satisfied, meaning the pledging cascade will not stop when the early-bird offers are sold out. It turns out this dual-pricing scheme raises the ex-ante success probability up to* $\text{Prob}(\tilde{L} \leq 2 \mid k_1 = 0.2, k_2 = 0.57) = 66.2\%$.

(2) (A Bad Design) If the entrepreneur over-provides the offer at a lower price $p_1 = 0.25$ *and to more buyers* $G_1 = 20$, k_1 *is further reduced to* 0.0667, *but the late-bird price becomes* $p_2 = 0.77$ *and the late-bird critical mass* $k_2 = 1$. *By Lemma 1 such* k_2 *can never be achieved while condition (1) is violated. The campaign at most captures 20 early-bird common buyers plus 30 dedicated buyers,* $50 < G^*$, *so* $\text{Prob}(\tilde{L} \leq 2 \mid k_1 = 0.0667, k_2 = 1) = 0$. *The campaign fails with certainty.*

5 Economic and Managerial Implications

In the Extension some applied suggestions have been given to the entrepreneur, namely about the best timing to utilize social network and the caveat in designing early-bird offers. In this Section, we summarize and discuss more economic and managerial implications derived from the model.

Succeed under uncertainty The campaign's success depends on two factors: sufficiently high demand (quality), and reaching critical mass early (luck). Quality comes first, thus even if the entrepreneur fakes funds in the beginning (which is restricted by most platforms), the campaign still fails if the true demand cannot cover production costs.[38] On the other hand, in standard economic setting, trade would happen (in terms of social-welfare improving) if the demand is sufficient to cover the production costs (as in the crowdfunding modelings by Chakraborty and Swinney, 2021; Chakraborty et al., 2021; Strausz, 2017). However, such assertion is valid only in an uncertainty-free environment. In Proposition 3, we have shown that sufficient demand (to cover production costs) is *not sufficient* for a strictly positive success probability; more demand than that

[38] In addition, faking funds causes risk on the entrepreneur side. If the target is reached only by combining buyers' pledges and the fake funds, it means the actual funds cannot cover production costs or conversely there will be excessive productions (corresponding to the fake funds), causing losses for the entrepreneur. Such risk is deepened if backers stop pledging once the campaign succeeds in fear of moral hazard (Strausz, 2017). Hence although early momentum is crucial for campaign success, faking early funds is very risky and also restricted by most platforms.

is needed for the campaign to be possible to succeed, and this additional requirement originates from buyers' uncertainty about the campaign's outcome. That is, traditional wisdom only works when buyers coordinate perfectly with each other. In an uncertain environment as crowdfunding, qualitative projects eligible to initiate production in traditional markets may still be doomed to failure in crowdfunding. Despite the benefits of crowdfunding financing especially for startups, it seems the fruits are only in the reach of sufficiently high-quality projects.

Target the right crowd at the right time The marketing literature strand abounds in the discussion of market segmentation and targeted marketing (e.g., Goldstein and Lee, 2005; Johnson, 2013; Weinstein, 2013), which seem to keep eluding the crowdfunding research strand. Targeted marketing is necessary in particular for capital-shy startups, as promotions and advertisements are usually costly. This paper provides a natural segmentation for crowdfunding buyers, namely by their hassle costs. In Subsection 3.1.2 we identified several potential groups of dedicated buyers: impulse buyers (Hausman, 2000), big fans (of the product category), engaging customers (Pansari and Kumar, 2017), internal and external social networks (Buttice et al., 2017), etc. These groups are not necessarily dedicated buyers ($k = 0$), but in general represent low-k types, depending on how low their hassle costs are, how much they gain from engaging in community activities, etc. The idea given by the model is to *bring dedicated (or low-k) buyers to the campaign as early as possible*, thus targeting the right crowd (while different groups may be reached in different online communities/subreddits) at the right time (in the early funding stage). Among all, dedicated buyers serve as the cornerstone of campaign success as they are the initial funding source, so without a solid foundation of dedicated buyers, the attempts to bring in more common buyers might be of no avail if the critical mass is yet to be reached.

Monitor the campaign The backers' goal being close to total demand, plus an overall stable arrival rate, imply that the funds should always grow proportionate to, or ahead of (if accounting for the denser early arrivals), time; while the opposite holds for failing campaigns (consistent with the empirical evidence, Figure 3 in Li et al., 2020). Sluggish growth, funds falling behind time suggest either the campaign is of low quality thus doomed to failure, or the early momentum did not form properly, which calls the entrepreneur to take action in trafficking dedicated buyers in. The earlier to notice the sluggish growth and take actions before the campaign loses too many common buyers, the more likely the campaign can come back to life, given it is a qualitative project.

Re-campaign or not A handful of studies have focused on entrepreneurial learning in crowdfunding (Buttice et al., 2017; Lee and Chiravuri, 2019; Peterson and Wu, 2021; Yang and Hahn, 2015), particularly about how past crowdfunding experiences affect entrepreneur's decisions about, and performances in, new campaigns. However, one important question remains neglected: whether it is worth to start the same failed campaign all over again (with potential modifications). Our model suggests that bad-quality projects (insufficient demand), or qualitative campaigns failing to form the early momentum (dedicated buyers arrive too late), both would fail in the end. As starting a new campaign involves preparation and costs, the former is not worthy re-campaigning unless the project is improved, while the latter might worth a trial. The question is how to distinguish bad luck from bad quality. The model does not give an assertive answer but provide some clues and insights. First, the number of dedicated buyers (T_d) is an indicator of the campaign's quality: As shown, higher T_d not only raises the lower bound of capturable buyers, but also improves the probability distribution. And as suggested by the model, they usually reveal themselves despite of the campaign's progress. Hence, if the campaign fails but receives a good amount of pledges, these are mostly dedicated buyers, and it is a strong signal for re-campaigning. On top of that, if these pledges did not show up early but gradually accumulate over time, it implies the absence of an early funding momentum, that dedicated buyers might have arrived too late, so a re-trial might be worthy. On the other hand, failures with a small amount of pledges rushing in only at the beginning is an indicator for bad-quality projects: Even the dedicated buyers have contributed early, the critical mass is still not reached for common buyers; it is probably unreachable in the first place because of insufficient demand. One caveat is that, since buyers are allowed to cancel their pledges, the entrepreneur needs to monitor the campaign closely to get an accurate estimation of dedicated buyers. If re-campaigning, the important task is to traffick dedicated buyers in as early as possible, thus forging a dynamic and interactive campaign community as a way to increase the campaign network's social embeddedness (Hong et al., 2018), and maintaining relationship with these supporters to transform them to one's internal social network (Buttice et al., 2017) might be a rewarding endeavor.

6 Concluding Remarks

We propose a behavioral crowdfunding model with boundedly rational buyers and success-probability maximizing entrepreneur, explain some widely observed crowdfunding features, and derive rich economic as well as managerial implications. It may seem desirable to calibrate the model, but there are two major hurdles: first, high degree-of-freedom, since many buyer attributes affect the model through one channel k, while there is no closed-form formula for $\tilde{L}$'s p.m.f for arbitrary k; second, varying T and T_d for different projects which are hard to observe for all crowdfunding campaigns. Calibration may be made feasible by changing some basic setups of the model, or by adding additional assumption about the campaign pool, which is left for future studies.

Other possible avenues to extend the current work include, for example, testing the model's robustness to more general ϕ functions (e.g., the exponential form derived from hazards model in Li et al., 2020), as we take a linear specification mainly for tractability. Also, all buyers are assumed to be bounded rational, while in reality some informed buyers may possess higher sophistication. So the coordination between naive and experienced buyers can be considered. Another way is to test the model's robustness when the entrepreneur's objective is not the success probability but the revenue. In all, our paper may open a valuable research strand for behavioral crowdfunding theory, aiming to provide better understandings of backers' behaviors, to help the entrepreneur navigating campaigns, and to give rise to new perspectives in evaluating crowdfunding mechanism as a whole.

References

Agrawal, A., Catalini, C., and Goldfarb, A. 2011. The geography of crowdfunding. National Bureau of Economic Research, No.w16820.

Agrawal, A., Catalini, C., and Goldfarb, A. 2014. Some simple economics of crowdfunding. *Innovation policy and the economy*, 14(1):63–97.

Alaei, S., Malekian, A., and Mostagir, M. 2016. A dynamic model of crowdfunding. Ross School of Business Paper, No.1307.

Allon, G. and Babich, V. 2020. Crowdsourcing and crowdfunding in the manufacturing and services sectors. *Manufacturing & Service Operations Management*, 22(1):102–112.

Astebro, T. B., Fernández Sierra, M., Lovo, S., and Vulkan, N. 2019. Herding in equity crowdfunding. In *Paris December 2018 Finance Meeting EUROFIDAI-AFFI*.

Babich, V., Marinesi, S., and Tsoukalas, G. 2020. Does crowdfunding benefit entrepreneurs and venture capital investors? *Manufacturing & Service Operations Management*.

Belleflamme, P., Lambert, T., and Schwienbacher, A. 2010. Crowdfunding: An industrial organization perspective. In *Digital Business Models: Understanding Strategies, held in Paris on June. 2010*, pages 25–26.

Belleflamme, P., Lambert, T., and Schwienbacher, A. 2014. Crowdfunding: Tapping the right crowd. *Journal of Business Venturing*, 29(5):585–609.

Belleflamme, P., Omrani, N., and Peitz, M. 2015. The economics of crowdfunding platforms. *Information Economics and Policy*, 33:11–28.

Burtch, G., Ghose, A., and Wattal, S. 2013. An empirical examination of the antecedents and consequences of contribution patterns in crowd-funded markets. *Information Systems Research*, 24(3):499–519.

Buttice, V., Colombo, M. G., and Wright, M. 2017. Serial crowdfunding, social capital, and project success. *Entrepreneurship theory and practice*, 41(2):183–207.

Chakraborty, S., Ma, A., and Swinney, R. 2021. Designing rewards-based crowdfunding campaigns for strategic (but distracted) contributors.

Chakraborty, S. and Swinney, R. 2021. Signaling to the crowd: Private quality information and rewards-based crowdfunding. *Manufacturing & Service Operations Management*, 23(1):155–169.

Chang, J. W. 2020. The economics of crowdfunding. *American Economic Journal: Microeconomics*, 12(2):257–80.

Chemla, G. and Tinn, K. 2019. Learning through crowdfunding. *Management Science*.

Colombo, M. G., Franzoni, C., and Rossi–Lamastra, C. 2015. Internal social capital and the attraction of early contributions in crowdfunding. *Entrepreneurship Theory and Practice*, 39(1):75–100.

Deb, J., Oery, A., and Williams, K. R. 2019. Aiming for the goal: Contribution dynamics of crowdfunding. National Bureau of Economic Research, No.w25881.

Ellman, M. and Fabi, M. 2019. A simple model of crowdfunding dynamics.

Gabaix, X. and Laibson, D. 2006. Shrouded attributes, consumer myopia, and information suppression in competitive markets. *The Quarterly Journal of Economics*, 121(2):505–540.

Gao, H., Zhao, H., and Tan, Y. R. 2019. When investors meet consumers: The interaction of different backers in crowdfunding markets. Available at SSRN 3467116.

Goldstein, D. and Lee, Y. 2005. The rise of right-time marketing. *Journal of database marketing & customer strategy management*, 12(3):212–225.

Hausman, A. 2000. A multi-method investigation of consumer motivations in impulse buying behavior. *Journal of consumer marketing.*

Hong, Y., Hu, Y., and Burtch, G. 2018. Embeddedness, pro-sociality, and social influence: Evidence from online crowdfunding. *MIS Quarterly, Forthcoming.*

Hu, M., Li, X., and Shi, M. 2015. Product and pricing decisions in crowdfunding. *Marketing Science*, 34(3):331–345.

Johnson, J. P. 2013. Targeted advertising and advertising avoidance. *The RAND Journal of Economics*, 44(1):128–144.

Lee, C. H. and Chiravuri, A. 2019. Dealing with initial success versus failure in crowdfunding market: Serial crowdfunding, changing strategies, and funding performance. *Internet Research.*

Li, Z., Duan, J. A., and Ransbotham, S. 2020. Coordination and dynamic promotion strategies in crowdfunding with network externalities. *Production and Operations Management*, 29(4):1032–1049.

Loch, C. H. and Wu, Y. 2007. *Behavioral Operations Management.* Now Publishers Inc.

Mollick, E. 2014. The dynamics of crowdfunding: An exploratory study. *Journal of Business Venturing*, 29(1):1–16.

Pansari, A. and Kumar, V. 2017. Customer engagement: the construct, antecedents, and consequences. *Journal of the Academy of Marketing Science*, 45(3):294–311.

Peterson, A. and Wu, A. 2021. Entrepreneurial learning and strategic foresight. *Strategic Management Journal*, 42(13):2357–2388.

Radner, R. 1996. Bounded rationality, indeterminacy, and the theory of the firm. *The Economic Journal*, 106(438):1360–1373.

Rosenthal, R. W. 1993. Rules of thumb in games. *Journal of Economic Behavior & Organization*, 22(1):1–13.

Strausz, R. 2017. A theory of crowdfunding: A mechanism design approach with demand uncertainty and moral hazard. *American Economic Review*, 107(6):1430–76.

Wei, X., Fan, M., You, W., and Tan, Y. 2021. An empirical study of the dynamic and differential effects of prefunding. *Production and Operations Management*, 30(5):1331–1349.

Weinstein, A. 2013. *Handbook of market segmentation: Strategic targeting for business and technology firms*. Routledge.

Xiao, S., Ho, Y. C., and Che, H. 2021. Building the momentum: Information disclosure and herding in online crowdfunding. *Production and Operations Management*, 30(9):3213–3230.

Yang, L. and Hahn, J. 2015. Learning from prior experience: an empirical study of serial entrepreneurs in it-enabled crowdfunding. *Thirty Sixth International Conference on Information Systems*.

Zhang, J. and Liu, P. 2012. Rational herding in microloan markets. *Management science*, 58(5):892–912.

Zhang, J., Savin, S., and Veeraraghavan, S. 2017. Revenue management in crowdfunding.

Critical Mass and Campaign Success: A Behavioral Model of Reward-Based Crowdfunding

Supplement Document

Jingwen Tian

Department of Economics, The University of Iowa. gin-tian@uiowa.edu.

This supplement document (Sup) provides robustness checks for the baseline model in S1, and proofs for all propositions, lemmas and claims (unless already enclosed in the paper) in S2, and additional graph examples and wide tables for the proof in S3. Propositions 1 and 2 involve complex, rigorous math reasoning, so two lemmas are provided and proved first. For notation convenience, all G appearing in this material is an integer, which can be derived by taking the floor integer of the real backers' goal.

S1: Robustness check

This section conducts robustness check for the baseline model in four directions of perturbation: demand uncertainty, strategic waiting, random (and denser early) arrival, and higher level of buyer-side heterogeneity. Given the irregularity of the baseline maximization (discontinuous, only approximately convex, etc.), these assumption-perturbations are not analytically tractable, thus the robustness check is approached by a mixture of theoretical reasoning and simulation examples.

S1.1: Demand uncertainty

The baseline model assumes the entrepreneur has perfect knowledge of the product's market demand, T and T_d, calling for a robustness check. Instead of an accurate point estimation, the more general scenario is that the entrepreneur knows only the distribution of $\tilde{T}$ and $\tilde{T}_d$, with $F_1(T)$ and $F_2(T_d)$ denoting the respective cumulative density functions with supports $\mathbb{T}$ and $\mathbb{T}_d$.

To test the robustness of our main result, Corollary 5 in main file, let us first unravel the key factors leading to this result. A retrospect immediately reveals that (i) the fact that $\tilde{L}$'s p.m.f diminishes quickly to 0 (ii) while the change of $F(L \mid k)$ in k does not diminish to 0, two factors

Notation	Definition	Property
K	Production fixed cost	Parameter
c	Production marginal cost	Parameter
p	Product price	Decision Variable
G	Backers' goal, $p = \frac{K+cG}{G}$	Decision Variable
v	Buyers' valuation of the product, $v = 1$	Parameter
T	Total number of buyers	Parameter
T_d	Number of dedicated buyers	Parameter
h	Common buyers' hassle cost	Parameter
t	Buyer's arrival index, $t \in \{1, 2, ..., T\}$	Variable
n	Current number of pledges, $n \in \{0, 1, ..., t-1\}$	Variable
ϕ	Buyer's (behavioral) expectation of the campaign's success probability, $\phi = \rho\frac{n/G}{t/T}$, or $\phi = \frac{n}{rt}$ where $\rho = \frac{G}{rT}$	Variable
k	critical mass, $k = \frac{rh}{G/(K+cG)-1} \equiv k(G)$	Decision Variable
$\tilde{O}$	Buyers' arriving order, with a typical element $o \in \mathbb{O} \subset \{CB, DB\}^T$	Random Variable
$\tilde{N}$	Number of captured buyers, with a typical element $N \in \{T_d, T_d+1, ..., T\}$	Random Variable
$\tilde{L}$	Number of lost buyers, $\tilde{L} = T - \tilde{N}$, with a typical element $l \in \{0, 1, ..., T - T_d\}$	Random Variable
$\mathbb{K}$	Set of regular-form k's, $\mathbb{K} = \{\frac{1}{x}, \frac{y-1}{y} : x, y \in \{2, 3, 4, ...\}\}$	Set

Table 1: Variables and parameters used in the model

combined assure that lowering G after some point is not beneficial, thus leading to the assertion that G^* is close to T. The convexity of the maximization gives a stronger result that the *first* G satisfying (8) in main file is G^*, while (i)(ii) functions as the necessary condition for G^*.

Hence, if (i)(ii) still holds under demand uncertainty, the main result should be robust. Incorporate demand uncertainty to the maximization problem:

$$\max_{G \geq \frac{K}{1-c}} \int_{T_d \in \mathbb{T}_d} \int_{T \in \mathbb{T}} F(T - G \mid k) dF_1(T) dF_2(T_d) \qquad s.t. \;\; k = k(G) = \frac{rh}{(\frac{G}{K+cG} - 1)}.$$

Notice that $\tilde{T}$ and $\tilde{T_d}$ only affect $F(T - G \mid k)$, so the objective function is actually a (probabilistic) weighted sum of the $F(T - G \mid k)$ given some T and T_d, which has been studied afore in some depth. That is, given some G, $\tilde{N}$'s p.m.f here, should be the probabilistic weighted sum of the two-spike figures in Figure 3 in main file (using $\tilde{N} = T - \tilde{L}$), with two spikes occurring at capturing T buyers and capturing only T_d buyers. Now focus on $\tilde{N}$'s distribution given some G, as a weighted sum of $F(\cdot)$ given in the Maximization. Subsection 3.2.2 (C) in main file has shown that $\tilde{T}$ and $\tilde{T_d}$ affect $F(\cdot \mid k)$ in two ways. First, they affect its support $[T_d, T]$, while its density spikes exactly at these two polar, so for $\tilde{N}$'s distribution given some G, we should expect two probability masses forming in the support of $\tilde{T}$ and $\tilde{T}_d$ whose shapes resemble $F_1(T)$ and $F_2(T_d)$, since $\tilde{N} = T$ and $\tilde{N} = T_d$ are the dominating highest spikes given any T and T_d, as shown in Figure 3 in main file. Second, they affect the shape of $F(\cdot \mid k)$. We know that with a higher T, $\frac{T_d}{T}$ decreases so the probability tilts to T_d-polar, leaving less probability at T-polar. While with a higher T_d, $\frac{T_d}{T}$ increases so the probability tilts to T-polar, leaving less probability at T_d-polar. Therefore, the final distribution for $\tilde{N}$, as a weighted sum of $F(\cdot)$, modifies $F_1(T)$ and $F_2(T_d)$ in a way that, the higher T or T_d is, the less probability is allocated there, thus the probability density is right-skewed compared to $F_1(T)$ and $F_2(T_d)$.

The following Figure S1 shows the captured buyer, $\tilde{N}$'s p.m.f when $\tilde{T}$ and $\tilde{T}_d$ follow Poisson distributions with means 180 and 35. The two spikes reflect the weighted sum of the afore-studied two-spike figures, hump shape resembling Poisson (i.e., hump shapes for Poisson and Gaussian, rectangular for Uniform, etc.). The two humps are also right-skewed, with highest probability occurring at $T_d = 33$ and $T = 177$, instead of the original expectations $T_d = 35$ and $T = 180$. Finally, how widespread the two spikes are depends on the variance of $\tilde{T}$ and $\tilde{T}_d$; as long as the variance is not too high, we shall see $\tilde{N}$'s probability diminishes to 0 quickly as $\tilde{N}$ moves down from $\tilde{T}$'s mean. Therefore, (i) still holds if $\tilde{T}$ and $\tilde{T}_d$'s variances are not too high.[1]

Next, we need to check (ii). It is easily seen that the objective function's partial derivative with respect to k is the probabilistic weighted sum of $\frac{dF}{dk}(T - G \mid k) < 0$, because k only appears in $F(\cdot)$. Thus the weighted sum is close to 0 only if $\frac{dF}{dk}$ is close to 0 for all T and T_d, which is not the case given Lemma 4 in main file. Hence (ii) is satisfied. And the result in Corollary 5 in main file follows, i.e., G^* is set close to the expected $\tilde{N}$ for the all-captured mass (i.e., 177 in the Figure) which is approximately the mean of $\tilde{T}$ (i.e., 180 in the Figure) given the skewness is not too much, where $\tilde{N}$'s probability has not diminished to 0 (as G decreases). In Figure S1, the optimal G^* is solved to be 160.

In a nutshell, since demand uncertainty is essentially a probabilistic weighted sum of the original problem, the main intuition, that the entrepreneur should strive to set G^* close to the (expected) total demand $\tilde{T}$ to maximize the campaign's success probability, carries over.[2] However, this also

[1] That is, if the entrepreneur's estimation of market demand is too rough, or the market demand itself is overly variant, we may still observe the project to be far more than 100% funded.

[2] However, the question of how the entrepreneur's strategy G^* changes with demand uncertainty does not have a clear-cut answer according to (7) in main file. Demand uncertainty adds variance to the two spikes, so compared to the deterministic case, $\tilde{L}$'s ($\tilde{N}$'s here) p.m.f diminishes at a slower speed as l increases ($\tilde{N}$ decreases). But how $\frac{dF}{dk}$ responds to higher variance is unclear, as now $F(L \mid k)$ is a weighted sum for different L's given different realization of $\tilde{T}$'s. Compared to a deterministic case with $\tilde{T}$ equal the mean of its distribution, $F(L \mid k)$ is evaluated at higher L (thus higher $\frac{dF}{dk}$) when $\tilde{T}$ is greater than $\tilde{T}$'s mean, and vise versa, so whether $\frac{dF}{dk}$ as a weighted sum is higher or lower than the deterministic case is indeterminate. The same logic holds for random arriving time, which essentially adds variance to the two spikes along the time axis. Though analytically unclear, simulation results reveal that in

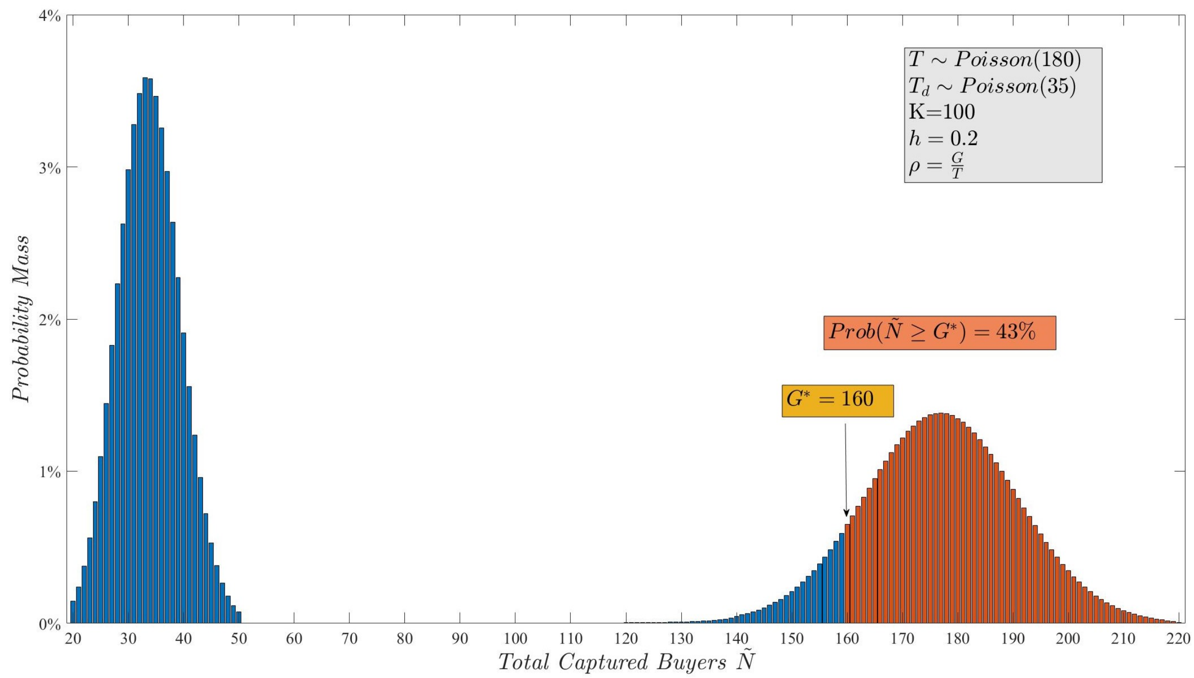


Figure S1: $\tilde{N}$'s p.m.f when T and T_d follow Poisson distribution

Note: This figure shows $\tilde{N}$'s p.m.f when $\tilde{T}$ and $\tilde{T}_d$ follow Poisson distributions with means 180 and 35. Marginal cost is simplified as $c = 0$, while other parameters are specified in the text box. G^* is solved to be 160 with the success probability 43%. The campaign is most likely to get $\frac{33}{160} = 20.6\%$ funded or $\frac{177}{160} = 110.6\%$ funded.

means that it is possible to observe almost-successful failures, i.e., $140 < \tilde{N} < 160$ in the Figure, which in reality is extremely unlikely to happen. Apparently, the current model, with a simplified framework of backers' incentive and behaviors, cannot directly explain the puzzle. We thus give two realistic explanations beyond our model setup, one related to strategic-waiting buyers, the other related to warm-glow donors.

First, buyers who have chosen the wait-and-see strategy may come back at the end of the campaign, and if the campaign is close to success, they will pledge and nudge the campaign to success (more on this point in the next Subsection). Second, it is found that many non-equity crowdfunders are warm-glow altruists with non-monetary incentives to support creative projects (Cecere et al., 2017). Specifically, Deb et al. (2019) wrote "There is a spike in donations and purchases [which may include those wait-and-see buyers] just before success, but once a campaign reaches its goal, donations drop..." Donors nudge almost-successful failure to 100% success, while

most times the former force dominates, namely the p.m.f decreases at a slower speed making the action of decreasing G (to include more l's) more beneficial, so G^* is lower than the deterministic counterpart in these cases. For example, under the same parameter setting as Figure S1 with $T = 180$ and $T_d = 35$, the optimal G^* is solved numerically to be 170. Again, how G^* changes with higher variance in the two spikes generally needs case-by-case analysis, depending on parameters as well as how much variance is added.

wait-and-see buyers and other normal buyers (who arrive after the donor's action) may further make last-minute pledges crossing the 100% threshold, resulting in slightly more than 100% funded. Eventually, we may observe the blue-bar probabilities to the left of G^* in Figure S1 load onto the orange bars to the right of G^*, almost-successful failures eliminated, while a downward sawtooth pattern may appear at the right of G^*, just as in Figure 1 in main file. Indeed, this sawtooth pattern (100% being most likely with decreasing likelihood over 100%) may due to a lot of realistic factors. Another explanation is provided by Strausz (2017), that in fear of moral hazard, some consumers stop pledging once the campaign reaches its goal and seek for purchase in the after-market, so the campaign is likely to stop right at 100%. For the sawtooth pattern at the barely-funded zone, since buyers are allowed to cancel their pledge when the campaign is live, they may well do so when the campaign unravels towards an apparent failure, thus tilting the probability bars to 0% funded.

All these nudging powers (wait-and-see buyers, donors, consumers fearing moral hazard, and canceling backers), combined with our basic model, give rise to predictions consistent with Figure 1 in main file. But these factors cannot replace the model, since the latter is essential in generating two probability masses around barely funded and just-adequately funded, explaining why halfway failure and blockbuster success are both small-probability events.

S1.2: Wait-and-see strategy

Many platforms facilitate strategic waiting for backers who have not made up their mind to pledge: They can follow the campaign and receive reminders when the campaign is about to close. That is, if a common buyer arrives at the campaign and decides not to pledge due to high uncertainty about whether the campaign can succeed, she can choose to wait and see and come back later.

Indeed, if wait-and-see strategies are used to reduce the uncertainty about the campaign's outcome, it is rational for buyers to come back in the very end, when such uncertainty is the smallest (which is also how major platforms design the timing of such reminders). Note that even if buyers come back in the end, the pledging decision may still involve hassle costs (sign up, make the order, release private information...), so they would not just pledge without thinking. Now, how would such strategic waiting affect the model's result? First, it is easily seen that these buyers would pledge if the campaign has already succeeded, not pledge if the campaign is far from success.

What if the campaign is close to success, but more than one pledge (from this buyer) is needed? Possibilities are: (i) They can pledge more than the price, up to $\frac{v}{1+h}$ if that helps the campaign to succeed (as analyzed in Hu et al., 2015). (ii) They may expect other lurking wait-and-see backers to collaboratively make the campaign successful. (iii) They may expect donors to help complete the campaign. All these possibilities call for the buyer to pledge, thus nudging the almost-successful campaigns to real success, while whether and which of the scenarios would happen is up to the buyer's characteristics. Deb et al. (2019) does document a last-minute surge of *purchases*.

To summarize, strategic waiting adds funds for nearly/already successful campaigns, while how much it adds depends on how many buyers choose to come back and to collaborate for close successes, thus making almost-successful failures less likely to happen.

S1.3: Random arriving time

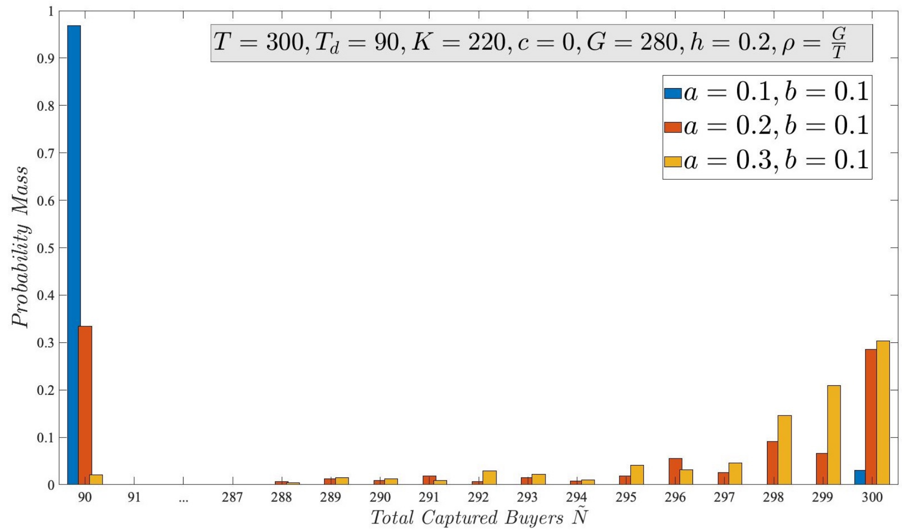
Many crowdfunding studies have documented a U-shape funding dynamic: The contribution tends to spike at the beginning and the end of the campaign (see e.g., Deb et al., 2019). The last-minute funding spike could owe to donors and strategic-waiting buyers. For the early spike, it may reflect high arrival rate when the campaign just starts, either because of the platform's algorithm for new campaign exposures or due to the entrepreneur's diligent early promotions. Therefore, it is desirable to relax the model's arrival timing to allow for denser arrivals in the beginning.[3] The question is whether the two probability spikes are still present with denser, random arrival in the beginning. Since the problem cannot be analytically solved, we use simulations to test its robustness by varying model parameters, and the simulation results support our earlier results: There are two probability spikes if there is no drastic difference for early- and late-period arrival rate; If there is, then the campaign may only have the all-captured spike because denser early arrival is favourable.

In the following Figure S2 we provide two simulation examples for (i) deterministic arrival with denser arrival in the beginning (which is a close analogue to our basic model), and (ii) random arrival with higher arrival rate in the beginning. The results show that $\tilde{L}$'s p.m.f in (i) resembles Figure 3 in main file, only that denser early arrival (while fixing T, so sparser late arrival) improves $\tilde{L}$'s p.m.f by tilting probabilities to the all-captured spike, because denser early arrival makes it easier to reach critical mass early on. Similarly, $\tilde{N}$'s p.m.f resembles Figure S1 because (ii) may be seen as a probabilistic weighted sum of (i) along the time axis, resulting in higher variance at the two spikes.

In theory, denser early arrival raises a new question that is absent in the baseline model: Does the pledging cascade stop when the arrival rate suddenly drops? Recall in the basic model the two probability spikes correspond to two scenarios: Pledging cascade starts very early, or it never starts. That is why a medium number of lost buyers is not likely—The cascade is unlikely to start halfway.

[3] Dense arrival at the deadline does not change the preceding funding dynamics, and has been discussed in the previous Subsection.

Now (a) if the cascade starts very early, by the time the arrival rate drops down, the campaign should have accumulated a good number of pledges, thus the cascade would continue even if the next buyer arrives after some while. The more drastic the arrival rate difference is, the later the next buyer arrives (unfavourably), but the more pledges have been collected within the short early period (favourably). It turns out the latter force always dominates. In fact, if one allows part of T buyers arrive earlier than before in the beginning, the rest evenly distributed along the time, it is easily seen that everyone arrives earlier than before except for the last buyer! (b) If the cascade has not started, it is unlikely to start after the arrival rate drops. Is it possible to have an additional spike where the campaign captures exactly the early-period buyers? If there are many such buyers, then reasoning (a) carrier over; if there are few of them, even if they form a spike, it should be near the dedicated-only spike because of their small amount. In all, our basic result seems robust to denser, random early arrival, accounting for the U-shape funding dynamics observed in data.



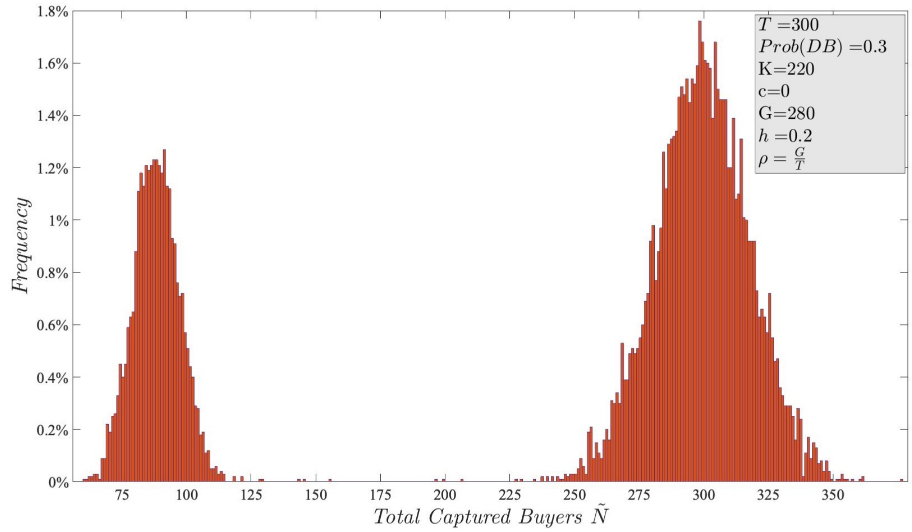


Figure S2: Denser early Arrival, deterministic time (top) and stochastic time (bottom)

Note: (1) **The top figure** displays simulation results with 1,000 repetition for each parameter pair. $a * T$ buyers arrive evenly (i.e., deterministic time) before $b * Duration$, while the other $(1 - a) * T$ buyers arrive evenly during the later $(1 - b) * Duration$, $a > b$ meaning denser early arrival, and T is the same for all scenarios. So the blue bars are the benchmark baseline model, while increasing a further tilts the arrivals to the early period, which increases the campaign's success probability, reflected in higher probability around the all-captured spike. (2) **The bottom figure** displays simulation results with 10,000 repetition. Buyers' arrivals are assumed to follow a Poisson process, i.e., the time interval (Δt) between two arrivals follows an exponential distribution. The duration is $T = 300$ periods. For the early 10% duration, Δt is generated from an exponential distribution with mean $\frac{1}{2}$, and $\frac{9}{8}$ for the later 90% duration. So on average, $E(\Delta t) = \frac{0.1}{1/2} + \frac{0.9}{9/8} = 1$, thus 300 expected total buyers. Since exponential distribution is memoryless, each arrival is generated independently; and upon each arrival, the buyer's type is drawn from a Bernoulli trial with probability 0.3 to be a dedicated buyer. So this example is the stochastic variation of the orange bars $a = 0.2(= \frac{10\%}{1/2}), b = 0.1$ in top figure. Random arrival increases the variance of the two probability spikes.

S1.4: Buyer heterogeneity

In Subsection 3.2.2 (C) in main file we have demonstrated that buyers' heterogeneity may emerge from different aspects (e.g., v, h, r), but all affecting the model via one variable, their critical masses k. Hence, higher heterogeneity means more types of buyers with different k's.

In the following Figure S3 we give an example of four types: $k_0 = 0$ (dedicated buyer), $k_1 = \frac{1}{3}$, $k_2 = \frac{1}{2}$, $k_3 = \frac{2}{3}$, with $n_0 = n_1 = n_2 = n_3 = 25$ being the number of each type. We use simulation to get $\tilde{L}$'s p.m.f, which turns out to retain the two-spike feature. Further varying the k's and n's gives us similar results. Why isn't additional spike forming? The answer is similar to that for denser early arrivals, that once the cascade starts for low-critical-mass (say, k_1) buyers, the funds accrue from k_0 and k_1 buyers and by the time k_2/k_3 buyer arrives their critical masses are likely to have been achieved. For example, when the third k_1 buyer observes $\frac{n}{t} = \frac{1}{3}$ and starts to pledge, the next buyer would observe $\frac{n}{t} = \frac{2}{4}$ and pledge if he is k_0, k_1 or k_2-type. Indeed, the probability that cascade starts for low-k buyers but stops for high-k buyers is extremely low (the cascade stops only if there are consecutive arrivals of high-k buyers). On the other hand, the probability is low to lose all high-k buyers *before* the cascade starts, because these buyers must all arrive very early before other buyers show up and such arriving order has low probability. Even if it happens, then the campaign is unlikely to regain momentum after losing so many buyers and would end up in the dedicated-only spike. Therefore, both simulation and theoretic reasoning support that there would still be two spikes in the presence of higher heterogeneity among buyers, thus our main results are robust.

With higher heterogeneity, the ideal scenario is to have buyers arrive in the order of their k's (low to high). Using the previous example, dedicated buyers still need to arrive very early to accumulate funds, but the best case is to have $k = \frac{1}{3}$ arrive soon after, because these buyers' required critical mass is the lowest thus the least dedicated buyers are required to arrive before them. Then low-k buyers accumulate funds and help achieving the critical masses for all types of buyers, leading to a potential campaign success. However, this is just the ideal scenario while in reality buyers' arrivals are subjected to many random factors. Nevertheless, the theory provides a trafficking benchmark for the entrepreneur: The early trafficking target should be low-k buyers, i.e., buyers with higher product valuation (category fans), lower hassle costs (impulse buyers or engaging customers), better optimism (social networks).

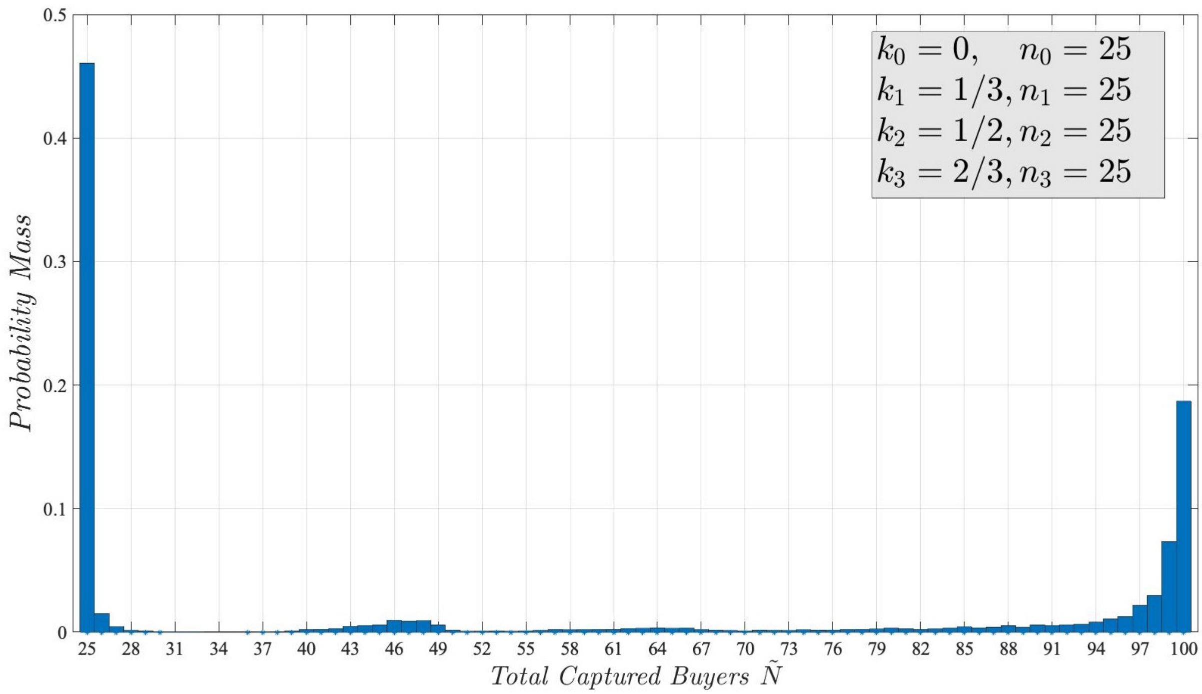


Figure S3: Baseline Model with Four Types of Buyers

Note: This figure shows $\tilde{N}$'s p.m.f when there are 100 buyers of 4 types, each type's pledging threshold k and amount n shown in text box. Each type has 25 buyers. The reason to have the same amount of each type is to (i) maximize the likelihood for additional spikes to show up (which turns out to be not the case, agreeing with the model's robustness) (ii) simplify the theoretic enumeration. We use theoretic enumeration method to derive Prob($\tilde{N} = 100$), and Prob($\tilde{N} = 99$). For Prob($\tilde{N} = 100$) or losing 0 buyer, it must be: at $t = 1$, k_0-type arrives; at $t = 2$, since $\frac{n}{t} = \frac{1}{2}$, $k_0/k_1/k_2$-type arrives, and the pledging cascade starts at $t = 3$ when $\frac{n}{t} = \frac{2}{3}$. So there are 3 possible arriving orders. Using combinatorics, the probability is 18.7%. Similarly, for Prob($\tilde{N} = 99$), the 1 lost buyer is possible to be lost at $t = 1$ or $t = 2$, and manual enumeration gives rise to 72 possible arriving orders, while the total probability is 7%. As seen, enumeration's complexity increases rapidly, and we cannot do this for all N. So we use simulation to generate the graph: Instead of calculate the possibility manually, we shuffle (Fisher–Yates shuffling) the arriving order for 10,000 times and record the number of captured buyers for each shuffle using the behavioral rule. The figure shows that the shuffle gives a consistent result as the theoretic enumeration for Prob($\tilde{N} = 100$) and Prob($\tilde{N} = 99$), thus the simulation is unbiased. To conclude, the two-spike result (thus the model) is robust to higher heterogeneity, and the reason is that the cascade is unlikely to stop when high-k buyer arrives once started by low-k buyers in the beginning.

S2: Proofs

S2.1: $\tilde{L}$'s Distribution for $k = \frac{1}{x}$

Definition 1. *Given k, define $\frac{n}{n+l+1}$ as the* ***pledging threshold*** *for the l, such that $\frac{n}{n+l+1} \geq k$ where n is the smallest integer satisfying this inequality. $\tilde{L}$'s p.m.f is directly calculated from these pledging thresholds.*

2.1.1 An illustration with $k = \frac{1}{3}$

Fixing the number of buyers (T) and dedicated buyers (T_d), we are interested in the distribution of the lost buyers $\tilde{L}$ when the entrepreneur sets a backers' goal such that the critical mass $k(G) = \frac{1}{3}$. Specifically, we want to derive Prob($\tilde{L} = l$) for all $l \in \{0, 1, ..., T - T_d\}$.

First, consider Prob($\tilde{L} = 0$). If the first arrival is a CB (common buyer), as $n = 0$ and $t = 1$, she would not pledge since $\frac{0}{1} < \frac{1}{3}$. For $\tilde{L} = 0$, the first arrival must be a dedicated buyer (DB). If so, the second buyer will pledge regardless of her type, as $\frac{1}{2} > \frac{1}{3}$, and meanwhile the pledging cascade starts given $\frac{n+1}{t+1} > \frac{n}{t}$. So, the first arrival being a DB is necessary and sufficient for $\tilde{L} = 0$, no matter in what order the rest of buyers arrive. Therefore, Prob($\tilde{L} = 0$) $= C_{T-1}^{T_d-1}/C_T^{T_d}$.

Next, if $\tilde{L} = 1$, the first arrival must be a CB who would not pledge, otherwise $\tilde{L} = 0$. The second arrival must be a DB, otherwise $\tilde{L} \geq 2$. Then the third arrival will always pledge, as $\frac{1}{3} \geq \frac{1}{3}$, and the pledging cascade starts. Therefore, the first two arrivals being (CB, DB) is necessary and sufficient for $\tilde{L} = 1$, and Prob($\tilde{L} = 1$) $= C_{T-2}^{T_d-1}/C_T^{T_d}$.

Similarly, let the initial 4 arrivals be ordered as (CB, CB, DB, DB), then the 5^{th} buyer observing $\frac{2}{5} > \frac{1}{3}$ will pledge and the pledging cascade starts. Readers can check that this is necessary and sufficient for losing exactly 2 buyers. So Prob($\tilde{L} = 2$) $= C_{T-4}^{T_d-2}/C_T^{T_d}$.

The key technique to derive such probabilities is to place exactly l CBs in the front who do not pledge, accompanied by n DBs such that $\frac{n}{l+n+1} \geq k$, so the $(l + n + 1)^{th}$ buyer will pledge and the cascade starts afterwards.[4] Such $\frac{n}{l+n+1}$ is defined as the *pledging threshold* for l given some k, from which $\tilde{L}$'s p.m.f is directly calculated. The complexity of such ordering grows when l becomes larger. For instance, if $\tilde{L} = 3$, we need to place 3 CBs in the front accompanied by 2 DBs so that the 6^{th} buyer who faces $\frac{2}{6} \geq \frac{1}{3}$ will pledge. However, the first arrival must be a CB, otherwise $\tilde{L} = 0$. Similar restrictions need to be considered for $\tilde{L} \neq 1, 2$. There turns out to be two ways to order the front 5 buyers: (CB, CB, CB, DB, DB) and (CB, CB, DB, CB, DB). These orderings

[4] The smallest such n would suffice because it already secures the cascade to happen, and it does not matter in what order the rest of buyers arrive.

are the necessary and sufficient conditions for $\tilde{L} = 3$. Therefore, Prob($\tilde{L} = 3$) $= 2 * C_{T-5}^{T_d-2}/C_T^{T_d}$.

For any critical mass $k \in (0, 1)$, $\tilde{L}$'s p.m.f can be derived similarly. For example, if $k = \frac{1}{2.5}$, Prob($\tilde{L} = 0$) is the same as that of $k = \frac{1}{3}$, since $\frac{1}{2} > \frac{1}{2.5}$; but Prob($\tilde{L} = 1$) is different, since $\frac{1}{3} < \frac{1}{2.5}$. For the latter, the initial 3 buyers must be ordered as (CB, DB, DB), for the 4^{th} buyer to observe $\frac{2}{4} > \frac{1}{2.5}$ and to start the pledging cascade. So Prob($\tilde{L} = 1$) $= C_{T-3}^{T_d-2}/C_T^{T_d}$ when $k = \frac{1}{2.5}$, which is lower than $C_{T-2}^{T_d-1}/C_T^{T_d}$ for $k = \frac{1}{3}$. Intuitively, a higher critical mass worsens $\tilde{L}$'s distribution in the sense that the campaign is likely to lose more buyers.

When T and T_d are fixed, the campaign's outcome depends entirely on how these buyers arrive in order. The campaign must achieve the critical mass, or in other words, the funding progress must gain a certain level of momentum (as often used in the financial market) to convince the common type to pledge. The entrepreneur can lower the critical mass by lowering the price, which partially cancels out the common buyers' hassle cost and makes them more prone to pledge, but meanwhile it is harder to achieve the higher backers' goal.

2.1.2 Lemma 1

Definition 2. *Let B_x be a matrix with a typical entry $b(i, j)$. B_x is defined recursively as follows. For the first row, $b(1, 1) = x$, $b(1, 2) = 1$, and $b(1, j) = 0$ for all $j \geq 3$. For the $(i + 1)^{th}$ row where $i \geq 1$, $b(i + 1, 1) = \sum_{s=1}^{x} C_x^s b(i, s)$ and $b(i + 1, j) = \sum_{s=0}^{x} C_x^s b(i, j - 1 + s)$ for all $j \geq 2$.*

Definition 3. *For some $n \in \{1, 2, 3, ...\}$, let a_n^m be a sequence defined via the matrix B_x, indexed by m where $m \in \{1, 2, 3, ..., x - 1\}$. If $n = 1$ or 2, $a_1^m = 1$ and $a_2^m = m$ for all m. If $n \geq 3$, $a_n^m = \sum_{s=1}^{m} C_m^s b(n - 2, s)$ for all m.*

Lemma 1. *Suppose $k = \frac{1}{x}$ for some $x \in \{2, 3, 4, ...\}$. For notation convenience, divide l by $(x - 1)$ and let $(n - 1)$, $(m - 1)$ denote the quotient and remainder.*

1. When $T_d \leq G - 1$

(i) For all l in $0 \leq l \leq min(T_d(x - 1) - 1, T - T_d - 1)$,

$$Prob(\tilde{L} = l) = a_n^m \frac{C_{T-(x(n-1)+m)}^{T_d-n}}{C_T^{T_d}}$$

(ii) If $T_d(x - 1) \leq T - T_d - 1$, for those l in $T_d(x - 1) \leq l \leq T - T_d - 1$, $Prob(\tilde{L} = l) = 0$.

2. When $T_d \geq G$

(i) For those l in $0 \leq l \leq min((G - 1)(x - 1) - 1, T - T_d - 1)$, $Prob(\tilde{L} = l)$ is derived the same as 1(i).

(ii) If $(G-1)(x-1) \leq T - T_d - 1$*, for those* l *in* $(G-1)(x-1) - 1 \leq l \leq T - T_d - 1$*, write* l *as* $l = (G-1)(x-1) + e$ *where* $e \in \{0, 1, ...(T - T_d - 1) - (G-1)(x-1)\}$*, and*

$$Prob(\tilde{L} = l) = (\sum_{s=1}^{x+e} C_{x+e}^{s} b(G-3, s)) \frac{C_{T-(x(G-1)+e)}^{T_d-(G-1)}}{C_T^{T_d}}.$$

3. In both cases, $Prob(\tilde{L} = T - T_d) = 1 - \sum_{l=0}^{T-T_d-1} Prob(\tilde{L} = l)$*.*

2.1.3 Proof of Lemma 1

Let $Cases(\tilde{L} = l)$ denote the total number of arriving orders that map to an outcome of l many lost buyers. If readers refer to proof of Proposition 3 in the main paper, $Cases(\tilde{L} = l)$ is $\parallel f^{-1}(\{l\}) \parallel$, where $f : \mathbb{O} \to \{0, 1, ..., T - T_d\}$ is the function mapping any arriving order to an outcome of l many lost buyers. We start with an example of $k = \frac{1}{3}$.

An Example of $x = 3$

For $l = 0$, the first arrival must be a DB (dedicated buyer) because all initial arrivals of CBs (common buyer) will not pledge before a DB shows up ($\frac{0}{t} < k$). Assume the first arrival is a DB, then the second buyer facing $\frac{1}{2} \geq k = \frac{1}{3}$ will pledge regardless of her type. Thus the pledging cascade starts, as $\frac{n+1}{t+1} > \frac{n}{t}$. So, $Cases(\tilde{L} = 0) = C_{T-1}^{T_d-1}$ and $Prob(\tilde{L} = 0) = Cases(\tilde{L} = 0)/C_T^{T_d} = \frac{T_d}{T}$.

Notice the key technique to derive $Cases(\tilde{L} = l)$ for some l is to find the smallest n such that $\frac{n}{n+l+1} \geq k$ ($n = 1$ in the previous case). That is, the campaign loses l many CBs who had arrived early before the critical mass is reached, accompanied by n many DBs to support the loss by forming and reaching the critical mass $\frac{n}{n+l+1} \geq k$, so that the $(l+n+1)^{th}$ buyer will pledge regardless of her type, and thus the pledging cascade starts, in which case the campaign loses exactly l many CBs eventually. The smallest such n would suffice because it already secures the cascade to happen, and it does not matter who is the next to arrive. Henceforth, given k,we define the key threshold $\frac{n}{n+l+1}$ as the *pledging threshold* for each l.

However, having l many CBs and n many DBs in the front is only a necessary condition for $\tilde{L} = l$. They must be additionally placed in a nice order such that the cascade starts exactly at the $(l+n+1)^{th}$ arrival, not **earlier**. That is to say, when deriving $Cases(\tilde{L} = l)$, we need to carefully exclude those cases such that $\tilde{L} = 0, 1, ..., l-1$. So, to derive $Cases(\tilde{L} = l)$ we need to take into account: (1) the necessary condition for $\tilde{L} = l$ associated with its pledging threshold (l CBs and n DBs being placed before the cascade starts at the $(l+n+1)^{th}$ arrival), and (2) the sufficient conditions for $\tilde{L} \neq 0, 1, ..., l-1$. These requirements naturally lead to a recursive method to solve for $\tilde{L}$'s p.m.f.

To simplify the notation, we have the next two definitions.

Definition 4. *Define "$\{\nu\} = / \geq \kappa$" to be the statement "in the initial ν arrivals, the number of CBs is equal to/ no less than κ".*

Definition 5. *Define "$\{\nu\}^2 = / \geq \kappa$" to be the statement "in the initial $\nu+2$ arrivals, the number of CBs is equal to/ no less than $\kappa + 2$".*[5]

Recall we have written l as a multiple of $(x-1)$, in this case $x = 3$ and thus $l = 2(n-1)+(m-1)$. It turns out that for each l, such n derived from the division of l and $(x-1)$ is exactly the n defined in the pledging threshold (the smallest n that makes the inequality to be true). The examination is left to readers. In Table 2, we derived conditions on the arriving order of DB and CB for $\tilde{L} = l$ for each l. Because n represents the number of DBs that must be placed in the front to reach the critical mass, it is called **DB hit** in the table.

l	**Pledging threshold**	**DB hit (n)**	**m**	**Necessary condition for $\tilde{L} = l$**	**Sufficient condition for $\tilde{L} \neq l$**
0	1/2	1	1	$\{1\} = 0$	$\underline{\{1\} = 1}$
1	1/3	1	2	$\{2\} = 1$	$\underline{\{2\} = 2}$
2	2/5	2	1	$\{2\}^2 = 0$	$\underline{\{2\}^2 \geq 1}$
3	2/6	2	2	$\underline{\{3\}^2 = 1}$	$\underline{\underline{\{3\}^2 \geq 2}}$
4	3/8	3	1	$\{5\}^2 = 2$	$\{5\}^2 \geq 3$
5	3/9	3	2	$\{6\}^2 = 3$	$\underline{\underline{\{6\}^2 \geq 4}}$
6	4/11	4	1	$\{8\}^2 = 4$	$\{8\}^2 \geq 5$
7	4/12	4	2	$\{9\}^2 = 5$	$\underline{\underline{\{9\}^2 \geq 6}}$
8	5/14	5	1	$\{11\}^2 = 6$	$\{11\}^2 \geq 7$
9	5/15	5	2	$\{12\}^2 = 7$	$\underline{\underline{\{12\}^2 \geq 8}}$
...	...	...	...	...	...
$2(n-2)$	$(n-1)/(3n-4)$	$n-1$	1	$\{3n-7\}^2 = 2n-6$	$\{3n-7\}^2 \geq 2n-5$
$2(n-2)+1$	$(n-1)/(3n-3)$	$n-1$	2	$\{3n-6\}^2 = 2n-5$	$\underline{\underline{\{3n-6\}^2 \geq 2n-4}}$
$2(n-1)$	$n/(3n-1)$	n	1	$\{3n-4\}^2 = 2n-4$	$\{3n-4\}^2 \geq 2n-3$
$2(n-1)+1$	$n/3n$	n	2	$\{3n-3\}^2 = 2n-3$	$\underline{\underline{\{3n-3\}^2 \geq 2n-2}}$
...	...	...	...	...	...

Table 2: Conditions for $\tilde{L} = l$, $x = 3$

The last two columns of Table 2 represent the conditions that are crucial to calculate $Cases(\tilde{L} = l)$. The necessary condition for $\tilde{L} = l$ directly follows the pledging threshold. For instance, for $\tilde{L} = 1$, the necessary condition is $\{2\} = 1$, i.e., 1 CB among the initial 2 arrivals. However, for $\tilde{L} \neq 0$, we need to have its sufficient condition $\{1\} = 1$, i.e., the first arrival being a CB. Combining these two,

[5] For instance, $\{2\} = 2$ means the initial 2 arrivals are all CBs; and $\{2\}^2 = 0$ means the initial 4 arrivals include 2 CBs and 2 DBs.

the sufficient and necessary condition for $\tilde{L} = 1$ becomes $\{2\} = 1$ and $\{1\} = 1$, uniquely pinning down the initial two arrivals as (CB, DB).

Table 2 shows all such conditions for each $\tilde{L} = 2(n-1) + (m-1)$, where $n = 1, 2, ...$ and $m = 1, 2$. The sufficient and necessary conditions for $\tilde{L} = l$ include (1) the necessary condition for $\tilde{L} = l$ and (2) the sufficient conditions for $\tilde{L} \neq 0, 1, ..., l-1$. For instance, $\tilde{L} = 3$ **if and only if** all 4 single-underlined conditions in Table 2 are satisfied.

Readers may notice that for $\tilde{L} \neq 0$ or 1, the first two arrivals must be (CB, CB), i.e., $\{2\} = 2$. As it must be satisfied for all $l = 2, 3, 4...$, we change the notation for all $l \geq 2$ to "$\{\cdot\}^2 = \cdot$" as defined above, thus only considering the placement from the third arrival.

The necessary condition is easily taken care of. The tricky part is to take care of all sufficient conditions for $\tilde{L} \neq 0, 1, ..., l-1$, and the complexity increases with l. How to approach $Cases(\tilde{L} = l)$ while taking into account all these conditions? First, notice these sufficient conditions can be reduced. $\{\nu\}^2 \geq \kappa \Rightarrow \{\nu - 1\}^2 \geq \kappa - 1$, that is, if there are at least $\kappa + 2$ CBs in the initial $\nu + 2$ arrivals, it must be true that there are at least $\kappa + 1$ CBs in the initial $\nu + 1$ arrivals. So, it suffices to consider all the double-underlined conditions, that is, $\{3(n-1)\}^2 \geq 2(n-1)$, as each of them implies the one condition above it in the table.

We need to generate an auxiliary matrix $B_{x=3} = (b(i,j))_{i,j \in \{1,2,3,...\}}$ which helps to identify $Cases(\tilde{L} = l)$ with all the double-underlined conditions in Table 2 considered. The definition for an entry $b(i,j)$ is "the number of cases to place the initial $3i + 2$ buyers, such that $\{2\} = 2$, $\{3i\}^2 = 2i + j - 1$ and $\{3n\}^2 \geq 2n$ for all $n < i$". That is, the initial 2 arrivals are CBs, the following $3i$ arrivals contain $2i + j - 1$ CBs, and for all $n < i$, the following $3n$ arrivals (from the third arrival) contain no less than $2n$ CBs. With the definition, $j \leq i + 1$ follows naturally, as $2i + j - 1 \leq 3i$ from the expression $\{3i\}^2 = 2i + j - 1$. So, let $b(i,j) = 0$ for all $j > i + 1$. We will discuss shortly how to connect B_x and $Cases(\tilde{L} = l)$.

Definition 6. *Let* $\{3i\}^2_B = 2i + j - 1$ *denote the number of cases to place the initial* $3i + 2$ *many buyers such that* $\{2\} = 2$, $\{3i\}^2 = 2i + j - 1$ *and* $\{3n\}^2 \geq 2n$ *for all* $n < i$. *Then,* $b(i,j) = Cases(\{3i\}^2_B = 2i + j - 1)$.[6]

Now, we are going to generate B_x based on the definition of its entry, and show that the B_x generated in this way is the same as defined in Definition 2.

When $i = 1$ (the first row), $b(1,1) = Cases(\{3\}^2_B = 2) = C_3^2 = 3$. Place 2 CBs in the first and second, and choose 2 out of the 3 following arrivals (C_3^2) to place the other 2 CBs. Since $i = 1$,

[6]Notice, the notation $\{3i\}^2_B = 2i + j - 1$ represents a stricter placement requirement than $\{3i\}^2 = 2i + j - 1$, as it takes into account extra conditions.

no more conditions regarding $n < i$ need to be considered. Similarly, $b(1,2) = Cases(\{3\}_B^2 = 3) = C_3^3 = 1$, and $b(1,j) = 0$ for all $j \geq 3$. So, the first row of B_x has specified how to place the initial 5 buyers as defined.

When $i = 2$ (the second row) and $j = 1$, we need to consider $\{2\} = 2$, $\{6\}^2 = 4$, and also $\{3\}^2 \geq 2$. For the last one, there are two possibilities, $\{3\}^2 = 2$ and $\{3\}^2 = 3$. Notice $\{3\}^2 = 2 \Leftrightarrow \{3\}_B^2 = 2$, and $\{3\}^2 = 3 \Leftrightarrow \{3\}_B^2 = 3$, which coincide with $b(1,1)$ and $b(1,2)$. Therefore, $b(2,1) = Cases(\{6\}_B^2 = 4) = Cases(\{3\}^2 = 2)C_{6-3}^{4-2} + Cases(\{3\}^2 = 3)C_{6-3}^{4-3} = b(1,1)C_3^2 + b(1,2)C_3^1$.

Similarly, $b(2,2) = Cases(\{6\}_B^2 = 5) = Cases(\{3\}^2 = 2)C_{6-3}^{5-2} + Cases(\{3\}^2 = 3)C_{6-3}^{5-3} = b(1,1)C_3^3 + b(1,2)C_3^2$, and $b(2,3) = Cases(\{6\}_B^2 = 6) = Cases(\{3\}^2 = 3)C_{6-3}^{6-3} = b(1,2)C_3^3$. The second row of B_x has specified how to place the initial 8 buyers as defined.

When $i = 3$ (the third row) and $j = 1$, we need to consider $\{2\} = 2$, $\{9\}^2 = 6$, $\{3\}^2 \geq 2$ and $\{6\}^2 \geq 4$. Notice for the last one, there are 3 possibilities, $\{6\}^2 = 4, 5, 6$, corresponding to $b(2,1)$, $b(2,2)$ and $b(2,3)$, which have already taken into account $\{3\}^2 \geq 2$ and $\{2\} = 2$. Therefore, $b(3,1) = Cases(\{9\}_B^2 = 6) = Cases(\{6\}_B^2 = 4)C_{9-6}^{6-4} + Cases(\{6\}_B^2 = 5)C_{9-6}^{6-5} + Cases(\{6\}_B^2 = 6)C_{9-6}^{6-6} = b(2,1)C_3^2 + b(2,2)C_3^1 + b(2,3)C_3^0$.

Similarly, $b(3,2) = Cases(\{9\}_B^2 = 7) = Cases(\{6\}_B^2 = 4)C_{9-6}^{7-4} + Cases(\{6\}_B^2 = 5)C_{9-6}^{7-5} + Cases(\{6\}_B^2 = 6)C_{9-6}^{7-6} = b(2,1)C_3^3 + b(2,2)C_3^2 + b(2,3)C_3^1$, etc.

It is left to readers to check that, the recursive formula for the $(i+1)^{th}$ row of B_x is, $b(i+1,1) = \sum_{s=1}^{3} C_3^s b(i,s)$, and $b(i+1,j) = \sum_{s=0}^{3} C_3^s b(i, j-1+s)$ for $j \geq 2$,[7] which coincides with Definition 2.

Based on matrix B_x, the last step is to derive $Cases(\tilde{L} = l)$ for each $l = 2(n-1) + (m-1)$, $m = 1, 2$. From the last two rows of Table 2, the necessary condition is $\{3n-5+m\}^2 = 2n-5+m$, and the sufficient conditions are $\{3(n-2)\}^2 \geq 2(n-2)$, ..., $\{3\}^2 \geq 2$ and $\{2\} = 2$.

When $l = 2(n-1)$, conditions are $\{3n-4\}^2 = 2n-4$ (to place the initial $3n-2$ buyers, with $2n-2$ CBs and n DBs), $\{3(n-2)\}^2 \geq 2(n-2)$, $\{3(n-3)\}^2 \geq 2(n-3)$, ..., $\{3\}^2 \geq 2$ and $\{2\} = 2$. Combining the first two, we get $\{3n-6\}^2 = 2n-4$. Notice, $\{3n-6\}^2 = 2n-4$ coupled with other conditions, $\{3(n-3)\}^2 \geq 2(n-3)$, ..., $\{3\}^2 \geq 2$ and $\{2\} = 2$, correspond to $b(n-2,1)$ as it is defined to be $Cases(\{3n-6\}_B^2 = 2n-4)$. Therefore, $Cases(\tilde{L} = 2(n-1)) = b(n-2,1)C_{T-(3n-2)}^{T_d-n}$, with $C_{T-(3n-2)}^{T_d-n}$ ways to place the rest buyers.

Similarly, when $l = 2(n-1)+1$, conditions are $\{3n-3\}^2 = 2n-3$ and $\{3n-6)\}_B^2 \geq 2n-4$. For the latter, there are two possibilities: $\{3n-6\}_B^2 = 2n-3$, that is, $b(n-2,2)$, leaving 0 CB in the following three arrivals; or $\{3n-6\}_B^2 = 2n-4$, that is, $b(n-2,1)$, leaving 1 CB in the

[7] Notice $C_\alpha^\beta = C_\alpha^{\alpha-\beta}$, and we define $C_\alpha^0 \equiv 1$.

following three arrivals (but notice the last arrival cannot be CB, otherwise the pledging cascade should have started from this instead of the next buyer). Therefore, $Cases(\tilde{L} = 2(n-1)+1) = \big(\sum_{s=1}^{2} C_2^s b(n-2,s)\big) C_{T-(3n-1)}^{T_d-n}$.

These expressions are the same as in Lemma 1 1.(i) when $x = 3$.

Analogue to General x

[Insert Table 4 here.]

Table 2 is a special case of Table 4.[8] Now readers can see why we write l as multiples of $(x-1)$: The pledging thresholds show a cyclic pattern with a period of $(x-1)$. The necessary condition for $\tilde{L} = l$ follows directly from the pledging threshold, and the sufficient condition comes from the contrapositive statement. Again, the sufficient and necessary conditions for $\tilde{L} = l$ include the necessary condition for $\tilde{L} = l$ and the sufficient conditions for $\tilde{L} \neq 0, 1, ..., l-1$. An example is: $\tilde{L} = x$ if and only if all the single-underlined conditions are true in Table 4.

Define the notations and matrix B_x's entry similarly.

Definition 7. *Define "*$\{\nu\}^{(x-1)} = / \geq \kappa$*" to be the statement "in the initial* $\nu + (x-1)$ *arrivals, the number of CBs is equal to/ no less than* $\kappa + (x-1)$*".*

Definition 8. *Let* $\{ix\}_B^{(x-1)} = i(x-1)+j-1$ *denote the number of cases to place the initial* $ix+(x-1)$ *many buyers, such that* $\{x-1\} = x-1$*,* $\{ix\}^{(x-1)} = i(x-1)+j-1$ *and* $\{nx\}^{(x-1)} \geq n(x-1)$ *for all* $n < i$*. Then,* $b(i,j) = Cases(\{ix\}_B^{(x-1)} = i(x-1)+j-1)$*; and* $b(i,j) = 0$ *if* $j \geq i+1$*.*

It is left to readers to check that, by Definition 8, B_x can be generated recursively as follows: $b(1,1) = C_x^{x-1} = x$, $b(1,2) = C_x^x = 1$; $b(i+1,1) = \sum_{s=1}^{x} C_x^s b(i,s)$ and $b(i+1,j) = \sum_{s=0}^{x} C_x^s b(i,j-1+s)$ for all $i \geq 1$, the same as defined in Definition 2.

Lastly, we need to derive $Cases(\tilde{L} = l)$ for each $l = (x-1)(n-1)+(m-1)$. When $n = 1$, $l \in \{0, 1, ..., x-2\}$, and the required DB hit in the front is 1. There is only one way to place these $(l+1)$ buyers: placing the DB as the $(l+1)^{th}$ arrival, otherwise the cascade would have started earlier. Therefore, for these l, $Cases(\tilde{L} = l) = 1 * C_{T-(l+1)}^{T_d-1} = C_{T-m}^{T_d-1}$.

When $n = 2$, l is in $\{x-1, x, ..., 2x-3\}$. Notice, as before, the necessary condition for $\tilde{L} = l$ combined with the sufficient condition for $\tilde{L} \neq l-1$ reduces the later to an equality, $\{m\}^{x-1} = m-1$, resulting in $C_m^{m-1} = C_m^1$ ways to place the initial $l+2$ buyers. Then $Cases(\tilde{L} = (x-1)+(m-1)) = C_m^1 * C_{T-(x+m)}^{T_d-2}$.

[8] Tables 4,5,6,7 are wide tables and are thus put at the end of this material for better display.

When $n \geq 3$, $l = (x-1)(n-1) + (m-1)$. Take $l = (x-1)(n-1)$ (i.e., $m = 1$) as an example. Conditions are $\{(n-2)x+2\}^{(x-1)} = (n-2)(x-1)$, $\{(n-2)x\}^{(x-1)} \geq (n-2)(x-1)$, ..., $\{x\}^{(x-1)} \geq x-1$ and $\{x-1\} = x-1$. Combine the first two we get $\{(n-2)x\}^{(x-1)} = (n-2)(x-1)$; coupled with other conditions, it corresponds to $b(n-2,1)$. Therefore, $Cases(\tilde{L} = (x-1)(n-1)) = b(n-2,1) * C_{T-(x(n-1)+1)}^{T_d-n}$. Analogously, we derive for other m, $Cases(\tilde{L} = (x-1)(n-1)+(m-1)) = \left(\sum_{s=1}^{m} b(n-2,s)C_m^s\right) * C_{T-(x(n-1)+m)}^{T_d-n}$.

These expressions are the same as stated in Lemma 1 1(i).

When There Are Unproportionally Too Many CB

From the pledging threshold expressions in Table 4, n DBs can support at most $n(x-1)-1$ many CB losses. Since there are in total T_d DBs, they can support up to $((x-1)T_d - 1)$ CB losses. We are interested in deriving $Prob(\tilde{L} = l)$ for $l \in \{0, 1, 2, ..., T - T_d - 1\}$.[9] Therefore, if $T - T_d - 1 \leq (x-1)T_d - 1$, the algorithm in Section covers all $l \in \{0, 1, ..., T - T_d - 1\}$.

What if $T - T_d - 1 > (x-1)T_d - 1$? Then, it is impossible to lose more than $(x-1)T_d - 1$ CBs, if not losing all of them, as T_d DBs are not sufficient to reach the critical mass after the l^{th} CB loss and to start the pledging cascade. But it is still possible to lose all buyers, as the cascade need not happen. These arguments are demonstrated in Lemma 1 1(ii) and 3.

Change of Pledging Rule Once $n \geq G-1$

Common buyers use $\frac{n}{t} \geq k$ as the pledging rule only when $n+1 < G$. If the buyer can complete the campaign by herself, she would always pledge. However, this change of rule does not make any difference if the cascade has already started after reaching the critical mass. It only makes a difference before the critical mass is reached (before $\frac{n}{n+l+1} \geq k$), in which case the cascade will happen earlier than when the critical mass is reached.[10] That is, the DB hit takes the minimum of the smallest n such that $\frac{n}{n+l+1} \geq k$, and $G-1$. Also notice, as the critical mass is reached by dedicated buyers, this is only possible if $T_d \geq G$.

[Insert Table 5 here.]

The last four rows of Table 5 represent the change of rule, since DB hit must be no more than $G-1$. First, notice at $n = G-1$, two rules are equivalent. So, the fourth to last row is the same as it is in Table 4. From the next row, only $G-1$ DBs are required to start the cascade. Write these l as $l = (G-1)(x-1) + e$, where $e \in \{0, 1, ...(T - T_d - 1) - (G-1)(x-1)\}$ is a new

[9] $Prob(\tilde{L} = T - T_d)$ is simply calculated by Lemma 1 3., as the pledging cascade need not happen.

[10] This point reminds that the critical mass reflects both the percentage funded and the fraction elapsed time, instead of the former alone.

index represented by the underlined number in the column corresponding to m. The necessary and sufficient conditions change accordingly.

Take $l = (G-1)(x-1)$ as an example. The front $(G-1)(x-1)$ CB losses and $G-1$ DBs need to be placed nicely. Combining the necessary condition for $\tilde{L} = (G-1)(x-1)$ and the sufficient condition for $\tilde{L} \neq (G-1)(x-1)-1$, we have $\{(G-2)x\}^{(x-1)} = (G-2)(x-1)$. Other sufficient conditions are $\{x-1\} = x-1$ and $\{nx\}^{(x-1)} \geq n(x-1)$ for all $n \leq G-3$, which correspond to the $(G-3)^{th}$ row of B_x. Compare $\{(G-2)x\}^{(x-1)} = (G-2)(x-1)$ and $\{(G-3)x\}^{(x-1)} \geq (G-3)(x-1)$; the former has x more arrivals and $x-1$ more CBs. Possible placements are $\{(G-3)x\}^{(x-1)} = (G-3)(x-1)$, i.e., $b(G-3,1)$; $\{(G-3)x\}^{(x-1)} = (G-3)(x-1)+1$, i.e., $b(G-3,2)$; $\{(G-3)x\}^{(x-1)} = (G-3)(x-1)+2$, i.e., $b(G-3,3)$ ... and place the rest of CBs ($x-1$, $x-2$, $x-3$, ... respectively) in the additional x arrivals. In total, there are $\sum_{s=1}^{x} C_x^s b(G-3,s)$ ways, thus $Cases(\tilde{L} = (G-1)(x-1)) = \left(\sum_{s=1}^{x} C_x^s b(G-3,s)\right) * C_{T-x(G-1)}^{T_d-(G-1)}$. Similar arguments apply to other e. The results are presented in Lemma 1 2(ii).

2.1.4 Lemma 2

Lemma 2. *The matrix B_x and sequences a_n^m defined in Section have the following expressions:*

1. $b(i,j) = \frac{jC_{x(i+1)}^{i-j+1}}{i+1}$ *when* $j \leq i+1$, *and* $b(i,j) = 0$ *when* $j > i+1$.
2. $a_n^m = \frac{mC_{x(n-1)+(m-1)}^{n-1}}{(x-1)(n-1)+m}$, *for all* $m \in \{1,2,...,x-1\}$ *and* $n \in \{1,2,3,...\}$.

2.1.5 Proof of Lemma 2

Proof for Matrix B_x

We want to show B_x defined recursively in Definition 2 has the following explicit form: $b(i,j) = \frac{jC_{x(i+1)}^{i-j+1}}{i+1}$ when $j \leq i+1$ and $b(i,j) = 0$ when $j \geq i+2$.

First, $b(1,1) = \frac{C_{2x}^1}{2} = x$, $b(1,2) = \frac{2C_{2x}^0}{2} = 1$, $b(1,j) = 0$ when $j \geq 3$, the same as in Definition 2. The first row of B_x is proved, and we use induction to complete the proof.

Assume the expression is true for the i^{th} row of B_x.

By B_x's definition, the first element of the next row is defined recursively by $b(i+1,1) = \sum_{s=1}^{x} C_x^s b(i,s)$; plug in $b(i,s)$, and $b(i+1,1) = \sum_{s=1}^{min(x,i+1)} C_x^s \frac{sC_{xi+x}^{i-s+1}}{i+1}$. Therefore, we want to show $\frac{C_{x(i+2)}^{i+1}}{i+2} = \sum_{s=1}^{min(x,i+1)} C_x^s \frac{sC_{xi+x}^{i-s+1}}{i+1}$.

Notice: $\frac{C_{x(i+2)}^{i+1}}{i+2} = \sum_{s=1}^{min(x,i+1)} C_x^s \frac{sC_{xi+x}^{i-s+1}}{i+1}$

$\Leftrightarrow \frac{C_{x(i+2)}^{i+1}}{i+2} = \frac{x}{i+1} \sum_{s=1}^{min(x,i+1)} C_{x-1}^{s-1} C_{xi+x}^{i-s+1}$

$\Leftrightarrow \frac{C_{x(i+2)}^{i+1}}{i+2} = \frac{x}{i+1} C_{x(i+2)-1}^{i}$

$\Leftrightarrow \frac{i+1}{x(i+2)} C_{x(i+2)}^{i+1} = C_{x(i+2)-1}^{i}$

$\Leftrightarrow C_{x(i+2)-1}^{i} = C_{x(i+2)-1}^{i}$. Thus the equality is proved.

When $2 \le j \le i+1$, $b(i+1, j)$ is defined recursively to be

$$\sum_{s=0}^{x} C_x^s b(i, j-1+s) = \sum_{s=0}^{min(x,i-j+2)} C_x^s \frac{(j-1+s)C_{xi+x}^{i-j-s+2}}{i+1}.$$

Therefore, we want to show

$$\frac{jC_{x(i+2)}^{i-j+2}}{i+2} = \sum_{s=0}^{min(x,i-j+2)} C_x^s \frac{(j-1+s)C_{xi+x}^{i-j-s+2}}{i+1}.$$

Notice: $\frac{jC_{x(i+2)}^{i-j+2}}{i+2} = \sum_{s=0}^{min(x,i-j+2)} C_x^s \frac{(j-1+s)C_{xi+x}^{i-j-s+2}}{i+1}$

$\Leftrightarrow \frac{jC_{x(i+2)}^{i-j+2}}{i+2} = \frac{1}{i+1}(\sum_{s=0}^{min(x,i-j+2)} sC_x^s C_{xi+x}^{i-j-s+2} + (j-1)\sum_{s=0}^{min(x,i-j+2)} C_x^s C_{xi+x}^{i-j-s+2}$

$\Leftrightarrow \frac{jC_{x(i+2)}^{i-j+2}}{i+2} = \frac{1}{i+1}(x\sum_{s=0}^{min(x,i-j+2)} C_{x-1}^{s-1} C_{xi+x}^{i-j-s+2} + (j-1)C_x^s C_{x(i+2)}^{i-j+2})$

$\Leftrightarrow (i+1)jC_{x(i+2)}^{i-j+2} = (i+2)((j-1)C_{x(i+2)}^{i-j+2} + xC_{x(i+2)-1}^{i-j-1})$

$\Leftrightarrow (i-j+2)C_{x(i+2)}^{i-j+2} = (i+2)xC_{x(i+2)-1}^{i-j-1}$

$\Leftrightarrow \frac{(i-j+2)}{x(i+2)} C_{x(i+2)}^{i-j+2} = C_{x(i+2)-1}^{i-j-1}$

$\Leftrightarrow C_{x(i+2)-1}^{i-j-1} = C_{x(i+2)-1}^{i-j-1}$. Thus the equality is proved.

For $j \ge i+2$, by definition $b(i+1,j) = \sum_{s=0}^{x} C_x^s b(i, j-1+s)$. Notice $j-1+s \ge j-1 \ge i+2$, thus $b(i, j-1+s) = 0$ for all $s \in \{0, 1, ..., x\}$. Then $b(i+1, j) = 0$.

The proof is complete.

Proof for Sequences a_n^m

In Definition 3, a_n^m is defined via matrix B_x. We want to show $a_n^m = \frac{mC_{x(n-1)+(m-1)}^{n-1}}{(x-1)(n-1)+m}$.

When $n = 1$, $a_1^m = \frac{mC_{m-1}^0}{m} = 1$ ($C_0^0 \equiv 0$) for all m. When $n = 2$, $a_2^m = \frac{mC_{x+m-1}^1}{x+m-1} = m$ for all m. The equality is proved for $n = 1, 2$.

When $n \ge 3$, $a_n^m = \sum_{s=1}^{m} C_m^s b(n-2, s) = \sum_{s=1}^{min(m,n-1)} C_m^s \frac{sC_{x(n-1)}^{n-s-1}}{n-1}$. With simple transformation, $a_n^m = \frac{m}{n-1}\sum_{s=1}^{min(m,n-1)} C_{m-1}^{s-1} C_{x(n-1)}^{n-s-1} = \frac{m}{n-1} C_{x(n-1)+(m-1)}^{n-2}$. Notice:

$$\frac{m}{n-1} C_{x(n-1)+(m-1)}^{n-2} = \frac{mC_{x(n-1)+(m-1)}^{n-1}}{(x-1)(n-1)+m}.$$

Thus the equality is proved for $n \ge 3$.

The proof is complete.

2.1.6 Proposition 1

Proposition 1. *Suppose $k = \frac{1}{x}$ for some $x \in \{2, 3, 4, ...\}$. For notation convenience, divide l by $(x-1)$ and let $(n-1)$, $(m-1)$ denote the quotient and remainder.*

1. When $T_d \le G - 1$,

(i) For all l in $0 \le l \le min(T_d(x-1) - 1, T - T_d - 1)$,

$$Prob(\tilde{L} = l) = \frac{mC_{x(n-1)+(m-1)}^{n-1} C_{T-(x(n-1)+m)}^{T_d - n}}{((x-1)(n-1)+m)C_T^{T_d}}.$$

Notice:

(a) When $n = 1$,

$$Prob(\tilde{L} = m-1) = \frac{C_{T-m}^{T_d-1}}{C_T^{T_d}} \le \frac{T_d}{T}(1 - \frac{T_d - 1}{T-1})^{m-1}.$$

(b) When $n = 2$,

$$Prob(\tilde{L} = x+m-2) = \frac{mC_{T-(x+m)}^{T_d-2}}{C_T^{T_d}} \le (\frac{T_d}{T})^2(1 - \frac{T_d - 2}{T-2})^{x+m-2}.$$

(c) When $n \ge 3$,

$$Prob(\tilde{L} = l) \le \frac{T_d}{nT}.$$

(ii) If $T_d(x-1) - 1 < T - T_d - 1$, for those l in $T_d(x-1) \le l \le T - T_d - 1$,

$$Prob(\tilde{L} = l) = 0.$$

2. When $T_d \ge G$,

(i) For all l in $l \le min((G-1)(x-1) - 1, T - T_d - 1)$, $Prob(\tilde{L} = l)$ is derived the same as 1(i);

(ii) If $(G-1)(x-1) - 1 < T - T_d - 1$, for those l in $(G-1)(x-1) \le l \le T - T_d - 1$, rewrite l as $l = (G-1)(x-1) + e$ where $e \in \{0, 1, ...(T - T_d - 1) - (G-1)(x-1)\}$, then

$$Prob(\tilde{L} = l) = \frac{(x+e)C_{x(G-1)+e-1}^{G-3} C_{T-(x(G-1)+e)}^{T_d-(G-1)}}{(G-2)C_T^{T_d}}.$$

3. In both cases, $Prob(\tilde{L} = T - T_d) = 1 - \sum_{l=0}^{T-T_d-1} Prob(\tilde{L} = l)$.

Remark. *This proposition is an extension of Proposition 1 presented in the main paper. The* *inequalities in 1(i) are meant to show that the p.m.f decreases very quickly (of approximately an exponential form) as l increases and will not rise back to a significant level before $\tilde{L} = T - T_d$. One may also be curious whether the scenario of $T_d \geq G$ changes the two-spikes feature. As will be shown in Section , the mass at $\tilde{L} = 0$ is preserved, while the other mass is replaced by a small hump before l reaches $\tilde{L} = T - T_d$. Naturally, with a large number of dedicated buyers, the probability of losing all common buyers becomes small. However, notice all these cases have $T_d \geq G$, meaning the dedicated buyers secure the success. Thus such campaigns will end up in the outlier of Figure 1 (more than 100% funded) in the main paper as blockbusters.*

2.1.7 Proof of Proposition 1

Plug the expressions of a_n^m and $b(i,j)$ from Lemma 2 to Lemma 1, and one gets $\tilde{L}$'s p.m.f as stated in Proposition 1.

Next, we will prove the three inequalities (a)(b)(c) in 1(i).

When $n = 1$, $Prob(\tilde{L} = m - 1) = \frac{C_{T-m}^{T_d-1}}{C_T^{T_d}}$. Open the combinatorial numbers and rewrite the right hand side: $\frac{C_{T-m}^{T_d-1}}{C_T^{T_d}} = \frac{T_d}{T}\prod_{s=0}^{m-2}(\frac{T-T_d-s}{T-1-s})$. Because $T_d \geq 1$, we have $\frac{T-T_d-s}{T-1-s} \leq \frac{T-T_d}{T-1} = 1 - \frac{T_d-1}{T-1}$ for all $s \geq 0$. Therefore, $\frac{C_{T-m}^{T_d-1}}{C_T^{T_d}} \leq \frac{T_d}{T}(1 - \frac{T_d-1}{T-1})^{m-1}$. Thus (a) is proved.

When $n = 2$, $Prob(\tilde{L} = x + m - 2) = \frac{mC_{T-(x+m)}^{T_d-2}}{C_T^{T_d}}$. Open the combinatorial numbers and rewrite the right hand side: $\frac{mC_{T-(x+m)}^{T_d-2}}{C_T^{T_d}} = m(\frac{T_d}{T})(\frac{T_d-1}{T-1})\prod_{s=0}^{x+m-3}(\frac{T-T_d-s}{T-2-s})$. We have $\frac{T_d}{T} > \frac{T_d-1}{T-1}$. Because $T_d \geq 2$ (since $n = 2$), we have $\frac{T-T_d-s}{T-2-s} \leq \frac{T-T_d}{T-2} = 1 - \frac{T_d-2}{T-2}$ for all $s \geq 0$. Therefore, $\frac{mC_{T-(x+m)}^{T_d-2}}{C_T^{T_d}} \leq m(\frac{T_d}{T})^2(1 - \frac{T_d-2}{T-2})^{x+m-2}$. Thus (b) is proved.

For any $n \in \{1, 2, 3, ...\}$, $Prob(\tilde{L} = l) = \frac{mC_{x(n-1)+(m-1)}^{n-1}C_{T-(x(n-1)+m)}^{T_d-n}}{((x-1)(n-1)+m)C_T^{T_d}}$. Notice:

$$C_{x(n-1)+(m-1)}^{n-1}C_{T-(x(n-1)+m)}^{T_d-n} < C_{T-1}^{T_d-1} = \sum_{s=\underline{s}}^{\overline{s}} C_{x(n-1)+(m-1)}^{s}C_{T-(x(n-1)+m)}^{T_d-1-s}$$

where $\underline{s} = max(0, (T_d-1)-(T-(x(n-1)+m)))$ and $\overline{s} = min(T_d-1, x(n-1)+(m-1))$. Therefore, $\frac{mC_{x(n-1)+(m-1)}^{n-1}C_{T-(x(n-1)+m)}^{T_d-n}}{((x-1)(n-1)+m)C_T^{T_d}} < \frac{m}{((x-1)(n-1)+m)}\frac{C_{T-1}^{T_d-1}}{C_T^{T_d}} = \frac{m}{((x-1)(n-1)+m)}(\frac{T_d}{T})$. Lastly, $\frac{m}{(x-1)(n-1)+m} < \frac{x-1}{(x-1)(n-1)+x-1} = \frac{1}{n}$. Thus (c) is proved.

S2.2: $\tilde{L}$'s Distribution for $k = \frac{y-1}{y}$

2.2.1 Lemma 3

Definition 9. *Let A_y be a matrix with a typical entry $a(i,j)$. A_y is defined recursively as follows. For the first row, $a(1,j) = C^0_{jy} = 1$ for all j. For the $(i+1)^{th}$ row where $i \geq 1$, $a(i+1,j) = \sum_{s=1}^{min(jy,i)} C^s_{jy} a(i+1-s, s)$ for all j.*

Definition 10. *Let b_n be a sequence defined via A_y. $b_1 = 1$. $b_{n+1} = \sum_{s=1}^{min(y-1,n)} C^s_{y-1} a(n+1-s, s)$, for all $n \geq 1$.*

Definition 11. *Let C_y be a matrix with a typical entry $c(i,j)$. C_y is defined recursively as follows. For the first row, $c(1,j) = C^j_{y-1}$ for all $1 \leq j \leq y-1$, and $c(1,j) = 0$ for all $j \geq y$. For the $(i+1)^{th}$ row where $i \geq 1$, $c(i+1,j) = \sum_{s=0}^{min(y,j)} C^s_y c(i, j+1-s)$ for all $j \geq 1$.*

Lemma 3. *Suppose $k = \frac{y-1}{y}$ for some $y \in \{1,2,3,...\}$. If $y - 1 > T_d$, $Prob(\tilde{L} = T - T_d) = 1$. Otherwise,*

1. When $T_d \leq G - 1$

(i) For all l in $0 \leq l \leq min(\frac{T_d}{y-1} - 1, T - T_d - 1)$,

$$Prob(\tilde{L} = l) = b_{l+1} \frac{C^{T_d-(l+1)(y-1)}_{T-((l+1)y-1)}}{C^{T_d}_T}.$$

(ii) If $\frac{T_d}{y-1} - 1 < T - T_d - 1$, for all l in $\frac{T_d}{y-1} - 1 < l \leq T - T_d - 1$, $Prob(\tilde{L} = l) = 0$.

2. When $T_d \geq G$

(i) For all l in $0 \leq l \leq min(\frac{G-1}{y-1} - 1, T - T_d - 1)$, $Prob(\tilde{L} = l)$ is derived the same as 1(i).

(ii) If $\frac{G-1}{y-1} - 1 < T - T_d - 1$

(a) When $y - 1 > G - 1$,

$$Prob(\tilde{L} = l) = \frac{C^l_{G+l-2} C^{T_d-(G-1)}_{T-(G+l-1)}}{C^{T_d}_T}.$$

(b) When $y - 1 \leq G - 1$, let $\hat{l} \geq 0$ be such that $(\hat{l}+1)(y-1) \leq G-1$ and $(\hat{l}+2)(y-1) > G-1$. For all l in $\hat{l} < l \leq T - T_d - 1$, rewrite l as $l = \hat{l} + e$, $e \in \{1, 2, ..., T - T_d - 1 - \hat{l}\}$. Then,

$$Prob(\tilde{L} = \hat{l} + e) = \left[\sum_{s=0}^{\hat{s}} C^s_{G-(y-1)(\hat{l}+1)+e-2} c(\hat{l}+1, e-s)\right] \frac{C^{T_d-(G-1)}_{T-(G+\hat{l}+e-1)}}{C^{T_d}_T}$$

where $\hat{s} = min(G - (y-1)(\hat{l}+1) + e - 2, e - 1)$.

3. In both cases, $Prob(\tilde{L} = T - T_d) = 1 - \sum_{l=0}^{T-T_d-1} Prob(\tilde{L} = l)$.

2.2.2 Proof of Lemma 3

An Example of $y = 3$

This proof is similar to that of Lemma 1. However, now that k takes a different form, the pledging thresholds also have a different pattern as shown in Table 3. The necessary condition for $\tilde{L} = l$ comes directly from the pledging threshold, and the sufficient condition for $\tilde{L} \neq l$ comes from its contrapositive statement. Similarly, conditions for $\tilde{L} = l$ include the necessary condition for $\tilde{L} = l$ and the sufficient conditions for $\tilde{L} \neq 0, 1, ..., l-1$. For instance, $\tilde{L} = 3$ if and only if the single-underlined 4 conditions are true.

l	**Pledging threshold**	**DB hit**	**Necessary condition for $\tilde{L} = l$**	**Sufficient condition for $\tilde{L} \neq l$**
0	2/3	2	$\{2\} = 0$	$\underline{\{2\} \geq 1}$
1	4/6	4	$\{5\} = 1$	$\underline{\{5\} \geq 2}$
2	6/9	6	$\{8\} = 2$	$\underline{\{8\} \geq 3}$
3	8/12	8	$\underline{\{11\} = 3}$	$\{11\} \geq 4$
4	10/15	10	$\{14\} = 4$	$\{14\} \geq 5$
...	...	...	...	...
$l-1$	$(2l)/(3l)$	$2l$	$\{3l-1\} = l-1$	$\{3l-1\} \geq l$
l	$(2(l+1))/(3(l+1))$	$2(l+1)$	$\{3(l+1)-1\} = l$	$\{3(l+1)-1\} \geq l+1$
...	...	...	...	...

Table 3: Conditions for $\tilde{L} = l$, $y = 3$

For example, when $l = 0$, the first two arrivals must be (DB, DB) ($\{2\} = 0$), and the third buyer facing $\frac{n}{t} = \frac{2}{3} \geq k = \frac{2}{3}$ will pledge and thus the pledging cascade starts. So, $Prob(\tilde{L} = 0) = C_{T-2}^{T_d-2}/C_T^{T_d}$. Notice $\{2\} \geq 1$ is sufficient for $\tilde{L} \neq 0$. When $l = 1$, the pledging threshold is 4/6, that is, among the initial 5 arrivals there should be 1 CB and 4 DBs. Then, for $\tilde{L} = 1$ we need to have $\{5\} = 1$ and $\{2\} \geq 1$, leading to $\{2\} = 1$. It means there must be 1 CB among the initial 2 arrivals, and the third, fourth and fifth arrivals are all DBs. So, $Prob(\tilde{L} = 1) = (C_2^1) * (C_{T-5}^{T_d-4}/C_T^{T_d})$.

To take care of all the sufficient conditions for $\tilde{L} \neq 0, 1, ..., l-1$, we need to similarly generate some auxiliary matrix. There is more than one way to generate such matrices. At this point, a matrix named A_y (corresponding to Definition 9) in generated. Later in the scenario of $T_d \geq G$, another matrix named C_y (corresponding to Definition 11) will be utilized. The reason to have two auxiliary matrices is: A_y has a neat explicit expression (as will be proved in the next lemma) but it can not cater to $T_d \geq G$; C_y can take care of both scenarios but does not have an explicit expression.

Now, we first define matrix A_y by the placing requirements, and then show it is the same as Definition 9. In Table 3, notice all pledging thresholds are multiples of $\frac{2}{3}$, and the sufficient condition for $\tilde{L} \neq l$ is always of the form $\{3(l+1)-1\} \geq l+1$.

Definition 12. *Let $a(1,j) = 1$ for all j. When $i \geq 2$, for $a(i,j)$, consider $(i-1)$ groups of buyers. The first group has $3j$ arrivals; and each of the following $(i-2)$ groups has 3 arrivals. Thus, it contains $3j+3(i-2)$ arrivals in total. Then, $a(i,j)$ represents the number of cases, such that there are totally $i-1$ CBs in the $i-1$ groups, and for all $n < i-1$, the initial n groups contain at least n CBs.*

With the number of arrivals in each group specified, an alternative way to describe the definition is: the first group contains at least 1 CB ($\{3j\} \geq 1$), the initial 2 groups contain at least 2 CBs ($\{3j+3\} \geq 2$), the initial 3 groups contain at least 3 CBs ($\{3j+6\} \geq 3$), ... the initial $(i-2)$ groups contain at least $(i-2)$ CBs ($\{3j+3(i-3)\} \geq i-2$), and $(i-1)$ groups contain in total $(i-1)$ CBs ($\{3j+3(i-2)\} = i-1$).

For example, the second row of A_y corresponds to only 1 group, where 1 CB needs to be properly placed. Consequently, $a(2,1) = Cases(\{3\}{=}1) = C_3^1 = 3$. $a(2,2) = Cases(\{6\}{=}1) = C_6^1 = 6$. $a(2,3) = Cases(\{9\}{=}1) = C_9^1 = 9$, and so forth.

The third row corresponds to 2 groups, where 2 CBs need to be properly placed. For $a(3,1)$, both groups contain 3 arrivals, and there are two ways to place them by definition: 1 CB in the first group and 1 CB in the second group; or 2 CBs in the first group. Notice after placing 1 (or 2) CB(s) in the first group, the problem is reduced to a one-group (or zero-group) problem, and corresponds to the second (or first) row of A_y. That is, $a(3,1) = C_3^1 a(2,1) + C_3^2 a(1,2)$, recalling $a(1,j) = 1$ for all j.

For $a(3,2)$, the first group contains 6 buyers and the second group contains 3 buyers. We can have 1 CB in the first group and 1 CB in the second group; or 2 CBs in the first group. Thus, $a(3,2) = C_6^1 a(2,1) + C_6^2 a(1,2)$. Similarly, $a(3,3) = C_9^1 a(2,1) + C_9^2 a(1,2)$.

For the fourth row, say $a(4,1)$, it contains 3 groups, each having 3 arrivals, and 3 CBs need to be placed properly. If placing 1 CB in the first group, other conditions of $a(4,1)$ require "*at least* 1 CB in the second group, and in total 2 CBs in the second and third group", which is exactly the definition of $a(3,1)$. If placing 2 CBs in the first group, notice "at least 2 CBs in the initial 2 groups" is naturally satisfied, and the remaining requirements become "1 CB in the second and third group combined", which corresponds to $a(2,2)$ if we think of the second and third group, 3 buyers in each, as one group of 6 buyers. If placing 3 CBs in the first group, all requirements are satisfied and $a(1,3) = 1$ follows. Therefore, $a(4,1) = C_3^1 a(3,1) + C_3^2 a(2,2) + C_3^3 a(1,3)$.

Thus, Definition 12 leads to a recursive way to generate all entries of A_y. Consider $a(i+1,j)$ with i groups of buyers. If placing 1 CB in the first group, the conditions on the following $i-1$ groups, with 3 buyers in each, is the same as $a(i,1)$'s definition. If placing 2 CBs in the first group, "at least 2 CBs in the initial 2 groups" is naturally satisfied. Condition on the initial 3 groups then becomes "at least 1 CB in the second and third group combined" or "at least 1 CB in the 6-buyer group", which, coupled with other conditions, correspond exactly to $a(i-1,2)$. If placing 3 CBs in the first group, "at least 2 CBs in the initial 2 groups" and "at least 3 CBs in the initial 3 groups" are satisfied, and the remaining requirements are the same as $a(i-2,3)$'s definition, etc. That is to say, A_y is formulated recursively as $a(i+1,j) = \sum_{s=1}^{min(3j,i)} C_{3j}^{s} a(i+1-s,s)$, where $3j$ is the number of arrivals in the first group and s is the number of CBs placed in the first group. This formula is the same as Definition 9.

Now we can connect A_y to $Cases(\tilde{L}=l)$. For $\tilde{L}=l$, we need to place the front l CBs and $2(l+1)$ DBs nicely. Given in Table 3, the conditions include $\{3(l+1)-1\}=l$, $\{3l-1\}\geq l$, $\{3(l-1)-1\}\geq l-1$, ..., $\{5\}\geq 2$ and $\{2\}\geq 1$. Combining the first two conditions, we get $\{3l-1\}=l$. For the initial two arrivals we need $\{2\}\geq 1$, and starting from the third arrival, we have groups of 3 buyers in each. If $\{2\}=1$, we need $\{3\}^2\geq 1$, $\{6\}^2\geq 2$, ..., $\{3(l-1)\}^2=l-1$, which corresponds to $a(l,1)$. If $\{2\}=2$, $\{3\}^2\geq 1$ is satisfied, and we need $\{6\}^2\geq 1$, $\{9\}^2\geq 2$, ..., $\{3(l-1)\}^2=l-1$, which corresponds to $a(l-1,2)$. So, there are in total $\sum_{s=1}^{min(2,i)} C_{2}^{s} a(l+1-s,s) = b_{l+1}$ ways to place the front $3l+2$ buyers, by b_n's definition in Definition 10. It follows $Prob(\tilde{L}=l) = b_{l+1} * (C_{T-(3(l+1)-1)}^{T_d-2(l+1)} / C_{T}^{T_d})$ as in Lemma 3 1(i) with $y=3$.

Analogue to General y

[Insert Table 6 here.]

As shown in Table 6, for l CB losses, $(l+1)(y-1)$ DBs are needed. The sufficient and necessary conditions for $\tilde{L}=l$ include $\{(l+1)y-1\}=l$, $\{ly-1\}\geq l$, $\{(l-1)y-1\}\geq l-1$, ..., $\{2y-1\}\geq 2$ and $\{y-1\}\geq 1$. Similarly, we need to generate an auxiliary matrix A_y to get $Cases(\tilde{L}=l)$.

Let $a(1,j)=1$ for all j. Starting from the second row, for $a(i,j)$, consider $(i-1)$ groups. The first group has jy arrivals, and each of the following $(i-2)$ groups has y arrivals. Then, $a(i,j)$ represents the number of cases such that—the first group contains at least 1 CB ($\{jy\}\geq 1$); the initial 2 groups contain at least 2 CBs ($\{jy+y\}\geq 2$); the initial 3 groups contain at least 3 CBs ($\{jy+2y\}\geq 3$), ..., the initial $(i-2)$ groups contain at least $(i-2)$ CBs ($\{jy+(i-3)y\}\geq i-2$), and $(i-1)$ groups contain $(i-1)$ CBs in total ($\{jy+(i-2)y\}=i-1$).

Analogously, if placing s CBs in the first group, "at least $\hat{s}$ CBs in the initial $\hat{s}$ groups" is naturally satisfied for all $\hat{s} \leq s$, and readers can check that the remaining conditions correspond to $a(i+1-s, s)$ by its definition. Likewise, $a(i+1, j) = \sum_{s=1}^{min(jy,i)} C_{jy}^{s} a(i+1-s, s)$.

Lastly, the conditions for $\tilde{L} = l$ include $\{ly-1\} = l$, $\{(l-1)y-1\} \geq l-1$, ..., $\{3y-1\} \geq 3$, $\{2y-1\} \geq 2$ and $\{y-1\} \geq 1$. Consider $\{y-1\} \geq 1$. If $\{y-1\} = 1$, then we need $\{y\}^{y-1} \geq 1$, $\{2y\}^{y-1} \geq 2$, ..., $\{(l-1)y\}^{y-1} \geq l-1$, the same as $a(l,1)$'s definition. If $\{y-1\} = 2$, it calls for $a(l-1, 2)$, etc. In total, there are $\sum_{s=1}^{min(y-1,i)} C_{y-1}^{s} a(l+1-s, s) = b_{l+1}$ ways to place the front buyers. It follows that $Prob(\tilde{L} = l) = b_{l+1} * (C_{T-((l+1)y-1)}^{T_d-(l+1)(y-1)} / C_{T}^{T_d})$ as in Lemma 3 1(i).

When There Are Unproportionally Too Many CB

To support l CB losses, $(l+1)(y-1)$ DBs are needed to form the critical mass. There are in total T_d DBs. When $l = T - T_d - 1$, if the number of needed DBs, $(T - T_d)(y-1)$ is no more than T_d, the foregoing algorithm covers all l as needed. Otherwise, for those l such that $(l+1)(y-1) > T_d$ and $l \leq T - T_d - 1$, $Prob(\tilde{L} = l) = 0$. These arguments are reflected in Lemma 3 1(ii) and 3.

Change of Pledging Rule Once $n \geq G-1$

Let $\hat{l}$ be the largest CB loss that can be supported by no more than $(G-1)$ DBs, that is, $(\hat{l}+1)(y-1) \leq G-1$ and $(\hat{l}+2)(y-1) > G-1$. Similar to Section , for $l \leq \hat{l}$, the foregoing algorithm still works. As the cascade starts for sure after the $(G-1)^{th}$ DB's pledge, the column of DB hit in Table 6 needs to be modified for all $l \geq \hat{l}+1$. These are reflected in the last 4 rows of Table 7.

[Insert Table 7 here.]

First, an extreme case needs to be considered. When $y-1 > G-1$, the critical mass k is so close to 1 that it can not be reached before the actual target is achieved. In this case, $\hat{l}$ is not well defined, because even for $\hat{l} = 0$, $(\hat{l}+1)(y-1) \leq G-1$ is not true. Readers can check that, in this situation, conditions for $\tilde{L} = l$ are $\{G+l-1\} = l$, $\{G+l-2\} \geq l$, $\{G+l-3\} \geq l-1$, ..., and $\{G-1\} \geq 1$, all of which can be reduced to a single condition $\{G+l-2\} = l$, because $\{G+l-2\} = l \Rightarrow \{G+l-3\} \geq l-1 \ ... \Rightarrow \{G-1\} \geq 1$. Then $Cases(\{G+l-2\} = l) = C_{G+l-2}^{l}$, and consequently $Prob(\tilde{L} = l) = C_{G+l-2}^{l} * (C_{T-(G+l-1)}^{T_d-(G-1)} / C_{T}^{T_d})$.

Now suppose $y-1 \leq G-1$, $\hat{l}$ is well defined. For all $l \geq \hat{l}+1$, $(G-1)$ DBs are needed to start the cascade. Write these l as $l = \hat{l}+e$, $e \in \{1, 2, ..., T-T_d-1-\hat{l}\}$. The conditions for $\tilde{L} = \hat{l}+e$ include all the single-underlined conditions in Table 7.

First, notice $\{G+\hat{l}+e-1\} = \hat{l}+e$ and $\{G+\hat{l}+e-2\} \geq \hat{l}+e$ combined imply $\{G+\hat{l}+e-2\} = \hat{l}+e$. Second, $\{G+\hat{l}+e-2\} = \hat{l}+e \Rightarrow \{G+\hat{l}+e-3\} \geq \hat{l}+e-1 \Rightarrow \{G+\hat{l}+e-4\} \geq \hat{l}+e-2 \Rightarrow ... \Rightarrow \{G+\hat{l}\} \geq \hat{l}+2$. Third, for $e \geq 2$, conditions include $\{y-1\} \geq 1$, $\{2y-1\} \geq 2$, ...,

$\{(\hat{l}+1)y-1\} \geq \hat{l}+1$, and $\{G+\hat{l}+e-2\} = \hat{l}+e$. The last two conditions do not have an increment of y arrivals, meaning matrix A_y cannot be utilized.

For the purpose, we define matrix C_y (similar to matrix B_x) with a typical entry $c(i,j)$. $Cases(\tilde{L})$ directly relates to matrix C_y, but the latter does not have an explicit expression as B_x or A_y, meaning it can only be approached numerically.

The first row of C_y corresponds to $\{y-1\} \geq 1$. Define $c(1,1) = Cases(\{y-1\} = 1) = C_{y-1}^1$; $c(1,2) = Cases(\{y-1\} = 2) = C_{y-1}^2$; ...; $c(1, y-1) = Cases(\{y-1\} = y-1) = C_{y-1}^{y-1}$. And $c(1,j) = 0$ for all $j \geq y$.

The second row of C_y corresponds to $\{2y-1\} \geq 2$, with $\{y-1\} \geq 1$ implicitly required. Write it as $\{2y-1\}_C \geq 2$, meaning the second condition is implicitly required. Similar to B_x, $c(2,1) = Cases(\{2y-1\}_C = 2) = C_y^0 c(1,2) + C_y^1 c(1,1)$. The first term means placing 2 CB in the initial $y-1$ arrivals, i.e. $c(1,2)$, leaving 0 CB in the following y arrivals; the second term means placing 1 CB in the initial $y-1$ arrivals, i.e. $c(1,1)$, leaving 1 CB in the following y arrivals. Likewise, $c(2,2) = Cases(\{2y-1\}_C = 3) = C_y^0 c(1,3) + C_y^1 c(1,2) + C_y^2 c(1,1)$, and so forth.

For the $(i+1)^{th}$ row, $c(i+1,j)$ represents the number of cases such that $\{(i+1)y-1\} = i+1+j-1$, with $\{sy-1\} \geq s$ implicitly required for all $0 < s \leq i$; or as just defined, $c(i+1,j) = Cases(\{(i+1)y-1\}_C = i+1+j-1)$. Likewise, the recursive formula is $c(i+1,j) = \sum_{s=0}^{min(y,j)} C_y^s c(i, j+1-s)$, the same as Definition 11.

Lastly, relate C_y to the conditions for $\tilde{L} = \hat{l} + e$. When $e = 1$, conditions include $\{y-1\} \geq 1$, $\{2y-1\} \geq 2$, ..., $\{(\hat{l}+1)y-1\} = \hat{l}+1$, the same as $c(\hat{l}+1, 1)$'s definition. When $e \geq 2$, conditions include $\{y-1\} \geq 1$, $\{2y-1\} \geq 2$, ..., $\{(\hat{l}+1)y-1\} \geq \hat{l}+1$ and $\{G+\hat{l}+e-2\} = \hat{l}+e$; all but the last correspond to the $(\hat{l}+1)^{th}$ row of matrix C_y. If $\{(\hat{l}+1)y-1\} = \hat{l}+1$, $e-1$ CBs are left to be placed in the additional $G-(y-1)(\hat{l}+1)+e-2$ arrivals; if $\{(\hat{l}+1)y-1\} = \hat{l}+2$, $e-2$ CB are left to be placed in the additional $G-(y-1)(\hat{l}+1)+e-2$ arrivals, and so forth. Then $Cases(\{G+\hat{l}+e-2\}_C = \hat{l}+e) = \sum_{s=0}^{\hat{s}} C_{G-(y-1)(\hat{l}+1)+e-2}^s c(\hat{l}+1, e-s)$ where $\hat{s} = min(G-(y-1)(\hat{l}+1)+e-2, e-1)$. Lemma 3 2. is thus proved.

The proof is complete.

2.2.3 Lemma 4

Lemma 4. *The matrix A_y and sequence b_n defined in Section have the following expressions:*

1. $a(i,j) = \frac{jC_{(i+j-1)y}^{i-1}}{i+j-1}$ *for all* $i \geq 1$ *and* $j \geq 1$.
2. $b_n = \frac{C_{yn-2}^{n-1}}{n}$ *for all* $n \geq 1$.

2.2.4 Proof of Lemma 4

Proof for Matrix A_y

We want to show A_y defined recursively in Definition 9 has the following explicit form: $a(i,j) = \frac{jC^{i-1}_{(i+j-1)y}}{i+j-1}$ for all $i \geq 1$ and $j \geq 1$.

First, $a(1,j) = \frac{j}{j} = 1$, the same as Definition 9. Thus the first row of A_y is proved. We will use induction to complete the proof.

Assume the expression is true for the *initial* i rows of A_y. Then by definition, $a(i+1,j) = \sum_{s=1}^{min(jy,i)} C^s_{jy} a(i+1-s,s)$. Plug in $a(i+1-s,s)$, $a(i+1,j) = \sum_{s=1}^{min(jy,i)} C^s_{jy} \frac{sC^{i-s}_{iy}}{i}$. Notice $sC^s_{jy} = (jy)C^{s-1}_{jy-1}$. So $a(i+1,j) = \frac{jy}{i}\sum_{s=1}^{min(jy,i)} C^{s-1}_{jy-1}C^{i-s}_{iy} = \frac{jy}{i}C^{i-1}_{(i+j)y-1}$. Therefore, we need to show $\frac{jC^i_{(i+j)y}}{i+j} = \frac{jy}{i}C^{i-1}_{(i+j)y-1}$.

Notice: $\frac{jC^i_{(i+j)y}}{i+j} = \frac{jy}{i}C^{i-1}_{(i+j)y-1}$

$\Leftrightarrow \frac{i}{(i+j)y}C^i_{(i+j)y} = C^{i-1}_{(i+j)y-1}$

$\Leftrightarrow C^{i-1}_{(i+j)y-1} = C^{i-1}_{(i+j)y-1}$.

Thus the equality is proved.

Proof for Sequence b_n

In Definition 10, b_n is defined via matrix A_y. We want to show $b_n = \frac{C^{n-1}_{yn-2}}{n}$ for all $n \geq 1$.

When $n = 1$, the equivalence is trivial.

By definition 10, $b_{n+1} = \sum_{s=1}^{min(y-1,n)} C^s_{y-1} a(n+1-s,s)$ for $n \geq 1$. Plug $a(i,j)$ in, and we get $b_{n+1} = \sum_{s=1}^{min(y-1,n)} C^s_{y-1}\frac{sC^{n-s}_{ny}}{n}$. Notice $sC^s_{y-1} = (y-1)C^{s-1}_{y-2}$. So $b_{n+1} = \frac{y-1}{n}\sum_{s=1}^{min(y-1,n)} C^{s-1}_{y-2}C^{n-s}_{ny} = \frac{y-1}{n}C^{n-1}_{(n+1)y-2}$. Therefore we need to show $\frac{C^n_{(n+1)y-2}}{n+1} = \frac{y-1}{n}C^{n-1}_{(n+1)y-2}$.

Notice $\frac{C^n_{(n+1)y-2}}{n+1} = \frac{y-1}{n}C^{n-1}_{(n+1)y-2}$

$\Leftrightarrow \frac{n}{(n+1)(y-1)}C^n_{(n+1)y-2} = C^{n-1}_{(n+1)y-2}$

$\Leftrightarrow C^{n-1}_{(n+1)y-2} = C^{n-1}_{(n+1)y-2}$.

Thus the equality is proved and the proof is complete.

2.2.5 Proposition 2

Proposition 2. *Suppose* $k = \frac{y-1}{y}$ *for some* $y \in \{1,2,3,...\}$, *and consider the scenario when* $T_d \leq G-1$. *If* $y-1 > T_d$, $Prob(\tilde{L} = T - T_d) = 1$. *Otherwise,*

1. *$\tilde{L}$'s p.m.f:*

(i) For all l in $0 \le l \le min(\frac{T_d}{y-1} - 1, T - T_d - 1)$,

$$Prob(\tilde{L} = l) = \frac{C^{l}_{(l+1)y-2} C^{T_d-(l+1)(y-1)}_{T-((l+1)y-1)}}{(l+1)C^{T_d}_{T}}.$$

(ii) If $\frac{T_d}{y-1} - 1 < T - T_d - 1$, for all l in $\frac{T_d}{y-1} - 1 < l \le T - T_d - 1$, $Prob(\tilde{L} = l) = 0$.

(iii) $Prob(\tilde{L} = T - T_d) = 1 - \sum_{l=0}^{T-T_d-1} Prob(\tilde{L} = l)$.

2. Notice:

(i) When $l = 0$:

$$Prob(\tilde{L} = 0) = \frac{C^{T_d-(y-1)}_{T-(y-1)}}{C^{T_d}_{T}} \approx (\frac{T_d}{T})^{(y-1)}$$

*(ii) The p.m.f is **almost always** decreasing as l increases:*

(a) For those l such that $(l+2)(y-1) \le T_d - 2$ and $l + 1 \le T - T_d - 2$,

$$\frac{Prob(\tilde{L} = l + 1)}{Prob(\tilde{L} = l)} < 1.$$

(b) If both $l_1 = T - T_d - 2$ and $(l_1 + 1)$ have a non-zero probability mass, that is, $(T - T_d)(y-1) \le T_d$,

$$\frac{Prob(\tilde{L} = l_1 + 1)}{Prob(\tilde{L} = l_1)} < \frac{3}{e}.$$

(c) If such l_2 exists that $l_2 + 1 \le T - T_d - 1$ and $(l_2 + 2)(y-1) = T_d - 1$,

$$\frac{Prob(\tilde{L} = l_2 + 1)}{Prob(\tilde{L} = l_2)} < (\frac{3y}{2} - 1).$$

(d) If such l_3 exists that $l_3 + 1 \le T - T_d - 1$ and $(l_3 + 2)(y-1) = T_d$,

$$\frac{Prob(\tilde{L} = l_3 + 1)}{Prob(\tilde{L} = l_3)} < \frac{1}{e}(\frac{3y}{2} - 1).$$

Remark. *This proposition is an extension of Proposition of the main paper. Only the scenario of $T_d \le G - 1$ is included, following immediately from Lemma 3 and 4. The scenario of $T_d \ge G$ is fully characterized in Lemma 3 and is thus omitted.*

The new contents are 2(i)(ii), aiming to characterize the p.m.f in more details. 2(i) gives an approximation of $Prob(\tilde{L} = 0)$, clearly increasing in the proportion of dedicated buyers $\frac{T_d}{T}$ and decreasing in the critical mass $k = \frac{y-1}{y}$. Next, 2(ii) assures the monotonicity of $\tilde{L}$'s p.m.f *with small exceptions on the boundary in (b)(c)(d), enhancing the two-spikes pattern: If the p.m.f quickly and monotonically decreases to a negligible level, it naturally has two big masses around* $\tilde{L} = 0$ *and* $T - T_d$.

2.2.6 Proof of Proposition 2

Plug the expressions of b_n from Lemma 4 to Lemma 3, and 1(i) follows immediately. 1(ii)(iii) are identical to Lemma 3 1(ii) and 3. Next, we will prove the inequalities given in 2.

Plug $l = 0$ to the expression in 1(i), and one gets $Prob(\tilde{L} = 0) = C_{T-(y-1)}^{T_d-(y-1)}/C_T^{T_d}$. Open the combinatorial number and rewrite the expression. $C_{T-(y-1)}^{T_d-(y-1)}/C_T^{T_d} = \prod_{s=0}^{y-2} \frac{T_d-s}{T-s}$. Notice $\frac{T_d-s}{T-s} \approx \frac{T_d}{T}$ for some small $s \geq 0$, so $Prob(\tilde{L} = 0) \approx (\frac{T_d}{T})^{(y-1)}$ if y is small compared to T and T_d. Thus 2(i) is proved.

As for 2(ii), let $A(l) = C_{(l+1)y-2}^{l}/(l+1)$ and $B(l) = C_{T-((l+1)y-1)}^{T_d-(l+1)(y-1)}/C_T^{T_d}$, then $Prob(\tilde{L} = l) = A(l)B(l)$ from 1(i). When $l \geq 1$, $\frac{A(l+1)}{A(l)} = \left[\prod_{s=2}^{(l+1)} \frac{(l+1)y+y-s}{(l+1)y-s}\right](y-1)$. Notice $\frac{(l+1)y+y-s}{(l+1)y-s} > 1$, and it decreases in l for all s. So $\frac{A(l+1)}{A(l)}$ reaches its maximum at $l = 1$, $\frac{A(2)}{A(1)} = \frac{3y}{2} - 1$, for all $l \geq 1$. When $l = 0$, $\frac{A(1)}{A(0)} = y - 1 < \frac{3y}{2} - 1$. Therefore, for all $l \geq 0$, the maximum of $\frac{A(l+1)}{A(l)}$ is $(\frac{3y}{2} - 1)$.

Now examine $\frac{B(l+1)}{B(l)}$. Let $n_l \equiv (l+1)(y-1)$. It follows $\frac{B(l+1)}{B(l)} = C_{T-(n_l+y+l)}^{T_d-(n_l+y-1)}/C_{T-(n_l+l)}^{T_d-n_l}$. The first combinatorial number implies $T - (n_l + y + l) \geq T_d - (n_l + y - 1)$, so $T - T_d - l \geq 1$ is an underlying condition. Additionally, $B(l+1) = C_{T-(n_l+y+l)}^{T_d-(n_l+y-1)}$ by definition, so $T_d - (n_l + y - 1) \geq 0$ is also underlying condition, that is, $n_l \leq T_d - y + 1$. We need to discuss $\frac{B(l+1)}{B(l)}$ when $\underline{\underline{T - T_d - l \geq 1}}$ and $\underline{\underline{n_l \leq T_d - y + 1}}$.

If $\underline{\underline{T - T_d - l = 1}}$, that is, $l + 1 = T - T_d$. We are not interested in $\frac{B(T-T_d)}{B(T-T_d-1)}$, since there is a spike at $T - T_d$. So, consider $\underline{\underline{T - T_d - l \geq 2}}$.

When $n_l \leq T_d - y - 1$, it implies $T_d - (n_l + y - 1) \geq 2 \Rightarrow T - (n_l + l + y) \geq T - T_d - l + 1$; so $B(l + 1)$ and $B(l)$ can be opened and written as

$$\frac{B(l+1)}{B(l)} = \frac{\frac{(T-(n_l+l+y))(T-(n_l+l+y)-1)(T-(n_l+l+y)-2)...(T-T_d-l)}{(T_d-(n_l+y-1))!}}{\frac{(T-(n_l+l))(T-(n_l+l)-1)(T-(n_l+l)-2)...(T-T_d-l+1)}{(T_d-n_l)!}}.$$

Cancel identical terms to get

$$\begin{aligned}\frac{B(l+1)}{B(l)} &= (\frac{T-T_d-l}{T-n_l-l})\left[(\frac{T_d-n_l}{T-n_l-l-1})(\frac{T_d-n_l-1}{T_d-n_l-l-2})...(\frac{T_d-n_l-(y-2)}{T-n_l-l-(y-1)})\right] \\ &= (\frac{T-T_d-l}{T-n_l-l})\prod_{s=0}^{y-2}(\frac{T_d-n_l-s}{T-n_l-l-1-s}).\end{aligned}$$

Notice $\frac{T_d-n_l-s}{T-n_l-l-1-s} = 1 - \frac{T-T_d-l-1}{T-n_l-l-1-s} \leq 1$ because $T - T_d - l \geq 1$ and $T - n_l - l - 1 - s > 0$. So,

$\frac{T_d-n_l-s}{T-n_l-l-1-s} \le \frac{T_d-n_l}{T-n_l-l-1} = 1 - \frac{T-T_d-l-1}{T-n_l-l-1}$ for all $s \ge 0$. Then we get

$$\begin{aligned}\frac{B(l+1)}{B(l)} &\le (\frac{T-T_d-l}{T-n_l-l})(1-\frac{T-T_d-l-1}{T-n_l-l-1})^{y-1} \\ &< (\frac{T-T_d-l}{T-n_l-l-1})(1-\frac{T-T_d-l-1}{T-n_l-l-1})^{y-1} \\ &= (\frac{T-T_d-l-1}{T-n_l-l-1})(1-\frac{T-T_d-l-1}{T-n_l-l-1})^{y-1} + (\frac{1}{T-n_l-l-1})(1-\frac{T-T_d-l-1}{T-n_l-l-1})^{y-1}.\end{aligned}$$

Notice $(\frac{T-T_d-l-1}{T-n_l-l-1})(1-\frac{T-T_d-l-1}{T-n_l-l-1})^{y-1} \le \max_{a\in[0,1]} a(1-a)^{y-1} = \frac{(y-1)^{y-1}}{y^y}$. Since $T-T_d-l \ge 2$, it implies $T-T_d-l-1 \ge 1$, and so we have $\frac{B(l+1)}{B(l)} < (1+\frac{1}{T-T_d-l-1})(\frac{T-T_d-l-1}{T-n_l-l-1})(1-\frac{T-T_d-l-1}{T-n_l-l-1})^{y-1} \le (1+\frac{1}{T-T_d-l-1})\frac{(y-1)^{y-1}}{y^y}$.

So, when $\underline{\underline{n_l \le T_d-y-1}}$, we have $\frac{B(l+1)}{B(l)} < (1+\frac{1}{T-T_d-l-1})\frac{(y-1)^{y-1}}{y^y}$.

When $\underline{\underline{n_l = T_d-y}}$, $\frac{B(l+1)}{B(l)} = \frac{(T-T_d-l)y!}{(T-T_d-l+y)(T-T_d-l+y-1)(T-T_d-l+y-2)...(T-T_d-l+1)} = (\frac{T-T_d-l}{T-T_d-l+y})(\prod_{s=2}^{y}\frac{s}{T-T_d-l-1+s})$. Since $T-T_d-l-1 \ge 0$, we have $\frac{s}{T-T_d-l-1+s} \le 1$. Since $y \ge 2$, we have $\frac{T-T_d-l}{T-T_d-l+y} < 1$. Therefore, $\frac{B(l+1)}{B(l)} < 1$.

When $\underline{\underline{n_l = T_d-y+1}}$, $\frac{B(l+1)}{B(l)} = \frac{(y-1)!}{(T-T_d-l+y-1)(T-T_d-l+y-2)...(T-T_d-l+1)} = \prod_{s=1}^{y-1}\frac{s}{T-T_d-l+s} \le (\frac{(y-1)}{T-T_d-l+y-1})^{y-1}$. Since $T-T_d-l-1 \ge 0$, $\frac{B(l+1)}{B(l)} < (\frac{y-1}{y})^{y-1}$.

Hitherto, the part of $B(l)$ is complete.

Lastly, combine $A(l)$ and $B(l)$: $\frac{Prob(\tilde{L}=l+1)}{Prob(\tilde{L}=l)} = (\frac{A(l+1)}{A(l)})(\frac{B(l+1)}{B(l)})$.

(1) If $n_l \le T_d-y-1$ and $T-T_d-l-1 \ge 1$, $\frac{Prob(\tilde{L}=l+1)}{Prob(\tilde{L}=l)} < (\frac{3y}{2}-1)(1+\frac{1}{T-T_d-l-1})\frac{(y-1)^{y-1}}{y^y} = (\frac{3}{2}-\frac{1}{y})(1+\frac{1}{T-T_d-l-1})(\frac{y-1}{y})^{y-1}$. Notice $(\frac{y-1}{y})^{y-1} = (1-\frac{1}{y})^{y-1}$ is increasing in y, and $\lim_{y\to+\infty}(1-\frac{1}{y})^{y-1} = \lim_{y\to+\infty}(1-\frac{1}{y})^{y}\frac{y}{y-1} = \frac{1}{e}$. So, $(\frac{y-1}{y})^{y-1} < \frac{1}{e}$; it follows $\frac{Prob(\tilde{L}=l+1)}{Prob(\tilde{L}=l)} < \frac{3}{2e}(1+\frac{1}{T-T_d-l-1})$.

(a) When $T-T_d-l-1 = 1$, $\frac{Prob(\tilde{L}=l+1)}{Prob(\tilde{L}=l)} < \frac{3}{e}$.

(b) When $T-T_d-l-1 \ge 2$, $\frac{Prob(\tilde{L}=l+1)}{Prob(\tilde{L}=l)} < \frac{9}{4e} < 1$.

(2) If $n_l = T_d-y$, $\frac{Prob(\tilde{L}=l+1)}{Prob(\tilde{L}=l)} < (\frac{3y}{2}-1)$.

(3) If $n_l = T_d-y+1$, $\frac{Prob(\tilde{L}=l+1)}{Prob(\tilde{L}=l)} < (\frac{3y}{2}-1)(\frac{y-1}{y})^{y-1} < (\frac{3y}{2}-1)\frac{1}{e}$.

The proof is complete.

S2.3: $\tilde{L}$'s Distribution for Arbitrary $k \in (0,1)$

The way to numerically derive $\tilde{L}$'s p.m.f for arbitrary $k \in (0,1)$ is similar to $k \in \mathbb{K}$ by following the four steps: (i) finding the pledging threshold, (ii) deriving necessary condition for $\tilde{L} = l$, (iii) deriving sufficient conditions for $\tilde{L} \ne l$, (iv) using combinatorics to calculate the number of cases for $\tilde{L} = l$. The difference is that for arbitrary k, there is no pattern in the pledging thresholds, thus no general formula for the necessary conditions. Hence the process must be performed recursively by computer programs. Nevertheless, it is easily seen that for two k's of similar values, their pledging threshold, thus conditions, should be similar as well, leading to similar $\tilde{L}$'s p.m.f's. Therefore, the two regular-form k's serve as a good approximation for arbitrary k's.

Another way to get $\tilde{L}$'s distribution is by simulation. One can fix T and T_d, randomly draw a (T, T_d)-combination arriving order from the $C_T^{T_d}$ many possibilities, check the behavioral rule $\frac{n}{t} \geq k$ from the first arrival to the last, and determine how many pledges are received (equivalently, $\tilde{L}$) for this arriving order. Repeat the process for many times, and by the law of large number, the resulting histogram of $\tilde{L}$ is approximately $\tilde{L}$'s theoretical distribution.

S2.4: Bounded Rationality in the Behavioral Rule

Assume buyers believe the average percentage of duration needed to raise 1% of the target ($AveDur$) follows a uniform distribution over a range $(0, \Omega)$, where Ω stands for the longest needed time (or the lowest speed) to raise 1% of the target. Ω is not observable, but the current average speed prior to the buyer's arrival, $\frac{t/T}{n/G}$, delivers information about Ω. For instance, if $\frac{t/T}{n/G}$ is believed to be the mean of the random variable $AveDur$, then under the uniform distribution assumption, $\Omega = 2\frac{t/T}{n/G}$. More generally, Ω could be a multiple of the observed speed, $\Omega = c\frac{t/T}{n/G}$ where c is a constant. The campaign succeeds if less than 1% of its duration is needed to raise 1% of the target. So the success probability is $\text{Prob}(AveDur \leq 1) = \frac{1}{\Omega} = \frac{1}{c}\frac{n/G}{t/T}$, a linear function of the campaign's *funding progress*.

S2.5: Proof of Lemma 3

Recall $\tilde{O}$ is the random arriving order with a typical element $o \in \mathbb{O}$, and $\mathbb{O}$ is the set of all possible arriving orders. The size of $\mathbb{O}$ is $\parallel \mathbb{O} \parallel = C_T^{T_d}$. Notice, given T and T_d, there exists a mapping between $\tilde{O}$ and $\tilde{L}$ wherein the arriving order uniquely pins down the number of . Define $f_i : \mathbb{O} \rightarrow \{0, 1, ..., T - T_d\}$, $i \in \{A, B\}$, to be such mappings for campaigns A and B.

Now, for some $o \in \mathbb{O}$, suppose $f_B(o) = l$ for some $l \leq T - T_d - 1$ (the case of $l = T - T_d$ needs to be singled out because the pledging cascade does not happen in the case). Since campaign B loses only l buyers, for the $(l + n + 1)^{th}$ buyer, it must be $\frac{n}{l+n+1} \geq k_B$. Campaign B's pledging cascade starts at the $(l + n + 1)^{th}$ arrival. As $\frac{n}{l+n+1} \geq k_B \geq k_A$, campaign A's pledging cascade either starts now or has already started. So, $f_A(o) \leq l = f_B(o)$, which holds for all $o \in \mathbb{O}$. Therefore, $f_B^{-1}(\{0, 1, ..., l\}) \subseteq f_A^{-1}(\{0, 1, ..., l\})$. We know that $\text{Prob}(\tilde{L_i} \leq l) = \parallel f_i^{-1}(\{0, 1, ..., l\}) \parallel / C_T^{T_d}$, where $\parallel \cdot \parallel$ is the cardinality of the set. So, $\text{Prob}(\tilde{L_B} \leq l) \leq \text{Prob}(\tilde{L_A} \leq l)$ for all $l \leq T - T_d - 1$. When $l = T - T_d$, $\text{Prob}(\tilde{L_B} \leq l) = \text{Prob}(\tilde{L_A} \leq l) = 1$. Thus the FOSD is proved.

However, because n and t must be integers, even if $k_A < k_B$ (strictly less than), campaigns A and B's $\tilde{L}$'s p.m.f may still be the same. For example, let $T = 50$, $T_d = 10$, $k_A = \frac{1}{300}$ and $k_B = \frac{1}{200}$, so $l \leq T - T_d = 40$. As k_A and k_B are both extremely small, 1 DB suffices to reach the critical mass for both. Specifically, both campaigns have the same pledging thresholds "$\frac{1}{2}, \frac{1}{3}, ..., \frac{1}{41}$" for $l = 0, 1, ..., 39$ by definition. Therefore, $\tilde{L_A}$ and $\tilde{L_B}$ should be the same. For a strict improvement over $\tilde{L}$'s distribution, there must exist feasible l and n, $0 \leq l \leq T - T_d$ and $n \leq T_d$ such that $k_B > \frac{n}{n+l+1} \geq k_A$, which forms a real difference in the pledging thresholds.

S2.6: Clarifications for Maximization (6)

First, G^* must be an integer lying in $[K, T]$. Notice the problem can be rewritten as

$$\max_{G \in \left[\frac{K}{1-c}, T\right]} F(T - \lceil G \rceil \mid k) \qquad s.t. \;\; k = k(G) = \frac{rh}{(\frac{G}{K+cG} - 1)},$$

where $\lceil \cdot \rceil$ is the ceil function that takes the smallest integer greater than the argument, because $\tilde{L} \leq T - G$ is equivalent to $\tilde{L} \leq T - \lceil G \rceil$ if $\tilde{L}$ is an integer. Then for any non-integer G, the campaign's success probability can be increased by raising G towards $\lceil G \rceil$, which reduces k but keeps $T - \lceil G \rceil$ the same. So G^* must be an integer. This then leads to a finite feasible set of G between $[K, T]$, so a solution to (6) exists.

S2.7: Proof of Proposition 6

First, let $\tilde{L}$, $\tilde{L}'$ and $\tilde{L}''$ denote the random variable of in the scenarios of (a) no support, (b) early support and (c) deadline support. For three scenarios, the critical masses $k = \frac{rh}{(\frac{G}{K+cG}-1)}$ are the same. The campaign's ex-ante success probabilities are written as

(a) $P_s = \sum_{l=0}^{T-G} Prob(\tilde{L} = l)$,

(b) $P'_s = \sum_{l=0}^{\min(T-G+n_0, T-T_d)} Prob(\tilde{L}' = l)$, and

(c) $P''_s = \sum_{l=0}^{\min(T-G+n_0, T-T_d)} Prob(\tilde{L}'' = l)$.

Note the range of l. With $n_0 \geq 1$ many unconditional pledges, the campaign succeeds when $\tilde{N} + n_0 \geq G$ or $T - \tilde{L} + n_0 \geq G$. Also, it must be $\tilde{L} \leq T - T_d$, which is readily satisfied in (a) as $G > T_d$. So, in (b) and (c), $0 \leq l \leq \min(T - G + n_0, T - T_d)$.

Now, since we assumed late supports come after the last buyer arrives, buyers' pledging dynamics remains the same, and thus $\tilde{L}$ and $\tilde{L}''$ are the same. Therefore, $P_s = \sum_{l=0}^{T-G} Prob(\tilde{L} = l) \leq \sum_{l=0}^{\min(T-G+n_0, T-T_d)} Prob(\tilde{L} = l) = \sum_{l=0}^{\min(T-G+n_0, T-T_d)} Prob(\tilde{L}'' = l) = P''_s$. That is, $P_s \leq P''_s$. The inequality holds strictly if $\text{Prob}(\tilde{L} = T - G + 1) \neq 0$, which is often the case as the optimal goal

G^* is close to T, meaning $\tilde{L}$ has not decreased to a negligible level at $l = T - G^* + 1$.

As for $\tilde{L}'$, notice the first buyer would observe n_0 pledges at her arrival. The derivation of the pledging thresholds with which $\tilde{L}$'s p.m.f is calculated needs to be modified accordingly. If we think the first buyer arrives at $t = n_0 + 1$, the pledging threshold for some l becomes $\frac{n'}{n'+l+1}$ such that $\frac{n'+n_0}{n'+n_0+l+1} \geq k$ with the smallest such n', as the initial n_0 arrivals are fixed to pledge; if we think the first buyer arrives at $t = 1$ (friends and family swarm in at $t = 0$), it becomes $\frac{n'}{n'+l+1}$ such that $\frac{n'+n_0}{n'+l+1} \geq k$ with the smallest such n', as the initial n_0 pledges did not cost any time. Both cases give a lower $\frac{n'}{n'+l+1}$ compared to that of $\tilde{L}''$, which is defined as $\frac{n}{n+l+1}$ such that $\frac{n}{n+l+1} \geq k$ with the smallest n. Particularly, when $l = 0$, $\frac{n'}{n'+1}$ is strictly lower than $\frac{n}{n+1}$ because $n' < n$ by definition (the proof is left out). Therefore, with strictly lower pledging thresholds, $\tilde{L}'$ is strictly first-order stochastically dominated by $\tilde{L}''$ by Lemma 3 and its proof. Thus $P'_s = Prob(\tilde{L}' \leq \min(T - G + n_0, T - T_d)) > Prob(\tilde{L}'' \leq \min(T - G + n_0, T - T_d)) = P''_s$. That is, $P'_s > P''_s$. The proof is complete.

S2.8: Proof of Proposition 7

Condition (1) assures the pledging cascade does not stop when the first late bird arrives. To see it, suppose in the early stage the campaign had lost l early birds before the cascade started. Other early birds including the dedicated buyers and common buyers who arrived after k_1 had been achieved, in total G_1 many of them, pledged at the price p_1. When the $(G_1 + l + 1)^{th}$ buyer arrives, the price rises to p_2, and if she is a common type, she will compare $\frac{n}{t} = \frac{G_1}{G_1+l+1}$ against k_2. Considering those l such that $l \leq T - G$, condition (1) assures $\frac{G_1}{G_1+l+1} \geq k_2$, thus the first late bird would follow the cascade and pledge, in which case the campaign will eventually succeed as $T - l \geq G$. So, condition (1) guarantees k_2 does not stop the cascade once it starts with k_1. Condition (2), on the other hand, assures the k_1 is a strict improvement over k, since if two critical masses are too close their $\tilde{L}$'s p.m.f's may be the same. The proof is complete.

S3: Graphs and Examples

Figure S4 $\tilde{L}$'s p.m.f is invariant to scale (of T and T_d) but depends on their ratio ($\frac{T_d}{T}$).

Figure S5 More examples for $\tilde{L}$ p.m.f's with different k in the scenario $T_d < G$ (i.e., dedicated buyers cannot secure the target alone).

Figure S6 More examples for $\tilde{L}$ p.m.f's with different k in the scenario $T_d \geq G$.

Figure S7 $\tilde{L}$ p.m.f's for arbitrary $k \in (0, 1)$, $k \notin \mathbb{K}$.

Figure S8 $F(L \mid k)$ for different L and $\frac{T_d}{T}$, being a discontinuous, decreasing function of k.

Figure S9 The Maximization (6) is approximately convex in G.

Tables 4, 5, 6, 7 included in the end.

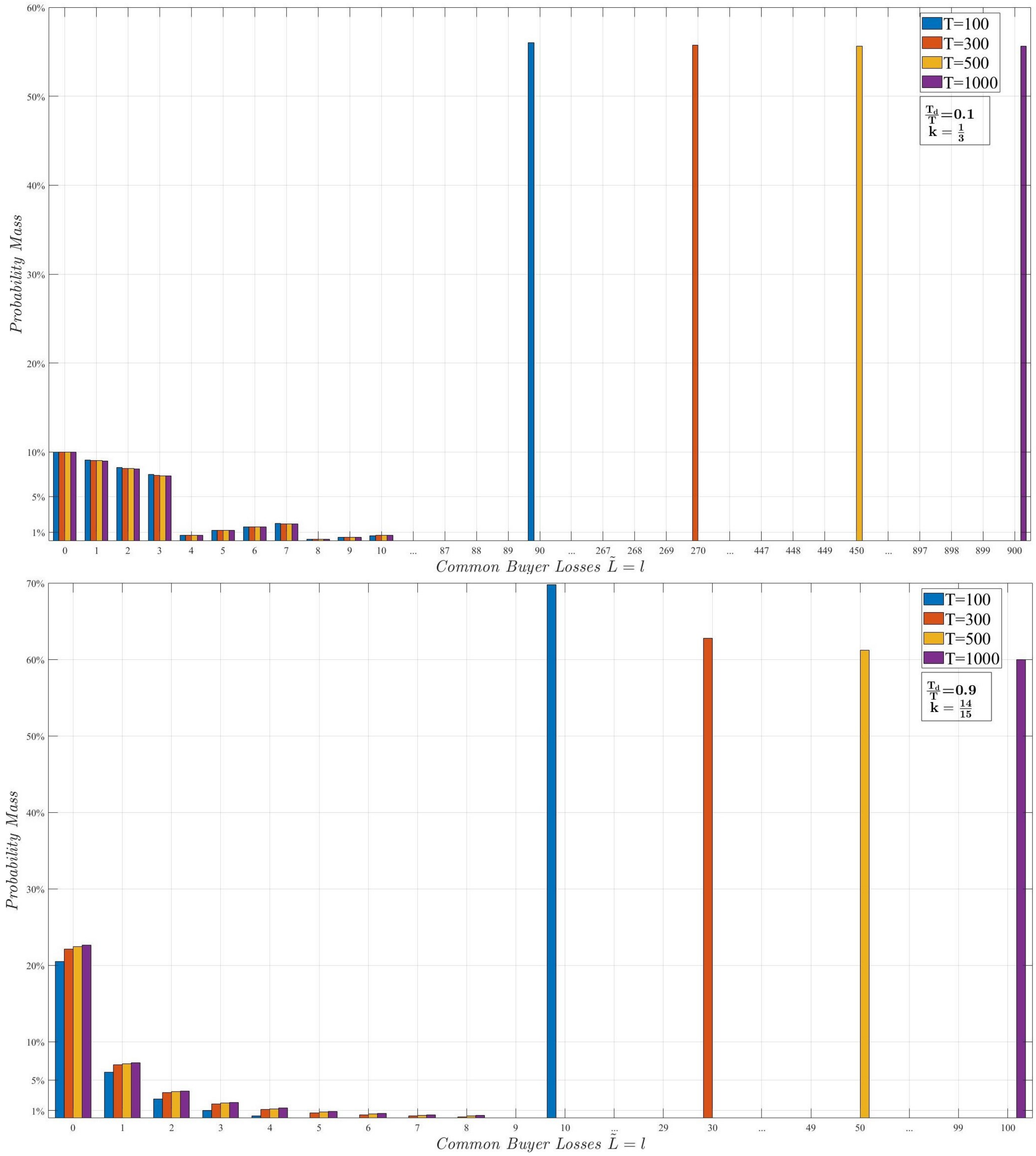


Figure S4: Invariant to scale

Note: This figure displays $\tilde{L}$'s p.m.f with varying scales. In the top (bottom) figure, $\frac{T_d}{T} = 0.1$ (0.9) and $k = \frac{1}{3}$ ($\frac{14}{15}$), while T takes the value of 100, 300, 500 and 1000. Notice the p.m.f is almost invariant to the scale except the tail's location $T - T_d$.

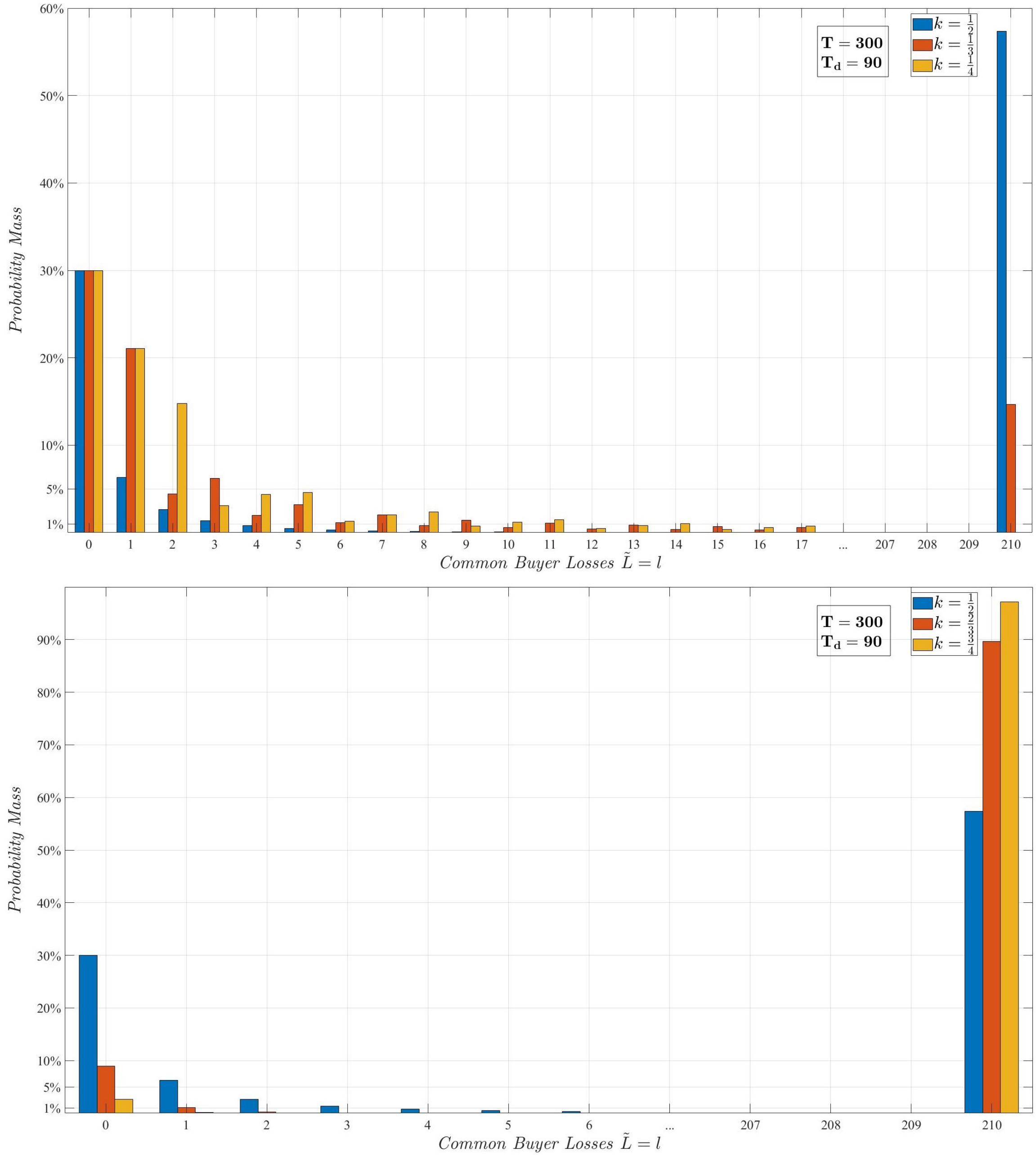


Figure S5: Varying k when $T_d < G$

Note: This figure displays $\tilde{L}$'s p.m.f with varying critical masses when $T_d < G$, the focal scenario of the main paper. In both figures, $T = 300$ and $T = 90$, while k takes the value of $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, $\frac{2}{3}$ and $\frac{3}{4}$. Because the p.m.f decreases quickly to a negligible level as l increases, there tend to be two probability spikes around $\tilde{L} = 0$ and $T - T_d$. Also, as the critical mass decreases, $\tilde{L}$'s p.m.f becomes better in the sense that the campaign tends to lose less buyers. Consequently, when k is small (e.g., $k = \frac{1}{4}$), the spike at $\tilde{L} = T - T_d$ may not exist—Campaigns with a very low critical mass are likely to capture almost all buyers.

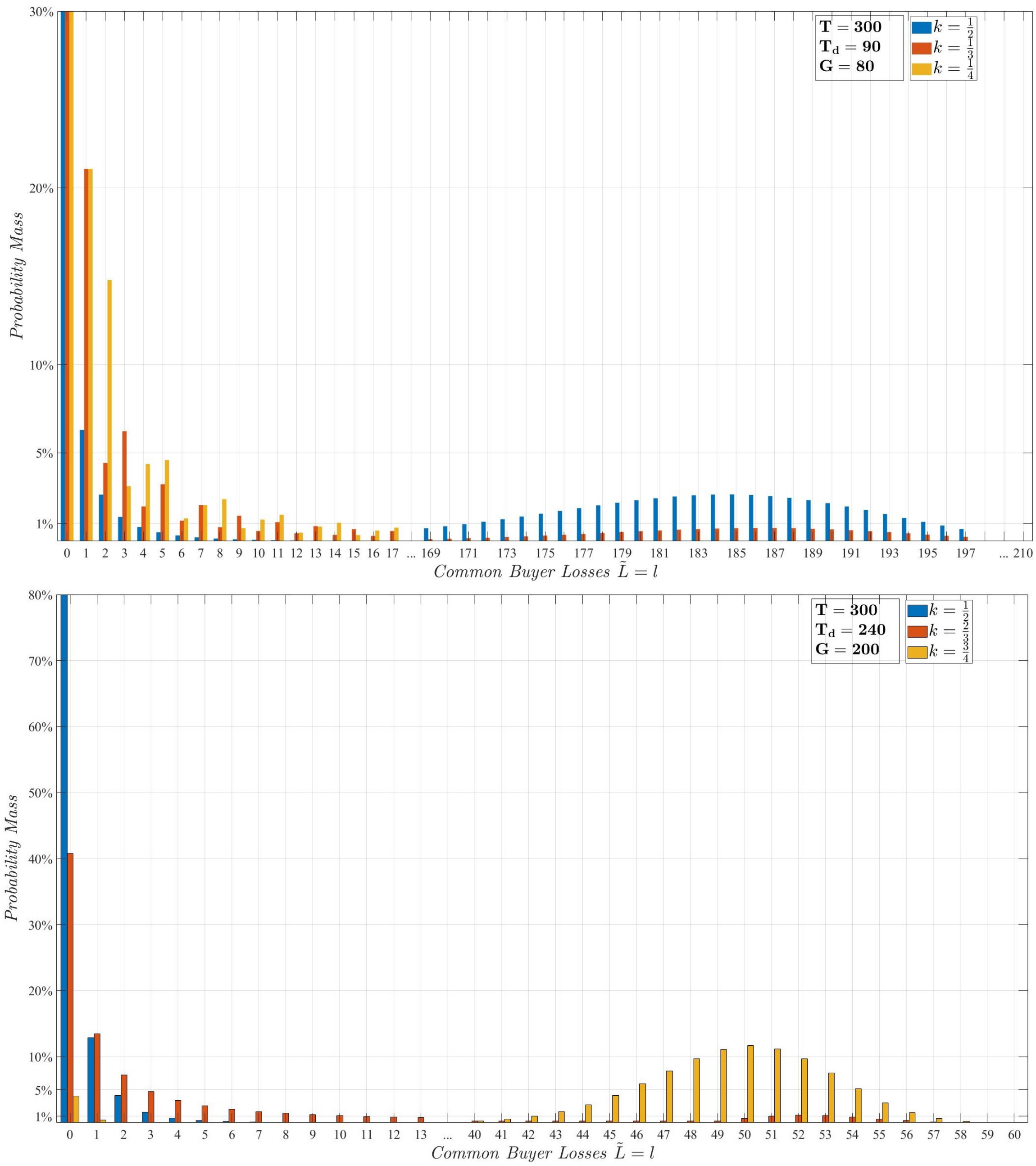


Figure S6: Varying k when $T_d \geq G$

Note: This figure displays $\tilde{L}$'s p.m.f with varying critical masses when $T_d \geq G$, a scenario not considered by the paper. The p.m.f is identical to that of $T_d < G$ when l is not too big by definition. For bigger l, the p.m.f forms a hump before l reaches its tail. The intuition is: If the number of dedicated buyers is sufficient to reach the goal even before the critical mass is reached, the probability at the tail moves to the left, as such campaigns are unlikely to lose all common buyers given the abundance of dedicated buyers. However, this scenario is rare, since all such campaigns will succeed for sure and thus fall in the outlier (more than 100% funded) as blockbusters. For example, if $k = \frac{1}{2}$ and $T = 300$, $T_d = 240$, $G = 200$, this campaign has 80% probability to get 150% funded. The number becomes more dramatic if the market demand T turns out to be very big for hot campaigns.

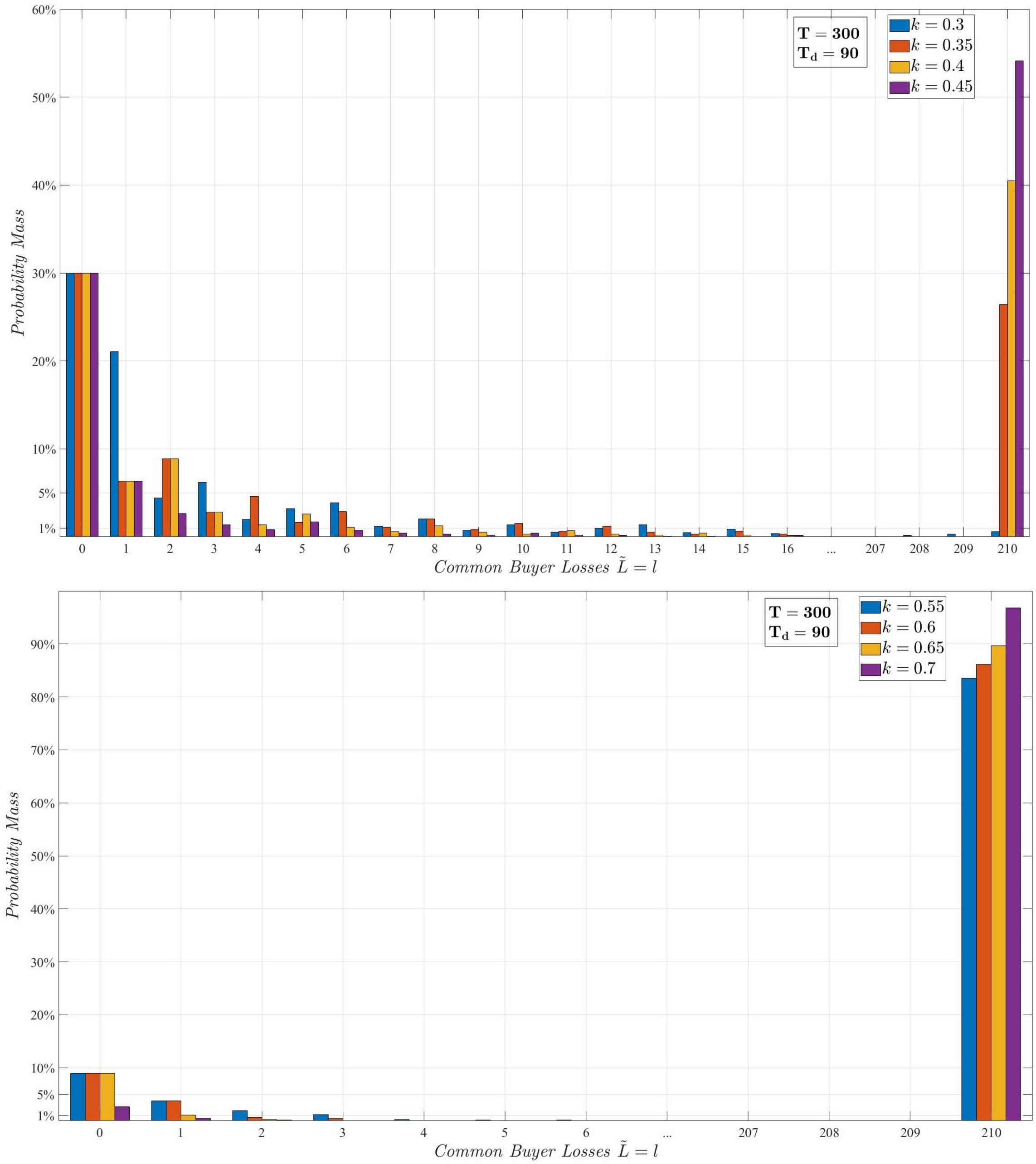


Figure S7: Arbitrary k when $T_d < G$

Note: This figure displays $\tilde{L}$'s p.m.f when $k \neq \frac{1}{x}$ and $k \neq \frac{y-1}{y}$. The two-spikes property is clearly retained.

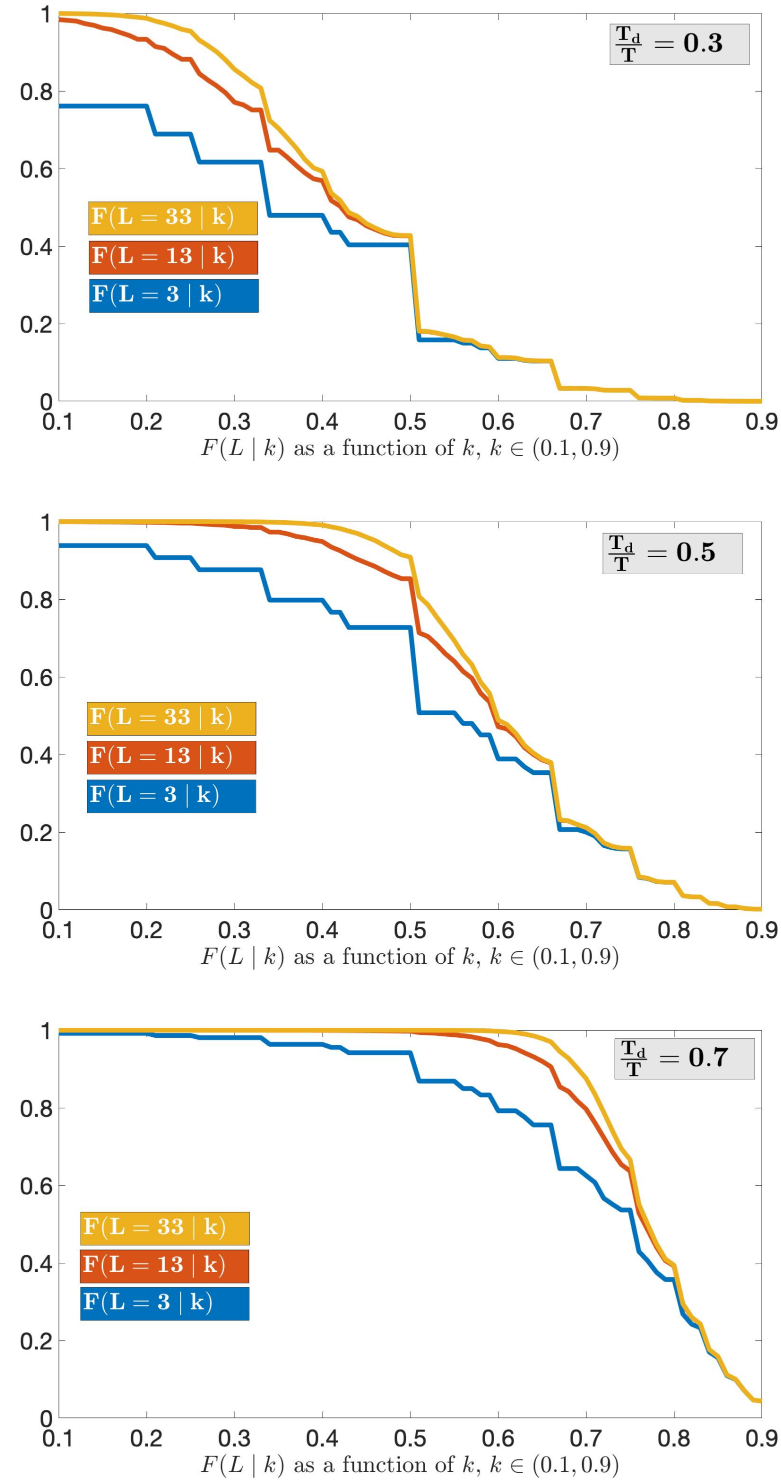


Figure S8: $F(L \mid k)$ for different L and $\frac{T_d}{T}$, corresponding to Lemma 4 in main paper

Note: We simulate $F(L \mid k)$ for $\frac{T_d}{T} = 0.1, 0.2, ..., 0.9$ and $k = 0.10, 0.11, ..., 0.90$, all exhibiting similar properties. The actual F is discontinuous in k but connected by line (seen as the kinks) in this graph.

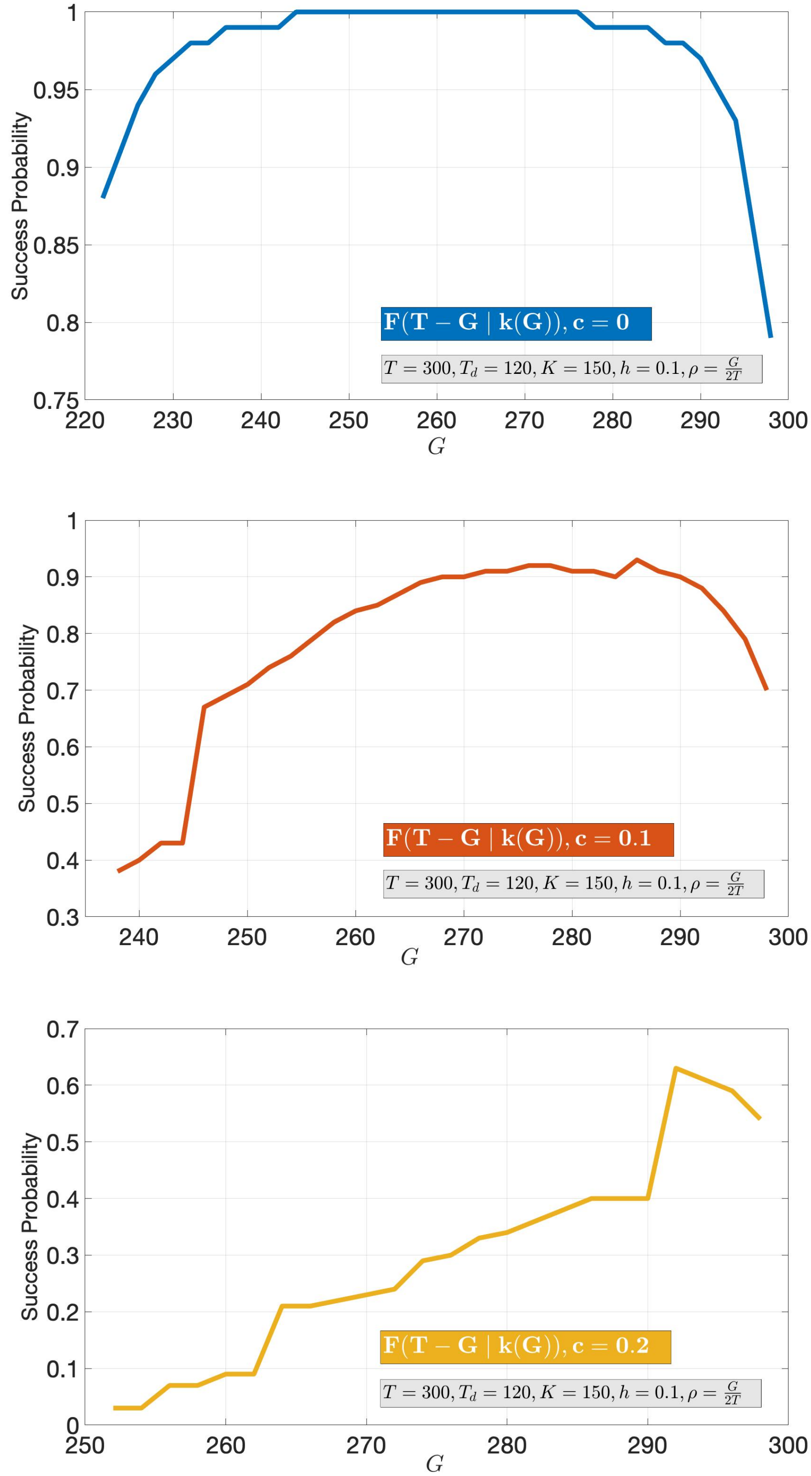


Figure S9: The maximization is approximately convex in G

Note: The figures also show that higher c lowers the overall success probability, i.e., the $F(\cdot)$ function shifts down.

l	**Pledging threshold**	**DB hit (n)**	**m**	**Necessary condition for $\tilde{L} = l$**	**Sufficient condition for $\tilde{L} \neq l$**
0	$1/2$	1	1	$\{1\} = 0$	$\{1\} = 1$
1	$1/3$	1	2	$\{2\} = 1$	$\{2\} = 2$
...	...	...	...	...	...
$x-2$	$1/x$	1	$x-1$	$\{x-1\} = x-2$	$\{x-1\} = x-1$
$x-1$	$2/(x+2)$	2	1	$\{2\}^{(x-1)} = 0$	$\{2\}^{(x-1)} \geq 1$
x	$2/(x+3)$	2	2	$\{3\}^{(x-1)} = 1$	$\{3\}^{(x-1)} \geq 2$
$x+1$	$2/(x+4)$	2	3	$\{4\}^{(x-1)} = 2$	$\{4\}^{(x-1)} \geq 3$
...	...	...	...	...	...
$2x-3$	$2/(2x)$	2	$x-1$	$\{x\}^{(x-1)} = x-2$	$\{x\}^{(x-1)} \geq x-1$
$2x-2$	$3/(2x+2)$	3	1	$\{x+2\}^{(x-1)} = x-1$	$\{x+2\}^{(x-1)} \geq x$
$2x-1$	$3/(2x+3)$	3	2	$\{x+3\}^{(x-1)} = x$	$\{x+3\}^{(x-1)} \geq x+1$
$2x$	$3/(2x+4)$	3	3	$\{x+4\}^{(x-1)} = x+1$	$\{x+4\}^{(x-1)} \geq x+2$
...	...	...	...	...	...
$3x-4$	$3/(3x)$	3	$x-1$	$\{2x\}^{(x-1)} = 2x-3$	$\{2x\}^{(x-1)} \geq 2(x-1)$
...	...	...	...	...	...
$(n-1)(x-1)-1$	$(n-1)/((n-1)x)$	$n-1$	$x-1$	$\{(n-2)x\}^{(x-1)}$ $= (n-2)x-n+1$	$\{(n-2)x\}^{(x-1)}$ $\geq (n-2)(x-1)$
$(n-1)(x-1)$	$n/((n-1)x+2)$	n	1	$\{(n-2)x+2\}^{(x-1)}$ $= (n-2)(x-1)$	$\{(n-2)x+2\}^{(x-1)}$ $\geq (n-2)(x-1)+1$
$(n-1)(x-1)+1$	$n/((n-1)x+3)$	n	2	$\{(n-2)x+3\}^{(x-1)}$ $= (n-2)(x-1)+1$	$\{(n-2)x+3\}^{(x-1)}$ $\geq (n-2)(x-1)+2$
$(n-1)(x-1)+2$	$n/((n-1)x+4)$	n	3	$\{(n-2)x+4\}^{(x-1)}$ $= (n-2)(x-1)+2$	$\{(n-2)x+4\}^{(x-1)}$ $\geq (n-2)(x-1)+3$
...	...	...	...	...	...

Table 4: Conditions for $\tilde{L} = l$, arbitrary $x \in \{3, 4, 5, ...\}$

l	**Pledging threshold**	**DB hit (n)**	**m (e)**	**Necessary condition for $\tilde{L} = l$**	**Sufficient condition for $\tilde{L} \neq l$**
...	...	...	...	...	...
$2x-3$	$2/(2x)$	2	$x-1$	$\{x\}^{(x-1)} = x-2$	$\underline{\underline{\{x\}^{(x-1)} \geq x-1}}$
...	...	...	...	...	...
$3x-4$	$3/(3x)$	3	$x-1$	$\{2x\}^{(x-1)} = 2x-3$	$\underline{\underline{\{2x\}^{(x-1)} \geq 2(x-1)}}$
...	...	...	...	...	...
$(G-2)(x-1)-1$	$(G-2)/((G-2)x)$	$G-2$	$x-1$	$\{(G-3)x\}^{(x-1)}$ $= (x-1)(G-3)-1$	$\underline{\underline{\{(G-3)x\}^{(x-1)}}}$ $\underline{\underline{\geq (x-1)(G-3)}}$
...	...	...	...	...	...
$(G-1)(x-1)-1$	$(G-1)/((G-1)x)$	$G-1$	$x-1$	$\{(G-2)x\}^{(x-1)}$ $= (G-2)(x-1)-1$	$\{(G-2)x\}^{(x-1)}$ $\geq (G-2)(x-1)$
$(G-1)(x-1)$	$(G-1)/((G-1)x+1)$	$G-1$	$\underline{0}$	$\{(G-2)x+1\}^{(x-1)}$ $= (G-2)(x-1)$	$\{(G-2)x+1\}^{(x-1)}$ $\geq (G-2)(x-1)+1$
$(G-1)(x-1)+1$	$(G-1)/((G-1)x+2)$	$G-1$	$\underline{1}$	$\{(G-2)x+2\}^{(x-1)}$ $= (G-2)(x-1)+1$	$\{(G-2)x+2\}^{(x-1)}$ $\geq (G-2)(x-1)+2$
$(G-1)(x-1)+2$	$(G-1)/((G-1)x+3)$	$G-1$	$\underline{2}$	$\{(G-2)x+3\}^{(x-1)}$ $= (G-2)(x-1)+2$	$\{(G-2)x+3\}^{(x-1)}$ $\geq (G-2)(x-1)+3$
...	...	...	...	...	...

Table 5: Conditions for $\tilde{L} = l$, considering $n \geq G-1$

l	**Pledging threshold**	**DB hit**	**Necessary condition for $\tilde{L} = l$**	**Sufficient condition for $\tilde{L} \neq l$**
0	$(y-1)/y$	$y-1$	$\{y-1\} = 0$	$\{y-1\} \geq 1$
1	$2(y-1)/2y$	$2(y-1)$	$\{2y-1\} = 1$	$\{2y-1\} \geq 2$
2	$3(y-1)/3y$	$3(y-1)$	$\{3y-1\} = 2$	$\{3y-1\} \geq 3$
3	$4(y-1)/4y$	$4(y-1)$	$\{4y-1\} = 3$	$\{4y-1\} \geq 4$
...	...	...	...	...
$l-1$	$(l(y-1)/ly$	$l(y-1)$	$\{ly-1\} = l-1$	$\{ly-1\} \geq l$
l	$(l+1)(y-1)/(l+1)y$	$(l+1)(y-1)$	$\{(l+1)y-1\} = l$	$\{(l+1)y-1\} \geq l+1$
...	...	...	...	...

Table 6: Conditions for $\tilde{L} = l$, arbitrary $y \in \{2, 3, 4, ...\}$

l	**Pledging threshold**	**DB hit**	**Necessary condition for $\tilde{L} = l$**	**Sufficient condition for $\tilde{L} \neq l$**
0	$(y-1)/y$	$y-1$	$\{y-1\} = 0$	$\underline{\{y-1\} \geq 1}$
1	$2(y-1)/2y$	$2(y-1)$	$\{2y-1\} = 1$	$\underline{\{2y-1\} \geq 2}$
2	$3(y-1)/3y$	$3(y-1)$	$\{3y-1\} = 2$	$\underline{\{3y-1\} \geq 3}$
3	$4(y-1)/4y$	$4(y-1)$	$\{4y-1\} = 3$	$\underline{\{4y-1\} \geq 4}$
...	...	...	...	...
$\hat{l}-1$	$(\hat{l}(y-1)/\hat{l}y$	$\hat{l}(y-1)$	$\{\hat{l}y-1\} = \hat{l}-1$	$\underline{\{\hat{l}y-1\} \geq \hat{l}}$
$\hat{l}$	$(\hat{l}+1)(y-1)/(\hat{l}+1)y$	$(\hat{l}+1)(y-1)$	$\{(\hat{l}+1)y-1\} = \hat{l}$	$\underline{\{(\hat{l}+1)y-1\} \geq \hat{l}+1}$
$\hat{l}+1$	$(G-1)/(G+\hat{l}+1)$	$G-1$	$\{G+\hat{l}\} = \hat{l}+1$	$\underline{\{G+\hat{l}\} \geq \hat{l}+2}$
$\hat{l}+2$	$(G-1)/(G+\hat{l}+2)$	$G-1$	$\{G+\hat{l}+1\} = \hat{l}+2$	$\underline{\{G+\hat{l}+1\} \geq \hat{l}+3}$
...	...	...	...	...
$\hat{l}+e-1$	$(G-1)/(G+\hat{l}+e-1)$	$G-1$	$\{G+\hat{l}+e-2\} = \hat{l}+e-1$	$\underline{\{G+\hat{l}+e-2\} \geq \hat{l}+e}$
$\hat{l}+e$	$(G-1)/(G+\hat{l}+e)$	$G-1$	$\underline{\{G+\hat{l}+e-1\} = \hat{l}+e}$	$\{G+\hat{l}+e-1\} \geq \hat{l}+e+1$
...	...	...	...	...

Table 7: Conditions for $\tilde{L} = l$, considering $T_d \geq G$

References

Cecere, G., Le Guel, F., and Rochelandet, F. 2017. Crowdfunding and social influence: an empirical investigation. Applied economics, 49(57):5802–5813.

Deb, J., Oery, A., and Williams, K. R. 2019. Aiming for the goal: Contribution dynamics of crowdfunding. National Bureau of Economic Research, No.w25881.

Hu, M., Li, X., and Shi, M. 2015. Product and pricing decisions in crowdfunding. Marketing Science, 34(3):331–345.

Strausz, R. 2017. A theory of crowdfunding: A mechanism design approach with demand uncertainty and moral hazard. American Economic Review, 107(6):1430–76.