On the Repeated Volunteer's Dilemma with Equal Cost Sharing

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Abstract

We study the symmetric volunteer's dilemma with binary actions and cost sharing, where the volunteering cost is split equally among volunteers. In the one-shot game, all pure-strategy Nash equilibria involve a single volunteer, while Pareto optimality allows any non-zero number. In the infinitely repeated game, all Pareto optima can be sustained in a subgame-perfect Nash equilibrium based on a grim-trigger strategy: trivially under undiscounted payoffs, and provided the discount factor exceeds a threshold under discounted payoffs. This threshold is non-monotonic in the number of volunteers; it is zero with one volunteer, highest with two, and decreases with both more volunteers beyond two and more players. Thus, the scope for tacit cooperation is universal with one volunteer, minimal with two, then improving as more join in, all the way to universal again only in the limit with more and more players and volunteers. Considering both equity and scope for cooperation as criteria, the grand coalition as volunteers emerges as the best cooperation scenario.

Keywords: volunteer's dilemma, cost sharing, repeated games, Pareto optimality, coalition formation.

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1 Introduction

The volunteer's dilemma (Diekmann, 1985) is an n-player, complete-information, simultaneous-move game dealing with the provision of a public good. Each individual chooses a binary action, to volunteer (incurring cost c) or to stand down, knowing that the provision of the public good only requires (at least) one player to volunteer, whereas all players receive a benefit of b upon provision. Therefore, more volunteers add cost but do not generate extra benefit. Because $b > c$, a lone individual would have the incentive to volunteer were he the only player in the game. But with n players, one might expect other players to volunteer, and such an incentive to free ride often results in socially inefficient outcomes.

Despite its simplicity, a diversity of political, military, economic, and biological settings may be seen as good fits for the volunteer's dilemma (VD). Well-known examples include the infamous story of Kitty Genovese who was murdered without any of many eye witnesses even calling the police (as in Manning, Levine, and Collins, 2007), alarm sounding in animal societies warning of approaching predators (Archetti, 2011), preemptive strikes to neutralize potential raiders (Konrad, 2024), and those discussed in Konrad and Morath (2021). Other examples put forth recently include dismantling piracy groups in international waters and whistle-blowing in private or government organizations (Amir, Machowska, and Tian, 2025).

Based on Diekmann's classical VD game, Weesie and Franzen (1998) introduced cost sharing, where the costs of providing the public good are split evenly among all volunteers.[1] Cost sharing arises in many volunteer dilemma settings, such as in counter-piracy coalitions and in corporate or political whistleblowing (see Amir, Machowska, and Tian, 2025, for more discussion). In the former case, each country in the coalition incurs a fraction of total naval costs, as it may patrol only the area of the danger zone closest to its coastline.[2] As for whistleblowing, the main cost incurred by any whistleblower is the risk of retaliation by the accused, but as more individuals join to report the wrongdoing, retaliation to any single

[1] Amir, Machowska, and Tian (2025) considered cost synergies where the cost-sharing function (of an exponential form) exhibits increasing returns to scale as the number of volunteers increases. As in Weesie and Franzen (1998), Amir, Machowska, and Tian (2025) assume a one-shot interaction.

[2] An example of such coalitions is the Contact Group on Piracy off the Coast of Somalia. See `https://2009-2017.state.gov/t/pm/rls/fs/2016/255175.htm`.

person becomes less and less likely.[3]

This paper studies the VD game with equal cost sharing (Weesie and Franzen, 1998) in an infinitely repeated setting. In Weesie and Franzen (1998), players only interact once and each player's action is unobservable by other players.[4] However, many VD settings (with or without cost sharing) involve repeated interaction among the players. For instance, in many animal societies, alarm calling to warn group members of approaching predators is part of daily behavior. Countries in a counter-piracy coalition face repeated interaction. Since the coalition itself may not have any effective coercive power over its members, members might choose to renege on their commitments to the group. In such settings, repeated interaction may mitigate free riding and strengthen cooperation, as players can tacitly coordinate on turn taking, contributing in some periods with the expectation of free riding in others.

In the one-shot VD game, all pure-strategy Nash equilibria (PSNE) involve a single volunteer and all single-volunteer outcomes are Pareto optimal. While cost sharing does not change PSNE relative to the original VD game, it does substantially enlarge the set of Pareto optima to consist of all scenarios where the public good is provided. This is due to the total cost being evenly split among the volunteers and thus invariant to their number.

We first consider the undiscounted repeated VD game. In line with the usual focus for this class of games, the full set of Nash equilibrium payoffs is directly identified using the Folk Theorem. The key observation is that the individually rational level of each player coincides with the PSNE of the one-shot game in which that player is the sole contributor.

For the discounted VD game, we characterize the scope for tacit cooperation on every Pareto optimum – that is, every possible volunteering coalition with $k = 1$ all the way to $k = n$ players. All players participate in equity-motivated rotations in being members of this coalition.[5] Cooperation is to be sustained with the threat of permanent reversion by

[3] In the same vein, *Psst* is a recently launched nonprofit platform designed to "collectivize" whistleblowing. See `https://time.com/7208911/psst-whistleblower-collective`.

[4] Closely related is the dynamic waiting game of Bliss and Nalebuff (1984), wherein the action of volunteering is observable, and each player decides when to volunteer, conditional on no one having done so yet. Common examples of the dynamic waiting VD include stopping a noise violation in a quiet study zone, rescuing a drowning person, confronting a perpetrator at a crime scene, etc.

[5] By exogenously fixing the turn-taking protocol between members of the volunteering coalition, we avoid otherwise likely coordination failures in playing according to equilibrium predictions. In the one-shot game, the most likely coordination failure is to end up with no volunteers, as illustrated by the Kitty Genovese story. Such failure remains a likely outcome even in the repeated version of the game.

all non-deviating players to not volunteering if any player deviates from the cooperation path, i.e., with the grim-trigger strategy. This threat forces the deviator to play as the lone volunteer in all subsequent periods following her deviation.

We find that the only volunteering coalition size that can sustain universal cooperation (i.e., for all discount factors) is the singleton coalition (i.e., $k = 1$). For coalitions with more than one volunteer, cooperation is only sustainable for sufficiently large discount factors, and the more volunteers in the coalition, the higher the resulting scope for cooperation. This is because cost sharing lowers the individual cost a player must incur in each period when assigned to volunteer, and the effect is stronger the larger the coalition. At the same time, the punishment payoff is independent of the coalition size. Hence, the larger the coalition size, the lower the player's incentive to deviate from the cooperative path. In the same vein, with a fixed coalition size, the scope for cooperation also increases with the number of players, since the cumulative individual cost along the cooperative path decreases when each player has to volunteer less frequently. However, as the number of players goes to infinity, only the grand volunteering coalition (i.e., $k = n$) can achieve universal cooperation (aside from the singleton coalition). Moreover, the grand coalition is the only one that ensures full equity, as all players bear identical costs every period.

A feature of interest of the repeated cost-sharing VD game is that if the Pareto optimum sequence of outcomes can be supported as a Nash equilibrium, the same sequence can also be supported as a subgame-perfect Nash equilibrium. This is because the individually rational level of each player coincides with the PSNE of the one-shot game in which that player is the sole contributor. Thus, the threat of reversion to the deviator's individually rational level amounts to repeated play of one of the Nash equilibria of the one-shot game, which means that the punishment path is itself an equilibrium path. Subgame perfection thus implies that the trigger strategies are based on credible threats and, hence, that the resulting tacit cooperation is relatively plausible.

It is worth noting that, besides leading to the emergence of tacit cooperation, a repeated game setup enjoys the ancillary benefit of allowing a more equitable distribution of the costly volunteer role. This equity-motivated intertemporal rotation serves as a substitute for transfers, which are usually not meaningful in our binary setting. While this is not part of the usual motivation for considering repeated games, in the present setting, it is a welcome remedy to the extreme inequity inherent in the PSNEs of the one-shot game.[6]

Due to its binary choice structure, the volunteer's dilemma is arguably the simplest model in the broader literature on voluntary public good provision. In many settings, provision is a continuous variable and depends not on a single contribution but on richer aggregation rules, such as total donations (Bergstrom, Blume, and Varian, 1986), weakest-link or best-shot scenarios (Hirshleifer, 1983), or threshold requirements on the number of contributors (Palfrey and Rosenthal, 1984). In contrast, this paper focuses on the case in which players face binary choices and a single volunteer suffices for full provision.

Other studies have extended Diekmann (1985) in various ways, including a volunteer timing game with asymmetric costs (Weesie, 1993) or incomplete information (Weesie, 1994). More recently, Shi (2025) introduced hyperbolic discounting into a dynamic volunteering setup, and Konrad (2025) studied how partitioning volunteers into teams in a one-shot game affects social welfare. The equal cost-sharing VD is related to the snowdrift (hawk–dove) game (Sugden, 2004), which is widely used to analyze evolutionary cooperation in biology and ecology (e.g., Gore, Youk, and Van Oudenaarden, 2009; Hauert and Doebeli, 2004).

There is also an experimental strand of literature dedicated to the original VD game. Kloosterman and Mago (2023) consider symmetric and asymmetric repeated VD games (with no cost sharing) and explore the emergence of turn taking and coordination on the PSNE of the two-player VD game and find substantial support for the theoretical predictions on this game. Leo (2017) has a similar setting but with privately known costs and assigned but tradable intertemporal duties between the two players. These two experimental studies also stress the importance and real-life relevance of turn taking as a natural protocol for equity/reciprocity. On the other hand, several studies report little support for the PSNE predictions of the game (or one of its variants with possible delays) as well as significant individual heterogeneity in observed volunteer rates (e.g., Otsubo and Rapoport, 2008; Goeree, Holt, and Smith, 2017).

The remaining part of this paper is organized as follows. Section 2 introduces the one-

[6] Repetition with turn taking is advocated as a way to remedy inequity in other settings where a planner finds it socially optimal to treat symmetric firms in a discriminatory manner, e.g., Salant and Shaffer (1999).

shot volunteer's dilemma with equal cost sharing and characterizes its pure-strategy Nash equilibria and Pareto-optimal outcomes. Section 3 examines the infinitely repeated game with undiscounted payoffs and shows how the Folk Theorem applies in this setting. Section 4 studies the repeated game with discounted payoffs, deriving the conditions under which each Pareto-optimal coalition can be sustained and highlighting the comparative statics of the discount factor thresholds. This is followed by a general intuitive discussion of the scope of the results in Section 5. Section 6 concludes.

2 The one-shot game

In this section, we describe the one-shot VD game with equal cost sharing of Weesie and Franzen (1998) and its pure-strategy Nash equilibria (henceforth, PSNE). We also characterize the set of its Pareto optima.

2.1 The game and its PSNE

Consider an n-player game ($n \geq 2$) wherein each player i chooses between volunteering ($a^i = V$) or not ($a^i = N$) to produce a public good or to accomplish a designated task – the two interpretations we are going to use interchangeably. If at least one person volunteers, each player gets benefit $b > 0$. In contrast to the standard VD game (Diekmann, 1985), the production process for the public good allows for equal cost sharing. Specifically, with a total of k volunteers, each volunteer incurs the cost $c/k > 0$. Thus, as more volunteers come forward, the individual cost of each volunteer falls, while the benefit to all players remains constant. The idea is that the full task in question requires the same overall cost to be performed as in the classical VD game, but the volunteers may engage in task-sharing or otherwise accomplish the task in joint work in a way that lowers the individual cost to each of them equally. There is no cost to any player who chooses not to volunteer, and no benefit to any player if the public good is not produced (i.e., in the case of no volunteers). In sum, the payoff to player i is

$$X^i = \begin{cases} b - c/k, & \text{if } a^i = V \text{ and } |\{j : a^j = V, j \neq i\}| = k - 1, \\ b, & \text{if } a^i = N \text{ and } a^j = V \text{ for some } j \neq i, \\ 0, & \text{otherwise.} \end{cases}$$

Assuming $b > c$ guarantees that a lone player would always choose to produce the good. The game is thus a classical symmetric anti-coordination game with binary actions.[7]

It is easy to verify that, just as in the classical model, there are n PSNE, each of which features one player as volunteer and the other $(n-1)$ players as free riders.

More precisely, the set of all PSNE is (Weesie and Franzen, 1998)

$$\{(V, N \ldots, N), (N, V \ldots, N), \ldots, (N, N \ldots, V)\}.$$

Due to the symmetry of the game, since all PSNE differ only in the identity of the sole volunteer, we may consider them as an equivalence class and refer to this class as the unique equilibrium.

2.2 The Pareto optima

In order to characterize the set of utilitarian Pareto optima, we first define the social welfare function. For $k \geq 0$ volunteers, the social welfare is

$$W(k) = \begin{cases} 0, & \text{if } k = 0, \\ nb - k \cdot \dfrac{c}{k} = nb - c, & \text{if } k \geq 1. \end{cases}$$

Since W is independent of k for $k \geq 1$ and lower for $k = 0$, every choice of $k \geq 1$ is optimal, and hence the solution set is $\{1, 2, \ldots, n\}$: every non-zero number of volunteers is socially optimal.[8]

[7] As such, this game is one of strategic substitutes, or equivalently a submodular game. It is well known that symmetric submodular games may possess only asymmetric PSNEs. For sufficient conditions on primitives leading to these properties for general symmetric games, see e.g., Amir, Garcia, and Knauff (2010).

[8] It is easy to see that there are no other (non-utilitarian) Pareto optima. Even if the social planner places

Proposition 1. *In the one-shot VD with equal cost sharing, there are n distinct (equivalence classes of) Pareto-optimal outcomes, each involving 1, 2, ..., or n volunteers.*

In other words, each equivalence class is defined by the number of volunteers involved, and every two elements of each class differ only in the identity of the volunteers involved.

Thus, for the cost-sharing VD game, the PSNE is Pareto optimal, but all other outcomes with any level of volunteering are also Pareto optimal. In other words, the set of Pareto optima is the entire set of possible outcomes of the game, excluding only the outcome with zero volunteers.

This is a highly uncommon outcome for any game, and in particular for one dealing with public good provision: As long as the public good is supplied, any outcome is Pareto optimal. The intuition behind this unusual feature is that it is a consequence of the conjunction of invariant benefit and equal cost sharing, i.e., the feature that total cost remains constant as the number of volunteers varies.

3 The undiscounted repeated game

In this section, we consider the infinitely repeated version of the game and determine its Nash equilibrium outcomes when rewards are not discounted. As is well known for such repeated games, the Folk Theorem applies neatly and in exact fashion.

For the undiscounted repeated game, the payoff is the long-run average one-stage payoff. Since the regular limit may fail to exist, the payoff of player i is formally defined as

$$\liminf_{T\to\infty} \frac{1}{T} \sum_{t=0}^{T} X_t^i,$$

where X_t^i is player i's payoff in period $t = 0, 1, 2, \ldots$.

unequal weights on different players, this does not create any new Pareto-optimal outcome beyond those with at least one volunteer. Pareto optimality does not allow trade-offs between persons: an outcome can only be Pareto optimal if no one can be made better off without someone else being made worse off. This means that an outcome with no volunteers remains inefficient even if the social planner places unequal weights on different individuals: each is better off if one player volunteers, even the volunteer herself. Conversely, once the public good is provided, changing the number or identity of volunteers will necessarily make at least one player worse off: Any shift in who contributes will lower the cost burden for some but raise it for others.

The Folk Theorem for repeated games with undiscounted payoffs states that the set of all Nash equilibrium payoff profiles coincides with the set of all feasible and individually rational payoff profiles. In the rest of this section, we characterize each of these two sets for the VD game with equal cost sharing.

The set of all feasible payoff profiles of a game is defined as the convex hull of all the pure strategy outcomes of the game. For the present game, this set is the convex hull of the origin and all the n points where the single volunteer gets $b - c$ and all the rest get b, i.e.,

$$F = \text{conv}\left\{(0, 0, 0, \ldots, 0), (b - c, b, b, \ldots, b), (b, b - c, b, \ldots, b), \ldots, (b, b, b, \ldots, b - c)\right\}.$$

Outcomes with $k \geq 2$ volunteers are already in this set, as they can be represented as a mixture of outcomes with a single volunteer.

The individually rational (henceforth, IR) level, also known as the maxmin level, of a player is defined as the payoff that the player can ensure herself in the worst-case scenario in the game – that is, when all other players act so as to minimize her payoff. For our game, the worst-case scenario occurs when all other players do not contribute, thus forcing said player to provide the good alone, thereby forsaking any cost sharing and obtaining a payoff of $b - c$. This simple observation is worth noting as a separate result for future use, as it plays a central role in the equilibrium analysis of the repeated game with discounting.

Proposition 2. *In the one-shot VD with equal cost sharing, the outcome that delivers the IR level of a player coincides with the PSNE in which that player is the sole volunteer.*

The property that the IR level of a player coincides with her payoff in a Nash equilibrium is satisfied by a number of different games arising in economic applications, including the prisoner's dilemma, Bertrand competition with homogeneous products (e.g., Tirole, 1988), and the crime game studied in Amir, Bose, Pal, and Topolyan (2025). This strong and somewhat unusual property defines a class of games that has been studied systematically by Pruzhansky (2011).

Proposition 2 implies that the joint IR vector is $(b - c, b - c, \ldots, b - c)$. The set of IR payoffs contains all the payoffs that weakly dominate it:

$$IR = \{(x_1, x_2, \ldots, x_n) \in \mathbb{R}^n \mid x_i \geq b - c \text{ for all } i\}.$$

Therefore, by the Folk Theorem, we get the following result.

Proposition 3. *In the undiscounted repeated VD with equal cost sharing, the set of all Nash equilibrium payoff profiles is*

$$E = F \cap IR.$$

The two sets F and IR and their intersection E are depicted in Figure 1 for the special case of two players ($n = 2$), with the set E being the triangle.

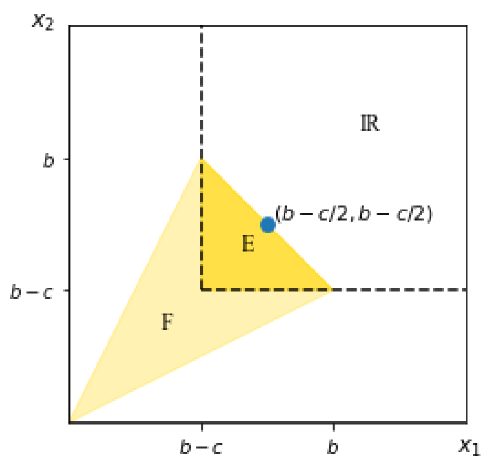

Figure 1: The set of Nash equilibrium payoff profiles (E) when $n = 2$

In line with the typical application of the Folk Theorem for undiscounted payoffs, the focus of the present analysis is on sustainable equilibrium payoffs, and not on their associated strategies. This will change in the case of the discounted repeated game, again in conformity with standard practice.

The analysis of the undiscounted case is useful as a benchmark to understand the maximal extent of cooperation in the long-run version of the game. However, in light of the fact that anything that takes place in finite time is irrelevant to the long-run welfare of the players, this is not a realistic approach to tacit cooperation in repeated games in economics. We next consider the more realistic case of discounted payoffs.

4 The discounted repeated game

In this section, we study the infinitely repeated version of the VD game with discounted rewards and determine the extent to which each of the n distinct Pareto-optimal outcomes (distinguished by the number of volunteers) is sustainable as equilibrium outcome. We evaluate sustainability using IR-level (minmax) threats implemented via grim-trigger punishments – that is, after any deviation, the deviator is held forever to her IR payoff. By Proposition 2, such threats can be implemented via the PSNE in which the deviator is the sole volunteer: any deviation by any player in any period would be met by permanent reversion to not volunteering in every subsequent period by all other players. Throughout the paper, we refer to the resulting equilibrium outcomes as "tacit cooperation".

Since the punishment path amounts to repeated play of one of the PSNE of the one-shot game, the punishment strategy profile constitutes a Nash equilibrium in the corresponding continuation subgame. Therefore, the resulting equilibrium for the full repeated game is subgame perfect.

For the discounted repeated game, the total payoff of player i is defined as

$$\sum_{t=0}^{\infty} \delta^t X_t^i,$$

where $\delta \in (0, 1)$ is the discount factor (assumed common to all the players).

Each of the n equivalence classes of Pareto-optimal outcomes, described in Proposition 1, has a fixed size for the coalition of volunteers. The associated cooperative path will keep this size fixed but allow its composition to vary from period to period as all the players in the game take turns in being members of this coalition according to an exogenously fixed order. For convenience, we shall refer to the group of volunteers in each period as "the volunteering coalition", even though only its size is fixed while its composition varies in periodic fashion.

When the size of the volunteering coalition is less than n, our chosen specification of the cooperation path – that is, the exogenously fixed order in which players take turns to volunteer – reflects considerations of equity, focality, realism and ease of implementation. We do not make any claims of uniqueness.

4.1 The Pareto optimum with one volunteer

In this subsection, we focus on the sustainability of any of the n Pareto-optimal outcomes with a single volunteer (distinguished by the identity of the volunteer).

Along the cooperation path, players are selected in some prescribed order for each of them to volunteer once in the first n periods (from period 0 to period $n-1$), after which this same contribution sequence is to be repeated indefinitely. This simple turn-taking protocol introduces novel non-stationarity and (equity-motivated) periodicity features for the cooperative path in a repeated game, whereas such paths typically specify the same Pareto-optimal outcome for the duration of the cooperative path in common repeated games.[9]

The main result is that, the single-volunteer Pareto-optimal outcome is sustainable *for any discount factor*. The reason is that the cooperation path amounts to a repeated play of different PSNEs of the one-shot game. A key implication of this unusual property is that no punishment is actually required for tacit cooperation and no player has an incentive to cheat on the agreement even if δ is arbitrarily close to 0. Therefore, an alternative possibility is to let the punishment path coincide with the cooperation path – that is, after any deviation, the predetermined turn-taking protocol remains intact and play proceeds as originally scheduled along the cooperative path.

While the periodic nature of the order for volunteering does not play any role in the equilibrium argument above, the assumption that this order is well-specified and commonly known among the players is a key point. Absent such a specified order, one would expect coordination failures if it is unclear who is volunteering in a given period.

Proposition 4. *For the VD game with equal cost sharing, the Pareto-optimal outcome with one volunteer is sustainable as a subgame-perfect Nash equilibrium of the discounted repeated game for all $\delta \in (0,1)$. Moreover, this is true for any exogenous order of volunteers.*

As we will see in the next subsections, the case of a single volunteer is the only one for which no punishment is needed to produce a Pareto-optimal outcome and cooperation is

[9] One exception in repeated games of duopolistic collusion is Herings, Peeters, and Schinkel (2005) who consider alternating monopoly instead of cartel as the cooperative path, and thus alternating one-stage profits between 0 and monopoly profits for each firm.

sustainable for all discount factors. To the best of our knowledge, these special features do not arise in other known applications of repeated games.

4.2 The Pareto optimum with n volunteers

In this subsection, we consider the other extreme case – the unique Pareto-optimal outcome with n volunteers – for which the cooperation path calls for every player to volunteer in every period. No additional protocol is needed here, as coordination failures are not an issue.

In any period, the continuation payoff for any player under maintained cooperation is

$$\sum_{t=0}^{\infty} \delta^t (b - c/n) = \frac{b - c/n}{1 - \delta}. \tag{1}$$

If a player deviates from the cooperative path by not contributing in one period, and then responds to the punishment by being the sole volunteer from the following period onwards, the corresponding payoff is

$$b + \sum_{t=1}^{\infty} \delta^t (b - c) = b + \frac{\delta(b - c)}{1 - \delta} = \frac{b - \delta c}{1 - \delta}. \tag{2}$$

Therefore, a player will choose to cooperate forever if the gains from cooperation (1) exceed or equal the gains from deviation (2). It is easy to see that this condition reduces to

$$\delta \geq \frac{1}{n} \triangleq \underline{\delta}.$$

We have just proved the following result.

Proposition 5. *For the VD game with equal cost sharing, the Pareto-optimal outcome with n volunteers is sustainable as a Nash equilibrium of the discounted repeated game if and only if $\delta \geq 1/n$. Moreover, this equilibrium is subgame perfect, with the punishment for any deviator consisting of subsequent permanent play of the PSNE with her as the sole volunteer.*

The set of discount factors $[\underline{\delta}, 1)$ for which the Pareto-optimal outcome is sustainable measures "the scope for tacit cooperation". Proposition 5 states that $\underline{\delta} = 1/n$, which is interesting for several reasons.

First, as more players enter the game, the scope for cooperation widens. To see the intuition, observe that the within-period incentive to deviate is captured by the deviation gain $b - (b - c/n) = c/n$, while the disincentive to deviate corresponds to a post-deviation punishment per-period loss of $(b - c/n) - (b - c) = c - c/n$. As the number of players n increases, a player's incentive to deviate declines while the loss from doing so rises. Therefore, these two effects reinforce each other in engendering a wider scope for tacit long-run cooperation as more players enter the game.

Second, with sufficiently many players, the scope for cooperation becomes universal. Intuitively, in the limit as n grows indefinitely, each player's individual per-period cost goes to zero, thus implying that the incentive to deviate converges to zero. At the same time, the punishment loss rises with n. Hence, in the limit as $n \to \infty$, the critical discount factor $\underline{\delta}$ converges to zero. This is tantamount to saying that, with a very large number of participants in the repeated discounted VD game, threat-based cooperation with everyone volunteering at every period is always sustainable as part of a subgame-perfect equilibrium.

Third, the scope for cooperation is independent of b and c. Intuitively, the benefit level b does not affect incentives to cooperate because the punishment path also provides the public good. The cost c also drops out because it affects both the short-term gain from shirking and the long-term loss from punishment in the same proportion.

Our next result shows that across all volunteering coalition sizes beyond 1, the grand coalition gives the largest scope for cooperation.

Proposition 6. *For the VD game with equal cost sharing, if a cooperation path in which at least two players volunteer in some period is sustainable as a subgame-perfect Nash equilibrium, then so is the cooperation path in which all players volunteer in all periods. Moreover, this cooperation path is the only one that achieves the maximal scope for cooperation ($\delta \geq 1/n$) among those paths in which at least two players volunteer in every period.*

The result in Proposition 6 is very general as the set of cooperation paths we consider only excludes the paths covered in Proposition 4 – that is, the paths in which only one player volunteers in every period. Intuitively, the grand coalition minimizes the per-period cost that each volunteer pays – and thus the benefit from shirking. At the same time, the grim-trigger punishment payoff – the IR level – is the same for all cooperation paths. Thus, including every player in the volunteering coalition yields the broadest scope for cooperation. To shed further light on this mechanism, the next subsection considers cooperation paths in which the size of the volunteering coalition remains constant, tracing how the scope for cooperation evolves as the coalition size changes.

4.3 The Pareto optimum with k volunteers

In this subsection, we restrict the coalition size to $k = 2, \ldots, n$ and see how the scope for cooperation changes with k.

Consider first $k = 2$. To specify the cooperation path along with its concomitant turn-taking protocol, assume for simplicity that n is even and assume that players 1 and 2 contribute in period 0, players 3 and 4 contribute in period 1, and so on until players $n-1$ and n contribute in period $n/2-1$. Then the same process is repeated starting in period $n/2$, again and again, indefinitely.

Since player 1 receives the benefit b in every period and pays the cost $c/2$ only at periods labeled $nt/2$, for $t = 0, 1, \ldots$, she obtains

$$\sum_{t=0}^{\infty} \delta^t b - \sum_{t=0}^{\infty} \delta^{\frac{nt}{2}} \frac{c}{2} = \frac{b}{1-\delta} - \frac{c}{2(1-\delta^{\frac{n}{2}})}. \tag{3}$$

In fact, this is the continuation payoff from the cooperative behavior for any player in any period in which this player is called to volunteer.

If any player deviates, she will obtain the payoff in (2).

Therefore, all players will cooperate forever if (3) is greater than or equal to (2), which simplifies to

$$3\delta - 1 - 2\delta^{\frac{n}{2}+1} \geq 0. \tag{4}$$

One may verify that there is a unique discount factor, denoted by $\underline{\delta}_{2,n} \in (0,1)$, such that (4) is satisfied with an equality sign and that cooperation is sustainable for any $\delta \geq \underline{\delta}_{2,n}$ (see Appendix for a general proof of the k-volunteer case). We note, as quick verification, that when $n = 2$, $\underline{\delta}_{2,n} = 1/2$, consistent with the case of n volunteers.

Now let us move to the general case of k volunteers. To specify the order of volunteers along cooperation path, assume for simplicity that n is a multiple of k and the process is analogous to the one described for two volunteers: players $1, 2, \ldots, k$ contribute in period 0, players $k+1, \ldots, 2k$ contribute in period 1, and so on until players $n-k+1, \ldots, n$ contribute in period $n/k - 1$; then the same process is repeated starting in period n/k.

Along the cooperation path, in any period in which a player is called to volunteer, this player obtains the continuation payoff of

$$\sum_{t=0}^{\infty} \delta^t b - \sum_{t=0}^{\infty} \delta^{\frac{nt}{k}} \frac{c}{k} = \frac{b}{1-\delta} - \frac{c}{k(1-\delta^{\frac{n}{k}})}. \tag{5}$$

If any player deviates, she will obtain (2).

Therefore, all players will cooperate forever if (5) is greater than or equal to (2), which simplifies to

$$(k+1)\delta - 1 - k\delta^{\frac{n}{k}+1} \geq 0. \tag{6}$$

In the Appendix, we prove that (6) with an equality sign admits a unique real root $\underline{\delta}_{k,n} \in (0,1)$ for any given $n \geq 2$ and $k \in \{2, \ldots, n\}$, and that inequality (6) holds if and only if $\delta \geq \underline{\delta}_{k,n}$. Therefore, $\underline{\delta}_{k,n}$ is the lowest bound on δ such that tacit cooperation with the described order for volunteering is sustainable.

In contrast to the typical characterization of the threshold discount factor in repeated games with stationary cooperative paths consisting of the same outcome in every period, the periodicity of this path here leads to the need to sign a higher-order polynomial.

It is easy to see that if $k = n$, then (6) is equivalent to $\delta \geq 1/n$, consistent with the case of n volunteers.

Proposition 7 summarizes the above discussion.[10]

Proposition 7. *For the VD game with equal cost sharing, the Pareto optimum with k volunteers in every period, for any $k = 2, \ldots, n$ such that n is a multiple of k, is sustainable as a subgame-perfect equilibrium of the repeated game if $\delta > \underline{\delta}_{k,n}$, where $\underline{\delta}_{k,n}$ is defined as the*

[10]Proposition 7 considers a specific order of volunteers along the cooperation path. Intuitively, this order is optimal in the sense that no other order with the same number of volunteers per period yields a wider scope for cooperation. The formal proof of this intuitive result is involved and is therefore omitted for brevity.

unique real root of

$$(k+1)\delta - 1 - k\delta^{\frac{n}{k}+1} = 0 \tag{7}$$

in the interval $(0, 1)$.

Proposition 8 provides the comparative statics of the threshold discount factor $\underline{\delta}_{k,n}$, as an inverse measure of the ease of sustaining cooperation.

Proposition 8. *For any* $n \geq 2$ *and any* $k = 2, \ldots, n$*, let* $\underline{\delta}_{k,n}$ *be the unique real root of* (7) *in* $(0, 1)$*. Then,*

(i) for fixed n*,* $\underline{\delta}_{k,n}$ *is decreasing in* k *on* $\{2, 3, \ldots, n\}$ *down to* $\underline{\delta}_{n,n} = 1/n$*;*

(ii) for fixed k*,* $\underline{\delta}_{k,n}$ *is decreasing in* n *on* $\{2, 3, \ldots\}$ *down to* $\underline{\delta}_{k,\infty} = 1/(k+1)$*.*

Hence, increasing either k *or* n *increases the scope for cooperation.*

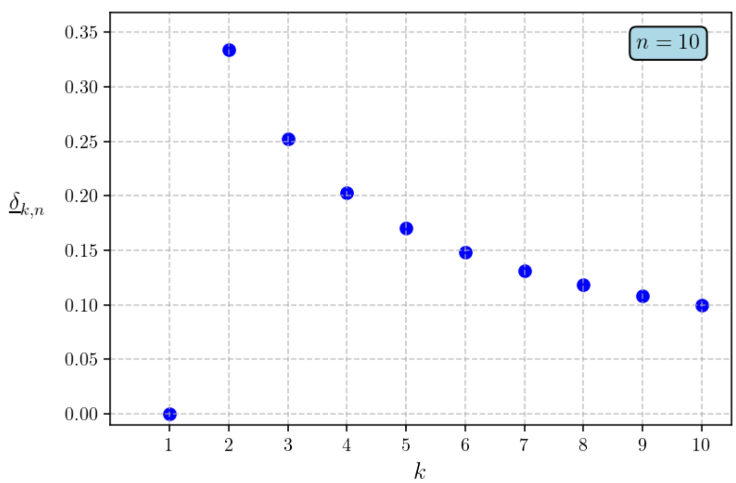

Figure 2: The scope for tacit cooperation is non-monotonic in k

Figure 2 illustrates point (i) of Proposition 8 which states that the scope for cooperation expands with more volunteers (beyond 2). This result is consistent with Proposition 6 in that the largest scope for cooperation is achieved with the grand coalition $k = n$. Intuitively, the larger the volunteering coalition size is, the smaller the incentive to deviate (in terms of the individual cost saving c/k) is. Thus, while the incentive to deviate shrinks with the coalition's size, the punishment itself (equal to the full c) remains invariant with respect to it. The notable exception to this result is that the scope for cooperation is maximal (not minimal) with a single volunteer – see Proposition 4. This follows from the total absence of any incentive to deviate due to the agreement itself being a PSNE, as noted earlier. In this case, a deviator has no one else in the coalition to rely on to volunteer, i.e., she only has herself to cheat in this case.

Point (ii) of Proposition 8 states that increasing the number of players in the game enhances the scope for cooperation. Intuitively, as the number of players increases, each individual volunteers less frequently. A deviation then leads to a greater long-term loss: the deviator forfeits more future periods in which she would have enjoyed the full cooperative benefit b as a non-volunteer. Since $\underline{\delta}_{k,n}$ converges to $1/(k+1)$ as $n \to \infty$, we conclude that even with very large numbers of players, tacit cooperation with a fixed number of volunteers (greater than 1) requires sufficiently strong discounting.

In the next section, we elaborate on the scope and implications of the results as well as on their general relationship to the literature on public good provision and to other well-known applications of repeated games.

5 General intuitive discussion of the results

In what follows, we step back from the formal analysis to draw out the broader intuition and implications of our findings. We discuss the inter-temporal inequity along the cooperation path, the trade-off between equity and the scope for cooperation, ease of implementation, the credibility of threat-based punishments, the robustness of the equilibrium to the length of the horizon, the role of perfect monitoring, and alternative ways tacit cooperation may arise. We also contrast our results to some standard findings in the existing literature.

5.1 On the lack of equal treatment

Although the game we study is symmetric across players, the cooperation path with $k < n$ volunteers every period features inter-temporal inequity. Later volunteers are favored over earlier ones over any provision cycle, due to discounting. Specifically, as long as cooperation is maintained throughout, all players receive the same discounted total benefit, but the discounted total cost is higher for early volunteers.

Unless one relies on randomization (according to a uniform distribution) to assign contributing periods to players, the inter-temporal inequity in the design of the cooperation path and its concomitant distinct discounted payoffs cannot be circumvented (see Konrad, 2024, for a one-shot VD setting where a similar issue arises and randomization is adopted to induce ex-ante equal treatment of players in terms of expected utility).[11] Another option is to consider mixed-strategy Nash equilibrium instead to restore symmetry, but Pareto optimality would then be forsaken.

In fact, besides allowing for tacit cooperation, a key advantage of invoking a repeated game setup for the VD game is to reduce the otherwise extreme form of asymmetry inherent in the PSNEs of the one-shot game. In particular, for the case of a single volunteer (Section 4.1), this reduction of asymmetry is the only benefit of a repeated game setup, as tacit cooperation is automatically present.

5.2 Equity versus scope for tacit cooperation

In this subsection, we compare the various equilibrium outcomes, distinguished by the size of the volunteering coalition, according to the three broadly accepted criteria: the scope for cooperation, equity, and ease of implementation.

The scenario with a single volunteer and turn taking achieves the maximal scope for cooperation – being sustainable for all discount factors – but entails the greatest degree of inter-temporal inequity among the scenarios we consider. While the scope for cooperation is worst with two volunteers, this scope improves, along with its concomitant equity level, as more volunteers are added. When the grand coalition is reached, it stands as the second-best scenario in terms of the scope for cooperation. However, in contrast to the first-best scenario with the singleton volunteering coalition, the grand coalition achieves perfect equality and has the advantage of not requiring a complex turn-taking protocol. While the cooperation scope of the grand coalition remains less than universal, it is close to maximal with modest

[11] This asymmetry may be partly mitigated, e.g., by reversing the order of volunteering across players from one set of n/k periods to the next, so that, say, players $1, \ldots, k$ who are first in the first set of n/k periods become last in the following set, and so on, indefinitely. However, while this and other possible re-orderings of players' moves would reduce the asymmetry, such steps would only amount to partial remedies.

values of n, and, as the grand coalition grows very large, it gets arbitrarily close to universal.

Therefore, the singleton and grand volunteering coalitions emerge as the most attractive options among all possible coalition sizes, with the final choice to be determined by the actual value of the discount factor, the population size and the relative ease of implementation of the relevant protocols. This is one of the main conclusions of the present analysis.

5.3 On the effects of group size on tacit cooperation

The conclusion that, as the number of players in the game grows, the scope for cooperation enlarges stands against multiple well-known results on the difficulty of sustaining cooperation and/or collusion in different repeated games as well as on the extent of free riding in various static public good models.

We begin by contrasting our result with its counterpart for static models. Using a standard model of (continuous) public good provision, Bergstrom, Blume, and Varian (1986) have shown that free riding worsens with larger group size. This is a core robust result across various public good models with continuous provision levels.

For the mixed-strategy Nash equilibrium of the classical VD game, Diekmann (1985) has shown that the probability of volunteering decreases in group size at such a strong rate that the overall probability of the public good being provided declines with group size. Weesie and Franzen (1998) concluded that the same result continues to hold for the present (cost-sharing) game.[12] This has been suggested as a rationalization of the well-known bystander effect (Darley and Latane, 1968) for which social psychologists offer other explanations.

The present analysis points to the opposite conclusion on the effect of group size than much of the rich literature on public good models. This is not surprising since repetition is generally known to engender more cooperation (i.e., remedy the free-riding problem) and a larger group size dilutes the incentive to deviate from the cooperation path, as seen earlier.

In repeated oligopoly games, whether based on Bertrand or Cournot competition, collusion or tacit cooperation between firms is well known to become more difficult to sustain (in the sense that the threshold discount factor increases) with more firms, e.g., Tirole (1988).

[12] As to the PSNE, they are Pareto optimal and trivially invariant to group size for both versions of the game, hence inadequate to address the key free-riding issue.

This is due to the fact that adding more firms enhances the incentive to deviate (going from $1/n$ to the entirety of the market) while the punishment remains the same (zero profit). An important exception is Matsumura and Matsushima (2005), who find a non-monotonic relationship between the critical discount factor and the number of firms in a spatial price discrimination model.

5.4 Subgame-perfect tacit cooperation

The fact that the Nash equilibrium that supports tacit cooperation is always subgame perfect is worth emphasizing; it enhances the plausibility of the threat-based equilibrium since the associated strategy relies on a credible threat.

Aside from the repeated VD game, the subgame-perfection property of a threat-based equilibrium is also shared by a few of the most important repeated games as far as economic applications are concerned. These include what are probably the two most-widely studied specific repeated games: the prisoner's dilemma and Bertrand competition with homogeneous products (Tirole, 1988). Interestingly, while the prisoner's dilemma and Bertrand competition are both characterized by some unusual structure in that they have (unique) PSNEs in strictly dominant strategies and in weakly dominated strategies, respectively, as a 2x2 coordination game, the VD game satisfies neither of these characteristics.

The singleton-coalition equilibrium in the repeated cost-sharing VD game combines two features rarely observed together in repeated games. First, the Pareto-optimal cooperation path consists of a repetition of PSNEs of the one-shot game. Second, the IR-inducing trigger-strategy punishment, though chosen a priori to inflict the most harm to a deviator, is itself a PSNE of the one-shot game (the worst of the PSNEs for the deviator) as well as a Pareto optimum. As a result, whether cooperation continues indefinitely or a deviation takes place, play always remains within the set of PSNEs of the stage game.

5.5 On the role of the length of the horizon

For many games of interest, the conclusion of tacit cooperation for sufficiently high discount factors relies critically on the horizon being infinite. This is the case of the prisoner's dilemma and of (homogeneous-good) Bertrand competition, among other examples. In contrast, in the VD game, for the case of the equilibrium with the singleton volunteering coalition, the same argument as given in the present paper would show that the same conclusion, namely that the Pareto-optimal outcome is sustainable via the threat of no volunteering, also extends to finitely repeated games.

Benoit and Krishna (1985) established a Folk Theorem for finite-horizon repeated games with multiple PSNE for the one-shot game, but their general result requires sufficiently long horizons. Their argument relies on alternating among different stage-game equilibria in the tail of the game to punish deviations. This reasoning is directly relevant to the present framework, as the cost-sharing VD game features multiple PSNE corresponding to different sole volunteers, implying that cooperation paths with any fixed volunteering coalition size $k > 1$ could, in principle, be sustained for long enough finite horizons. In contrast, the singleton-coalition case goes further: it requires no such assumption on the horizon, since both the cooperative outcome and the grim-trigger punishment are themselves PSNE of the one-shot game. The underlying mechanism therefore differs in a significant way from the analogous argument of Benoit and Krishna (1985).

5.6 On the role of perfect monitoring

In general, in repeated games, the issue of perfect monitoring of other players' actions is an important aspect that may critically affect the ability of players to punish deviators and therefore influence the extent to which the Folk Theorem may apply. However, as our analysis clearly indicates, for the VD game, the question of perfect monitoring is mostly immaterial. Indeed, were a deviation to take place, every player in the volunteering coalition would know with certainty that a deviation has taken place by observing a higher own cost to volunteer, even if actions are unobservable.[13] At least one player in the coalition needs to inform outsiders (who are first unaware of the deviation as the public good was supplied by the non-deviating coalition members) that a deviation has taken place, so the latter participate in the punishment phase in the periods after the deviation.

[13] The discussion is restricted to the cases with more than one volunteer in the coalition, as the agreement in the sole volunteer case is self-enforcing, as noted earlier.

There is no need to identify specifically the deviator to trigger the simple punishment of no volunteering in future periods following a single deviation. For the grand coalition, the requirement on monitoring is the mildest because there are no outsiders and all players can deduce that someone has deviated simply from observing their own stage payoff. Note that although the very nature of the punishment scheme implicitly targets the deviator, her identity remains hidden for other players if they only observe their own payoff.

Another uncommon feature of the repeated VD game with cost sharing is the observation that a deviator in a given period does not directly cheat the non-volunteers, but only her partners in that period's volunteering coalition. Indeed, the only direct harm of a deviation is to cause the cost of compliance for the other coalition members to increase. The non-volunteers are not affected directly in the deviation period since the other designated volunteers will continue to supply the public good. The non-volunteers are affected along the punishment phase, if any.

All in all, the informational or observational requirements of the cooperation scheme are quite minimal. This underlies another sense in which the threat mechanism here is very robust and broadly applicable, in particular in many settings where the lack of observability of others' actions is a realistic feature.

It is also worth noting that the tacit cooperation scheme described here is very intuitive, simple and so realistic that it sounds quite plausible as a way of dealing with free riders across a large sample of distinct societies and environments, in a robust and reliable way that eschews any critical role for a central authority in achieving the first-best outcome.

5.7 Other scenarios for tacit cooperation

In addition to the above turn-taking cooperation scenarios involving all players, there are other possible protocols/scenarios to achieve the desired outcome of the public good being supplied in every period. One option is to partition the set of players into an active subgroup and an inactive subgroup. The latter's members act as designated free riders, while the members of the active subgroup re-enact the analogous steps as in the previous section, using the same threat. It is easy to see that the scope for cooperation would then be determined by the size of the active subgroup, rather than by the total number of players.

In some tribal settings, being drafted into the active subgroup (of one or more persons) might correspond to a punishment for some transgression of community rules, which is milder than banishment or other harsh treatment.

6 Conclusion

This paper has investigated the extent of tacit cooperation in the infinitely repeated VD game with equal cost sharing (Weesie and Franzen, 1998), both with undiscounted and with discounted payoffs. We first observe that the main target of cooperation, i.e., the set of Pareto optima of the one-shot game, is much larger than in the original VD game, consisting of all n distinct (equivalence classes of) outcomes where the public good is provided. In contrast, the set of Nash equilibria of the one-shot game remains the same as in the original VD game.

For the undiscounted repeated game, we invoke the Folk Theorem directly, upon observing that the individually rational level of each player coincides with the PSNE of the one-shot game in which that player is the sole contributor.

For the discounted repeated game, we characterize the scope for tacit cooperation on every Pareto optimum, distinguished by the size $k = 1, \ldots, n$ of the volunteering coalition, sustained via the threat of permanent reversion by all non-deviating players to not volunteering in case of deviation. In all these scenarios, as this threat forces the deviator to act as the lone volunteer in all periods following a deviation, the resulting outcome constitutes a PSNE of the one-shot game (and at the same time achieves the deviator's individually rational payoff). Therefore, the resulting equilibria of the repeated game are subgame perfect.

We find that the only volunteering coalition size that can sustain universal cooperation (i.e., for all discount factors) is the singleton coalition (i.e., $k = 1$), where equity-motivated turn taking ensures that this role accrues to each player periodically. For coalitions with more than one volunteer, cooperation is only sustainable for sufficiently large discount factors. Overall, the scope for tacit cooperation is universal with one volunteer, minimal with two, then monotonically improving with higher numbers of volunteers and/or players in the game. For the grand coalition, the scope for cooperation asymptotically tends to universal again as more and more players join the game.

These comparative statics results form one of the main conclusions of this paper, a complete reversal of the standard takeaway from multiple static public goods models, for which a larger group size typically exacerbates free riding. A second key conclusion of this paper is that, invoking both equity and scope for cooperation as selection criteria, we have argued that the equilibrium with the grand volunteering coalition every period emerges as the cooperation scenario with the best overall performance, particularly in large populations.

References

Amir, R., A. Bose, D. Pal, and I. Topolyan (2025). Risk aversion and the punishment paradox in a crime game. *Journal of Economic Behavior & Organization 233*, 106985.

Amir, R., F. Garcia, and M. Knauff (2010). Symmetry-breaking in two-player games via strategic substitutes and diagonal nonconcavity: A synthesis. *Journal of Economic Theory 145*(5), 1968–1986.

Amir, R., D. Machowska, and J. Tian (2025). Volunteer's dilemma with cost synergies. *Journal of Conflict Resolution*. Forthcoming.

Archetti, M. (2011). A strategy to increase cooperation in the volunteer's dilemma: reducing vigilance improves alarm calls. *Evolution 65*(3), 885–892.

Benoit, J.-P. and V. Krishna (1985). Finitely repeated games. *Econometrica 53*(4), 905–922.

Bergstrom, T., L. Blume, and H. Varian (1986). On the private provision of public goods. *Journal of Public Economics 29*(1), 25–49.

Bliss, C. and B. Nalebuff (1984). Dragon-slaying and ballroom dancing: The private supply of a public good. *Journal of Public Economics 25*(1-2), 1–12.

Darley, J. M. and B. Latane (1968). Bystander intervention in emergencies: Diffusion of responsibility. *Journal of Personality and Social Psychology 8*, 377–383.

Diekmann, A. (1985). Volunteer's dilemma. *Journal of Conflict Resolution 29*(4), 605–610.

Goeree, J. K., C. A. Holt, and A. M. Smith (2017). An experimental examination of the volunteer's dilemma. *Games and Economic Behavior 102*, 303–315.

Gore, J., H. Youk, and A. Van Oudenaarden (2009). Snowdrift game dynamics and facultative cheating in yeast. *Nature 459*(7244), 253–256.

Hauert, C. and M. Doebeli (2004). Spatial structure often inhibits the evolution of cooperation in the snowdrift game. *Nature 428*(6983), 643–646.

Herings, P. J.-J., R. Peeters, and M. P. Schinkel (2005). Intertemporal market division:: A case of alternating monopoly. *European Economic Review 49*(5), 1207–1223.

Hirshleifer, J. (1983). From weakest-link to best-shot: The voluntary provision of public goods. *Public Choice 41*(3), 371–386.

Kloosterman, A. and S. Mago (2023). The infinitely repeated volunteer's dilemma: An experimental study. *Games and Economic Behavior 142*, 812–832.

Konrad, K. A. (2024). The collective security dilemma of preemptive strikes. *European Journal of Operational Research 313*(3), 1191–1199.

Konrad, K. A. (2025). Multiple teams and the volunteer's dilemma. *Rationality and Society 37*(3), 338–355.

Konrad, K. A. and F. Morath (2021). The volunteer's dilemma in finite populations. *Journal of Evolutionary Economics 31*(4), 1277–1290.

Leo, G. (2017). Taking turns. *Games and Economic Behavior 102*, 525–547.

Manning, R., M. Levine, and A. Collins (2007). The kitty genovese murder and the social psychology of helping: The parable of the 38 witnesses. *American Psychologist 62*(6), 555.

Matsumura, T. and N. Matsushima (2005). Cartel stability in a delivered pricing oligopoly. *Journal of Economics 86*(3), 259–292.

Otsubo, H. and A. Rapoport (2008). Dynamic volunteer's dilemmas over a finite horizon: An experimental study. *Journal of Conflict Resolution 52*(6), 961–984.

Palfrey, T. R. and H. Rosenthal (1984). Participation and the provision of discrete public goods: a strategic analysis. *Journal of Public Economics 24*(2), 171–193.

Pruzhansky, V. (2011). Some interesting properties of maximin strategies. *International Journal of Game Theory 40*(2), 351–365.

Salant, S. W. and G. Shaffer (1999). Unequal treatment of identical agents in cournot equilibrium. *American Economic Review 89*(3), 585–604.

Shi, Y. (2025). Dynamic volunteer's dilemma with procrastinators. *Journal of Public Economic Theory 27*(2), e70027.

Sugden, R. (2004). *The economics of rights, co-operation and welfare*. Springer.

Tirole, J. (1988). *The theory of industrial organization*. MIT press.

Weesie, J. (1993). Asymmetry and timing in the volunteer's dilemma. *Journal of Conflict Resolution 37*(3), 569–590.

Weesie, J. (1994). Incomplete information and timing in the volunteer's dilemma: A comparison of four models. *Journal of Conflict Resolution 38*(3), 557–585.

Weesie, J. and A. Franzen (1998). Cost sharing in a volunteer's dilemma. *Journal of Conflict Resolution 42*(5), 600–618.

A Appendix

This appendix contains the proofs of the results that were not given in the main text.

A.1 Proof of Proposition 2

The IR level of a player may be formally calculated as the value (a priori also allowing mixed strategies) of the auxiliary two-by-two zero-sum game presented in Table 1. Player 2's payoff in this construction is a proxy for the behavior of the $(n-1)$ other players whose goal is to minimize player 1's payoff. Therefore, their composite action set is composed of n actions specifying the number of volunteers from the coalition: N stands for no volunteer and action V_k corresponds to $k \geq 1$ out of $(n-1)$ other players volunteering.

It turns out that the resulting zero-sum game is dominance solvable. Indeed, for player 2, all V_k are strictly dominated by strategy N, so they may be deleted in the first round. Then strategy V is dominated by N for player 1 and deleted in the second round. Since mixed-strategy equilibria cannot put any mass on iteratively strictly dominated strategies, the leftover outcome (V, N) is the unique Nash equilibrium in both pure and mixed strategies. In this equilibrium, the value to player 1 is thus $b - c$.

Player 1 \ Player 2	V_{n-1}	V_{n-2}	$\dots$	V_1	N
V	$b-\frac{c}{n}, -\left(b-\frac{c}{n}\right)$	$b-\frac{c}{n-1}, -\left(b-\frac{c}{n-1}\right)$	$\dots$	$b-\frac{c}{2}, -\left(b-\frac{c}{2}\right)$	$b-c, -(b-c)$
N	$b, -b$	$b, -b$	$\dots$	$b, -b$	$0, 0$

Table 1: Auxiliary game

A.2 Proof of Proposition 6

Take any cooperation path in which at least two players volunteer in some period T and which is sustainable as a subgame-perfect Nash equilibrium. For any i, let U_i be player i's continuation payoff in period T along the cooperation path.

We first argue that for all i, U_i must be greater than or equal to (2):

$$U_i \geq \frac{b - \delta c}{1 - \delta}, \quad \text{for all } i. \tag{8}$$

If player i is called to volunteer in period T, she will cooperate if and only if U_i is greater than or equal to (2), her payoff from the deviation. If player i is not expected to contribute in period T, then her payoff in that period will be b, while her payoff in the rest of the periods must be greater than or equal to her IR level, $b - c$. Hence, her total payoff cannot be lower than (2).

We next show that there exists i such that U_i is less than or equal to (1):

$$U_i \leq \frac{b - c/n}{1 - \delta}, \quad \text{for some } i. \tag{9}$$

Since along the cooperation path the public good is always produced, in each period, the joint payoff of all players is $nb - c$. Thus,

$$\sum_{j=1}^{n} U_j = \sum_{t=0}^{+\infty} \delta^t (nb - c) = \frac{nb - c}{1 - \delta}. \tag{10}$$

Trivially, there exists i such that U_i does not exceed the average of all the players' payoffs:

$$U_i \leq \frac{1}{n} \sum_{j=1}^{n} U_j. \tag{11}$$

Together, (10) and (11) imply (9).

Statements (8) and (9) together imply that (1) is greater than or equal to (2). The latter condition is equivalent to $\delta \geq 1/n$. Hence, by Proposition 5, the cooperation path in which all players volunteer every period is sustainable as a subgame-perfect Nash equilibrium.

To show uniqueness, take $\delta = 1/n$ and take any sustainable cooperation path in which at least two players volunteer every period.

First, we argue that in any period, every player i gets the continuation payoff of at least as much as she would get in the cooperation path when all players volunteer every period:

$$U_i \geq \frac{b - c/n}{1-\delta}, \quad \text{for all } i. \tag{12}$$

If player i is not a volunteer in a given period, then her continuation payoff will be at least (2). Then, since

$$\frac{b - c/n}{1-\delta} = \frac{b - \delta c}{1-\delta} \quad \text{for } \delta = \frac{1}{n}, \tag{13}$$

we get (12). If player i is volunteering in a given period and (12) does not hold, then (13) implies that

$$U_i < \frac{b - \delta c}{1-\delta},$$

meaning that the continuation payoff of such volunteer is lower than her payoff from the deviation – a contradiction with sustainability.

Second, we argue that in every period, for every player, her continuation payoff is exactly equal to the continuation payoff she would get in the cooperation path when all players volunteer every period:

$$U_i = \frac{b - c/n}{1-\delta}, \quad \text{for all } i. \tag{14}$$

Indeed, (14) follows directly from (10) and (12).

Finally, observe that the only possible way to achieve (14) in every period is for every player volunteering all the time. Indeed, suppose that there is player i who does not volunteer in some period. Then, her continuation payoff in that period is greater than (14):

$$b + \delta \cdot \underbrace{\frac{b - c/n}{1-\delta}}_{\text{cont. payoff in the next period}} = \frac{b - \delta c/n}{1-\delta} > \frac{b - c/n}{1-\delta}.$$

A.3 Proof of Proposition 7

Lemma 1 below is a central step in the proof of Proposition 7 of the paper, which was omitted in the text.

Lemma 1. *For any $n \geq 2$ and any $k \in \{2, \ldots, n\}$, there exists $\underline{\delta}_{k,n} \in (0, 1)$ such that* *condition* (6) *on the discount factor* $\delta \in (0,1)$ *is equivalent to* $\delta \geq \underline{\delta}_{k,n}$.

Proof. Define the function $L(\delta) \triangleq (k+1)\delta - 1 - k\delta^{\frac{n}{k}+1}$.

Observing that $L(1) = 0$, careful factorization yields

$$L(\delta) = (1-\delta)(k\delta + k\delta^2 + \ldots + k\delta^{\frac{n}{k}} - 1).$$

Next, define

$$H(\delta) \triangleq k\delta + k\delta^2 + \ldots + k\delta^{\frac{n}{k}} - 1.$$

Since $H'(\delta) \triangleq k + k\delta + k\delta^2 + \ldots + k\delta^{\frac{n}{k}-1} > 0$ for $\delta \in [0,1]$ and since $H(0) < 0$ and $H(1) > 0$, $H(\delta)$ has a single zero on $[0,1]$, call it $\underline{\delta}_{k,n}$, with all the other solutions of $H(\delta) = 0$ therefore being either complex roots or lying outside the interval $[0,1]$.

Since $L(\delta) = (1-\delta)H(\delta)$, the equation $L(\delta) = 0$ has a real root at $\delta = 1$ and $\underline{\delta}_{k,n}$ as unique real root on the open interval $(0,1)$.

It remains to show that $L(\delta)$ crosses the horizontal axis from below. To this end, note that $L(0) = -1 < 0$ and $L(\underline{\delta}_{k,n} + \varepsilon) > 0$ for small ε. Hence, by the Intermediate Value Theorem, $L(\delta)$ crosses the horizontal axis from below at $\underline{\delta}_{k,n}$. Since $\underline{\delta}_{k,n}$ is the only real root on $(0,1)$, it follows that $L(\delta) > 0$ if and only if $\delta \in (\underline{\delta}_{k,n}, 1)$. □

A.4 Proof of Proposition 8

(i) We show that as k increases, the mapping $L(\delta) \triangleq (k+1)\delta - 1 - k\delta^{\frac{n}{k}+1}$ shifts upward. Treating k as a real number for simplicity, we have $\partial L/\partial k = \delta \,-\, \delta^{\frac{n}{k}+1} \,+\, \frac{n}{k}\,\delta^{\frac{n}{k}+1}\ln(\delta) > 0$. To establish the latter sign, consider the change of variable $t \triangleq \dfrac{n}{k}(-\ln\delta) > 0$. It follows then that $\delta^{\frac{n}{k}} = e^{-t}$ and

$$\frac{\partial L}{\partial k} = \delta\big[\,1 - e^{-t} - te^{-t}\,\big] = \delta\big[\,1 - (1+t)e^{-t}\,\big].$$

By the elementary inequality $e^t > 1 + t$ for $t > 0$, we have $(1+t)e^{-t} < 1$. It follows that

$$\frac{\partial L}{\partial k} = \delta\big[\,1 - (1+t)e^{-t}\,\big] \;>\; 0.$$

Since L crosses the horizontal axis from below (see the proof of Lemma 1), its resulting unique zero, i.e., $\underline{\delta}_{k,n}$, decreases in k.

If $k = n$, then $L(\delta) = (n+1)\delta - 1 - n\delta^2 = (1-\delta)(n\delta - 1)$, which implies that $\underline{\delta}_{n,n} = 1/n$.

(ii) Likewise, we show that the mapping L shifts upward as n increases. This follows directly from the fact that

$$\frac{\partial L}{\partial n} = -\delta^{\frac{n}{k}+1} \ln(\delta) > 0.$$

Hence, its unique zero, i.e., $\underline{\delta}_{k,n}$, decreases in n.

As $n \to +\infty$, $L(\delta) \to (k+1)\delta - 1$, which implies that $\underline{\delta}_{k,\infty} = 1/(k+1)$.